Application of the Nodal Displacement Method to Slope Stability Analysis

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SUMMARY: A simple procedure for predicting slope stability using the nodal displacements determined from finite element analyses is presented. It does not require a pre-assumed slip surface and all the assumptions associated with limit equilibrium methods are eliminated. The safety factor obtained reflects both safety against shear failure and large deformation. Application of the Nodal Displacement Method (N.D.M), as illustrated by examples, shows that the method is a promising alternative approach to slope stability problems.

1. INTRODUCTION

Limit equilibrium methods have been widely accepted for slope stability analysis because of the simplicity the methods offer. However, for a slope with complex, non-homogeneous and anisotropic material where its physical and mechanical nature changes with direction and time, limit equilibrium methods could be unreliable and may not give a convincing result. For instance, limit equilibrium methods do not distinguish between built-up or excavated slopes and would give the same critical slip surface in both cases. Brown and King (1969) have shown that the safety factor of an excavated slope is slightly higher than in the case of a built-up slope and the critical slip surface will be different for both situations.

As a consequence of the ready access to more powerful computers, efficient numerical models and the increased questioning of the validity of the limit equilibrium methods (Tavenas et al. (1980), Pilot et al. (1982), Ching and Fredlund (1983), Adikari and Cummins (1985)), the use of numerical methods has become very attractive.

The application of the finite element method to analysis of stresses and displacements in slopes has been well established and applied successfully by many authors. However, the use of the method in the evaluation of slope stability has not received wide attention. This paper presents a simple and rational method for the evaluation of slope stability using the nodal displacements determined from finite element analyses. The elasto-plastic model using the Mohr-Coulomb failure criterion and the Cambridge CRISP computer program (Gunn and Britto, (1981)) are used for the stress-strain calculation.

2. THE NODAL DISPLACEMENT METHOD. (N.D.M)

Donald et al. (1985) and, Tan and Donald (1985) have shown how a nodal displacement analysis can be used to determine the factor of safety, F, for a system. In this method, separate finite element analyses were performed, each with the strength parameters, (c and tan $\phi(= \mu)$), of all materials incrementally modified by multiplying with a common factor, the "strength modification factor, N". The factor of safety can then be obtained in terms of N when the modified strength parameters are associated with incipient failure. The displacements of selected nodal points provide a means of predicting this situation. The value of $\frac{1}{N}$ for which the displacements indicate a sharp increase in the rate of deformation is taken as the safety factor. Denoting the modified parameters by an asterisk and the strength modification factor by N, we have:

$$c^* = c(N) \tag{1}$$
$$\mu^* = \mu(N) \tag{2}$$

Factor of safety, $F = \left(\frac{1}{N}\right)$ when c^* and μ^* are associated with incipient failure. Trial and error is involved in the choice of the initial value of N and the number and size of its increments. An idealised plot of the nodal displacement curve is shown in Figure 1.
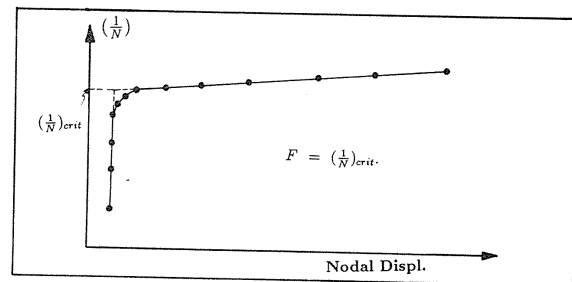

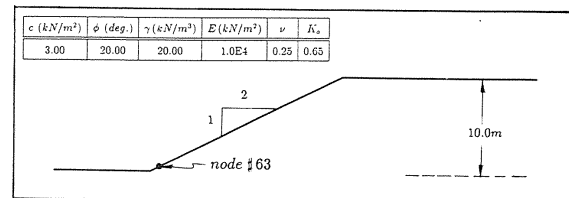Fig. 1: Typical Nodal Displacement Curve.

3. RESULTS

c (kN/m²)	φ (deg.)	γ (kN/m³)	E (kN/m²)	ν	K_s
3.00	20.00	20.00	1.0E4	0.25	0.65

Fig. 2: A simple homogeneous slope

First consider a homogeneous slope as shown in Figure 2.

A number of separate analyses each with c and μ reduced by a different value of the factor N as in equations (1-2) are carried out. The nodal displacement curve should, preferably, be plotted for numerous nodes within the potential failure zone. When the failure zone is not known, nodes corresponding to the toe region should be used. In this example, node #63 which is near the toe is selected. The nodal displacement curve for this node is shown in Figure 3. As seen in this figure, the rate of increase in $\left(\frac{1}{N}\right)$ after the sharp bend of the nodal displacement curve approaches an almost steady state. There is no perfectly delineated break-off point in the displacement curve which can be used to define the critical value of $\left(\frac{1}{N}\right)$; rather, the curve increases very gradually without apparent limit. However, if we adopt an alternate definition of the critical value of $\left(\frac{1}{N}\right)$, as indicated by the dotted lines, we obtain $\left(\frac{1}{N}\right)_{crit} = 1.06$. Although this alternative definition of critical value is not necessarily the true measure of the critical condition of the problem, it is nevertheless a good indication of the critical stage at which large deformation can be expected for a very small decrease in the soil parameters c and μ. The break-off point represents the beginning of rapid nodal displacement with a small reduction of N, a situation of instability with respect to c and μ which indicates the point of incipient failure. At this point, $F = \left(\frac{1}{N}\right)_{crit}$.
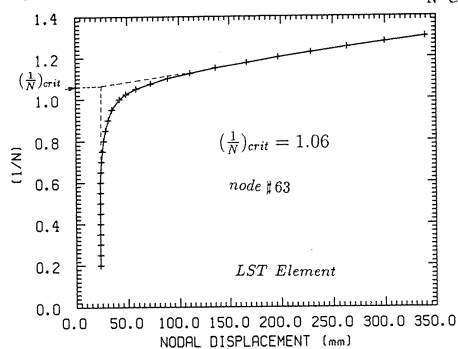

Fig. 3: $\left(\frac{1}{N}\right)$——Nodal Displ. Curve

It was found that the sharpness of definition of F is dependent upon the choice of constitutive model for the soil, the node in the mesh for which the curve is plotted, the type of element and mesh size used in the finite element formulation, the size of the increments in N and the method of plotting. Figure 4 is the nodal displacement curve for example 1 using the 15 noded triangular element (Cubic Strain Triangle, CuST) of Sloan and Randolph (1982). Comparing this curve with that of Figure 3 using the 6 noded triangular element (Linear Strain, LST), the CuST element produces a well defined critical value of $\left(\frac{1}{N}\right)$. However, the alternative definition indicated by the dotted lines in Figure 3 estimates the critical value closely.

$\left(\frac{1}{N}\right)_{crit}$ is a suitable choice for safety factor as it accounts for both the slope failure (shear failure) and safety against large deformation. The definition of safety factor is essentially the same as in limit equilibrium methods i.e. the factor by which the available strength must be reduced to just maintain the slope in equilibrium. Above

the $\left(\frac{1}{N}\right)_{crit}$ value, large deformations would occur for a slight reduction of strength parameters, whereas below it, considerable reduction of strength parameters is needed even for a small change in deformation.
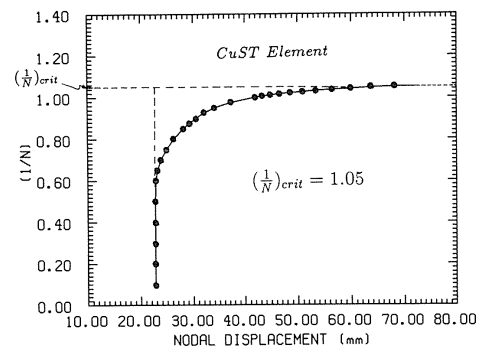

Fig. 4: Nodal Displacement Curve (CuST element)

Therefore, adoption of $\left(\frac{1}{N}\right)_{crit}$ as safety factor would reflect the overall stability of the system. The safety factor for this example with regard to the overall stability is $F_{NDM} = 1.05$ corresponding to $\left(\frac{1}{N}\right)_{crit}$. Bishop's simplified method gives $F_{Bishop} = 1.00$. Note that, in contrast to traditional limit equilibrium methods, no slip surface need be assumed for the N.D.M approach, the critical failure surface effectively being determined by the analysis. In addition, most of the other important assumptions required for the completion of limit equilibrium analyses are eliminated.

Example 2, taken from Baker (1980) includes a thin weak layer in the homogeneous simple slope as shown in Figure 5.
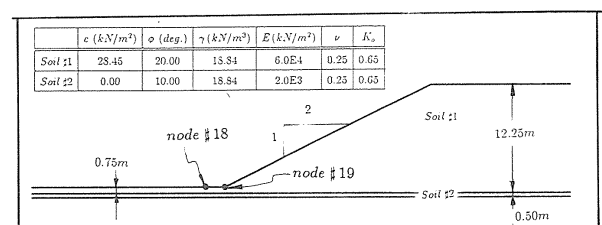

Fig. 5: Example 2.

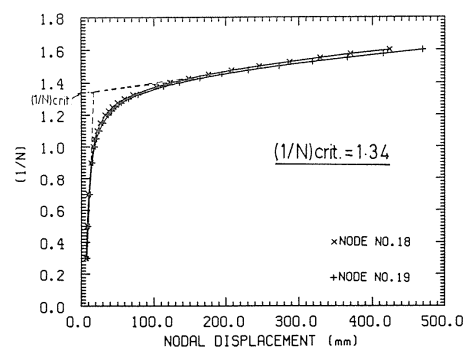The strength parameters (c and $\tan\phi$) for both layers were incrementally multiplied by N.

Fig. 6: $\left(\frac{1}{N}\right)$——Nodal Displ. Curve.

Typical nodal displacement behaviour is shown in Figure 6 for nodes #18 and #19 located near the toe region. The overall safety factor for this example is $\left(\frac{1}{N}\right)_{crit}$ = F = 1.34.

The displacement vector plots corresponding to $\left(\frac{1}{N}\right)$ of 1.25 and 1.34 are shown in Figure 7. No significant nodal displacements occur before $\left(\frac{1}{N}\right)_{crit}$. At $\left(\frac{1}{N}\right)_{crit}$, relatively large nodal displacement is observed and there exists a "discontinuity" between the displacement vectors particularly along the weak layer boundary. This is an indication of a possible location of a slip surface. (Figure 7(b)). Above $\left(\frac{1}{N}\right)_{crit}$, excessive nodal displacements occur with a similar failure pattern. Figure 8 shows the final incremental displacement vector plot for $\left(\frac{1}{N}\right)$= 1.34. Here the "discontinuity" along the weak layer is obvious.
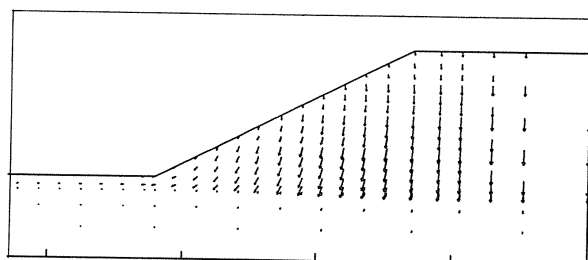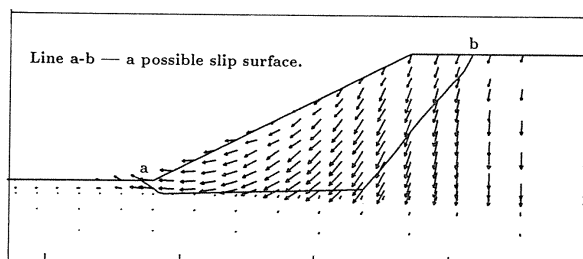

Fig. 7(a): $\left(\frac{1}{N}\right)$ = 1.25

Line a-b — a possible slip surface.

Fig. 7(b): $\left(\frac{1}{N}\right)$ = 1.34

Fig. 7: Displacement Vectors at Various Stages of $\left(\frac{1}{N}\right)$.
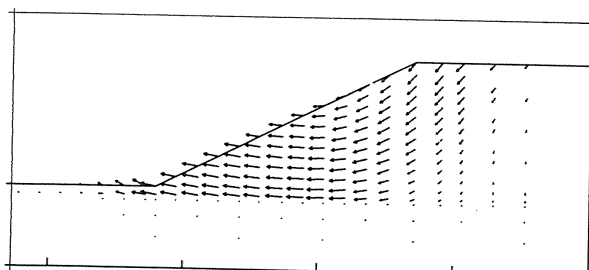

Fig. 8: Incremental Displacement Vectors at $\left(\frac{1}{N}\right)$ = 1.34

A comparison of results by various methods of analysis is presented in Table 1. This shows that the safety factor obtained by the N.D.M agrees well with the most commonly used limit equilibrium methods and also the finite element approach of CRISS (Giam and Donald, 1988). The comparison in Table 1 shows that simple limit equilibrium methods are capable of calculating accurate safety factors for this slope. Although these methods derive their factor of safety from a particular value of strength parameters only, the calculated F does provide the correct value by which the strength parameters must be reduced to bring the entire slope to a state of limit equilibrium. It also reveals that for such methods the definition of F, taken as the ratio of available shear strength over the mobilised shear stress does reflect the overall safety of the slope against complete collapse, although it does not provide any information on likely deformations. The usual assumption is that if F is sufficiently large, deformation will be acceptably small. For problems where deformations might be vital the N.D.M provides an alternative formulation where deformation limits may be used as a failure criterion in place of total collapse.

Table 1 also highlights that conventional methods could over estimate the safety factor if the correct critical failure surface is not used in the analysis. Different optimisation schemes employed yield their own value of safety factor corresponding to their selected critical slip surface. It is obvious that Bishop's method for a circular slip surface is not efficient for this problem.

Table 1: Comparison of factors of safety for example 2

	Method of Analysis	$F_{obtained}$
ϕ	Wedge Analysis (Simplex Optimisation)†	1.33
ϕ	Dynamic Programming (Baker)	1.29
ϕ	Morgenstern-Price (Krahn & Fredlund)‡	1.38
ϕ	Bishop's Method (Simplex Optimisation)	1.50
ϕ	Sarma's Method	1.39
*	CRISS	1.31
*	N.D.M (Nodal Displacement Method)	1.34

ϕ Limit Equilibrium analysis.
* Finite Element analysis.
† Ref: Donald (1987b)
‡ Ref: Krahn & Fredlund (1977)

4. SENSITIVITY OF STRENGTH PARAMETERS.

Not all parameters are equally important in providing strength for the system. A brief investigation into the sensitivity of the analysis to strength parameters (c , tan ϕ) is given below.

Consider Example 1. If we reduce c by 30% while keeping tan ϕ at its original value, using Bishop's method we obtain F = 0.94. On the other hand reducing tan ϕ by 30% while keeping c at its original value gives F = 0.76. The results are summarised in Table 2.

From Table 2 we see that the stability of this slope depends more on the coefficient of friction, tan ϕ, than cohesion, c. The nodal displacement behaviour with the reduction of c was observed for a number of nodes (μ kept constant) and typical behaviour is shown in Figure 9. There is no apparent "break-off" point. This indicates that the overall stability of the slope is not very sensitive to to the reduction of c. On the other hand, the reduction of μ as shown in Figure 10

produces a "break-off" point. The $(\frac{1}{N})_{crit}$ is 1.09 which is slightly higher than 1.06 found in Example 1 when both c and μ were simultaneously reduced. This shows that the strength component, μ is the controlling factor for the stability analysis and therefore, for this slope effort should be devoted to obtaining an accurate value of φ. For slopes with higher initial values of c this will not necessarily always be so.

Table 2: Effects of Reducing c or $\tan\phi$.

Actions	$c(kN/m^2)$	$\phi(deg.)$	F
Original values	3.0	20.0	1.00♣
Reduce c, 30%	2.1	20.0	0.94
Reduce $\tan\phi$, 30%	3.0	14.29	0.76
Reduce c, 100%	0.0	20.0	0.73♡
Reduce $\tan\phi$, 100%	3.0	0.0	0.09♠
Increase c, 30%	3.9	20.0	1.06
Increase $\tan\phi$, 30%	3.0	25.32	1.24
Increase c, 100%	6.0	20.0	1.16
Increase $\tan\phi$, 100%	3.0	36.05	1.80
Increase c, 200%	9.0	20.0	1.32
Increase $\tan\phi$, 200%	3.0	47.52	2.64

♣ Toe circle.

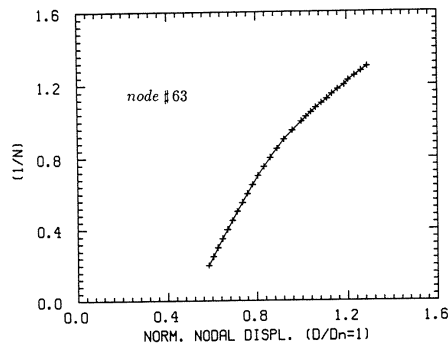♡ Converge to $\frac{\tan\phi}{\tan\beta}$; $\beta = angle\ of\ slope = 26.56°$

♠ Deep circle.

Fig. 9: Nodal Displ. Curve with c reduced incrementally.
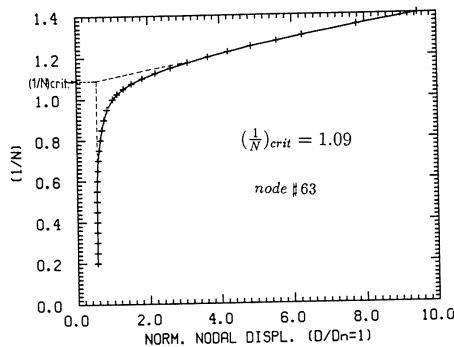
Fig. 10: Nodal Displ. Curve with μ reduced incrementally.

5. DISCUSSION

The above examples illustrate the use of the N.D.M as an improved alternative approach to slope stability analysis. Although the examples used in this paper do not involve considerations of ground water and pore water pressures there is no reason why the approach cannot be extended to such cases. Additional computing effort will certainly be required, but for complex problems where stress history influences could be important, conventional limit equilibrium analyses might prove inaccurate and a complete stress-strain distribution of the system is probably required anyway, justifying the use of the N.D.M.

It must be admitted that a considerable amount of computer effort is required if elasto-plastic analyses are to be performed for a sufficient number of N values to delineate the $(\frac{1}{N})$ nodal displacement curve accurately. However, since we are only particularly concerned with the initial and ending portions of the curve, a few separate analyses might be enough to construct the two tangent lines, particularly if the CuST element is used. Two or three analyses below $(\frac{1}{N})_{crit}$ and another three or four analyses above $(\frac{1}{N})_{crit}$ might be all that are required. $(\frac{1}{N})_{crit}$ can be approximated using F calculated by the simple limit equilibrium methods. Many finite element programmes are based on a small strain formulation and yet the analyses are continued until large deformations have been reached (e.g. Tan and Donald, 1985). However, once the turn-over point has been passed, which usually happens at reasonably low strains, the errors are of little consequence and for the examples presented in this paper strains remain small throughout.

A sensitivity study by the N.D.M allows us to decide which strength parameters will strongly govern the stability of the system. This is useful in laboratory and field investigations where considerable effort can then be devoted to obtaining accurate and representative value of the controlling strength parameters.

6. CONCLUSION

The evaluation of safety factors in slope stability analyses via stress-strain calculations is often a more convincing and satisfactory approach than using simpler conventional methods. This is because limit equilibrium analyses use stresses on the assumed failure surface which may bear little relationship to the actual stress distribution on that surface.

In limit equilibrium methods the safety factor expresses safety against shear failure and not against excessive deformations, which are assumed not to occur provided that the value of F is sufficiently high. In the N.D.M approach system deformations are calculated, providing an alternative acceptability criterion and the safety factor against shear failure is determined without the need to postulate a failure surface. Vector plots of displacement or incremental displacement indicate the likely position of the critical slip surface quite clearly. Various assumptions of limit equilibrium solutions, particularly regarding constancy of F around the failure surface, estimation of shear strength available at failure and nature of side forces between slices are not required in the N.D.M. The safety factor obtained reflects the excess strength component the design has against shear failure and excessive deformation and the analysis may readily be used to highlight significant material parameters.

The method currently requires significant computing power, but with continued advances in cheaper computing the N.D.M becomes an attractive alternative to limit equilibrium methods and for some problems may prove to be the only reliable analysis.

7. REFERENCES

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