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Determination of Critical Slip Surfaces for Slopes via Stress-Strain Calculations

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SUMMARY: An effective and systematic search scheme is presented to determine both the minimal factor of safety and the corresponding critical failure surface from a known stress distribution of the system. The scheme starts at a region of overstress and the slip surface propagates towards the boundaries of the slope. No arbitrary restrictions are placed on the shape of the failure surface and the analysis satisfies all conditions of equilibrium.

1. INTRODUCTION

The determination of the critical slip surface is of importance in slope stability analysis. It is particularly useful for remedial work, slope reinforcement, field instrumentation and performance monitoring. In limit equilibrium methods, a pre-assumed surface is necessary before the factor of safety can be evaluated. There are many methods currently available for slope stability analysis which are capable of assigning a safety factor to a given slip surface but do not attempt to search for the critical one. Often the geological features will dictate the shape of the slip surface but, when such information is not available, automatic search routines should be incorporated into the analysis to reduce the amount of computation. For limit equilibrium methods, many such techniques are available, notably - simplex reflection (Nguyen, 1985), grid search (Fredlund, 1981), Zigzag search (Wright, 1974), Dynamic programming (Baker, 1980) and minimization of multivariate function (Celestino and Duncan, 1981). All these methods seek to determine the global minimum although the local minimum is often unavoidable, particularly when the optimal surface is a mulitmodal one.

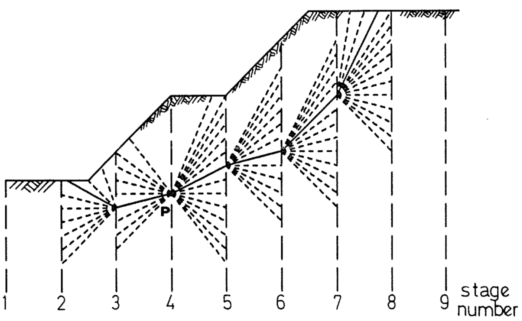

Fig. 1: The Systematic Search

Research on failure patterns in-situ and in soil elements has revealed that the location of the critical slip surface is primarily governed by the stress system in the soil (Janbu, 1973). This paper presents an effective and systematic procedure for obtaining the critical slip surface when the stress distribution of the slope is known. The minimum safety factor corresponding to the critical slip surface is determined simultaneously. No arbitrary restrictions are placed on the slip surface and the analysis satisfies all conditions of equilibrium. The finite element method is frequently used in deriving the stress distribution of the system.

2. THE SEARCH SCHEME

Starting at point P, (Fig. 1) the failure surface is propagated to the top and bottom of the slope. Point P is selected to represent the zone of maximum overstress. A useful way of doing this is to consider the value of stress level or fraction of strength mobilized, (Kulhawy et al., 1969) expressed in Eq. 1.

$$\text{Strength Mobilization Factor} = \frac{(\sigma'_1 - \sigma'_3)}{(\sigma'_1 - \sigma'_3)_f} \tag{1}$$

where

$$(\sigma'_1 - \sigma'_3)_f = \frac{2c' \cos\phi' + 2\sigma'_3 \sin\phi'}{1 - \sin\phi'} \tag{2}$$

Here, it is assumed that the effective minor principal stress is the same at failure, as for the mobilised stress state. The reciprocal of Eq. 1 is interpreted as being equal to the value of factor of safety against local failure. Thus, a good starting point P, would correspond to an area with low safety factor against local failure. The search commences by joining the adjacent stage lines with segments incremented about 1° radially as shown in Fig. 2. Starting with $\gamma°$ to $\beta°$, each segment extends to meet the neighbouring stage line. A typical segment is divided into six intervals (five control points). The normal and shear stresses, σ_{ni}, τ_{ni} are calculated at these points and hence the factor of safety of the segment can be evaluated using Eqs. (3-5).

$$\sigma'_{ni} = \frac{1}{2}(\sigma'_{yy} + \sigma'_{xx}) + \frac{1}{2}(\sigma'_{yy} - \sigma'_{xx})\cos 2\alpha - \tau_{xy}\sin 2\alpha \tag{3}$$

$$\tau_{ni} = \frac{1}{2}(\sigma'_{yy} - \sigma'_{xx})\sin 2\alpha + \tau_{xy}\cos 2\alpha \tag{4}$$

$$F_{segment} = \frac{\Sigma(c' + \sigma'_{ni}\tan\phi')\Delta l}{\Sigma(\tau_{ni})\Delta l} \tag{5}$$

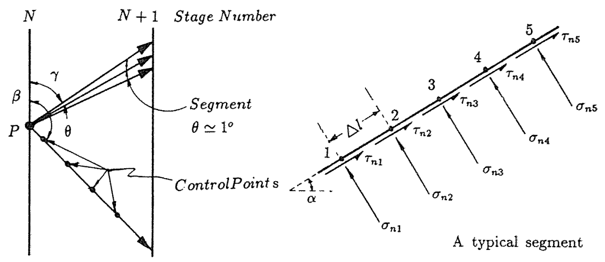

Fig. 2: Propagation of failure surface to the right

The summations (Σ) indicate that both the shear strength and the shear stress are summed over a number of increments of length (Δl) along the segment.

$$F_{overall} = \frac{\int_L (c' + \sigma'_n \tan \phi') dL}{\int_L (\tau_n) dL} \qquad (6)$$

where L = total length of failure surface.

The stresses, (σ'_{xx}, σ'_{yy} and τ_{xy}) at the control points are determined by interpolating the stresses at four closest integration points in the finite element mesh. The failure surface will propogate along the segment with the minimum factor of safety. The process of selecting a segment and calculating its safety factor is repeated until the failure surface intercepts the boundaries of the slope. The overall safety is then obtained from Eq. 6. Several different starting points, P, should be used so that the global minimum is obtained. In most cases, a few trials will be sufficient to ensure this.

3. RESULTS

The stress distributions for the examples presented here were calculated using an elasto-plastic model and the Cambridge CRISP computer program (Gunn and Britto, 1981).

Two cases analysed using the above search scheme with program CRISS (Critical Slip Surface) are presented here. A flow chart of CRISS computer program is shown in Fig. 3. Example 1, (Fig. 4) is a homogeneous slope with factor of safety close to unity. This example is selected to simulate the state of limiting equilibrium. The calculated F obtained from accurate limit equilibrium methods has absolute meaning only for F = 1. Values greater than 1 can only be used to compare the relative stability of two designs. The numerical value is a function of the definition employed by the method of analysis, rather than any aspect of basic structural behaviour. For this homogeneous simple slope, one would expect the critical slip surface to be circular or near circular. Bishop's simplified method using the simplex reflection optimisation (Nguyen, 1985) gives $F_{Bishop} = 0.983$ as compared to CRISS, $F_{CRISS} = 1.006$. (The 3 decimal places are for comparison purposes and do not necessarily reflect the validity of the analysis method.) It should be noted that both approaches are totally independent in their formulation. The latter is based on stress-strain calculations and the former on static limit equilibrium. Figure 5 compares the slip surface found by CRISS with the potential slip line directions. These slip lines are inclined at $\pm(\frac{\pi}{4} - \frac{\phi_{dev}}{2})$ to the major principal compressive stress. It can be seen that the failure surface is tangential to these slip line directions which confirms the location of the failure surface.

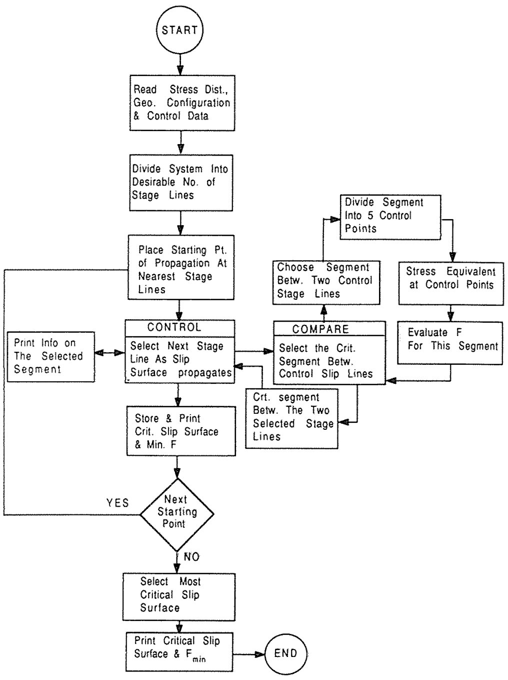

Fig 3: Flow chart

$c\ (kN/m^2)$	$\phi\ (deg.)$	$\gamma (kN/m^3)$	$E (kN/m^2)$	ν	K_o
2.93	19.55	20.00	1.0E4	0.25	0.65

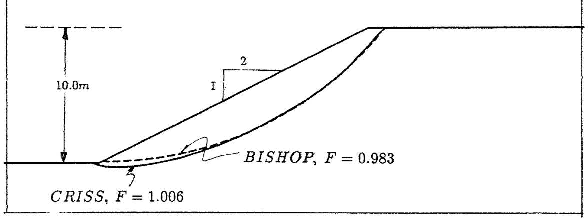

Fig. 4: Critical Slip Surfaces ($F_{Bishop} = 0.983$, $F_{CRISS} = 1.006$)

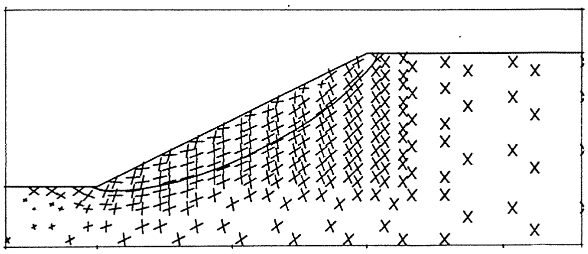

Fig. 5: Slip Line Directions and the Critical Slip Surface obtained by CRISS. (The relative sizes of the crosses correspond to the inverse of the local safety factor with respects to shearing.)

The second example, Fig. 6, is taken from Baker (1980). A thin weak layer is included in the homogeneous simple slope. The slip surface will be controlled by the presence of the thin weak zone. A Wedge analysis (Donald, 1987b) which uses either the Powell or Simplex optimisation algorithms (Kuester and Mize, 1973) is carried out to confirm both the location of slip surface and the calculated factor of safety. Comparison with the potential slip line directions is shown in Fig. 7. Again, the critical slip surface traces the path of slip line directions. As might be expected the slip surface passes through the locations where the stress level is the highest. A summary of the results by various methods of analysis (limit equilibrium methods and finite element analysis) is given in Table 1.

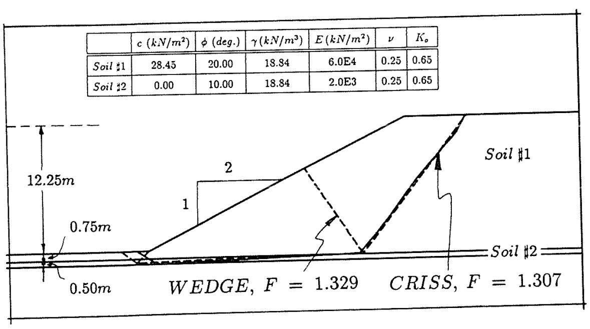

Fig. 6: Critical Slip Surfaces. F_{Wedge} = 1.329 F_{CRISS} = 1.307)

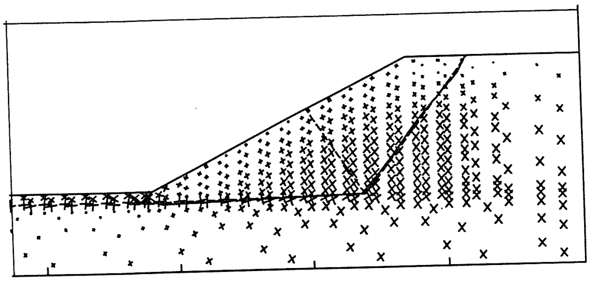

Fig. 7: Slip Line Directions with the Critical Slip Surfaces.

Table 1: Comparison of factors of safety for example 2

	Method of Analysis	$F_{obtained}$
ϕ	Wedge Analysis (Simplex Optimisation)⊕	1.33
ϕ	Dynamic Programming (Baker)	1.29
ϕ	Morgenstern-Price (Krahn & Fredlund)⊖	1.38
ϕ	Bishop's Method (Simplex Optimisation)	1.50
ϕ	Sarma's Method	1.39
*	CRISS	1.31
*	N.D.M (Nodal Displacement Method)⊗	1.34

ϕ Limit Equilibrium analysis.

* Finite Element analysis.

⊕ Ref: Donald (1987b)

⊖ Ref: Krahn & Fredlund (1977)

⊗ Ref: Donald & Giam (1988)

Example 2 involves a slip surface of a non-constant radius, hence any movement along such a surface will cause distortion of the soil mass enclosed by it. This distortion could occur only if the slip surface were intersected by a family of shear surfaces on which the factor of safety was constant and equal to that for the slip surface (Spencer, 1981). This is observed in Fig. 7 and 9 where a zone of distortion occurs around the sharp bend of the slip surface. In these figures, the relative sizes of the slip lines correspond to the inverse of the local safety factor.

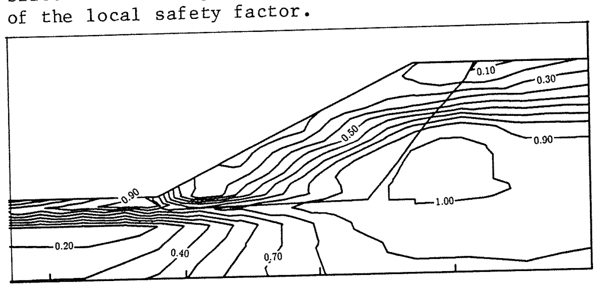

Fig. 8: Contours of Mobilized Strength. (Stress Level).

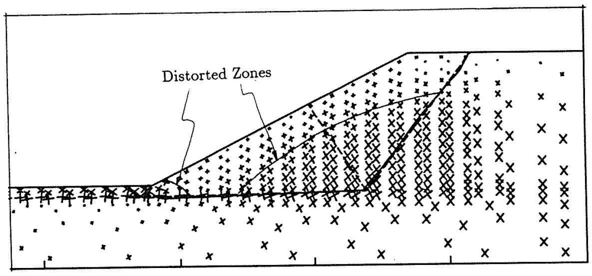

Fig. 9: Distorted Zones to allow for kinematically admissible slip

4. DISCUSSION

The above examples demonstrate the adequacy of the search scheme for the critical failure surface in slope stability analysis. It is capable of predicting circular (Example 1) as well as irregular failure surfaces (Example 2). For simple slope problems, as in these examples, the critical failure surface obtained by CRISS compares very well with those found by limit equilibrium methods. With the advance of powerful numerical methods and high speed computer technology, modelling complex, non-homogeneous and anisotropic material problems becomes possible. The stress distribution of such complex problems can be determined efficiently and the critical slip surface can be obtained by this search scheme.

In comparison with the stress level and potential slip line directions, the search scheme produces credible failure surfaces. Dynamic Programming or Simplex Optimisation can also be used for the search scheme. However, these techniques involve much more computational effort. During the CRISS search, most of the time is spent in selecting the four closest integration points for the calculation of normal and shear stresses at the control points. The Dynamic Programming or the Simplex Optimisation would involve more segmentation and thus require more computation effort than CRISS's search scheme.

5. CONCLUSION

The determination of the critical failure surface using the known stress distribution is the most satisfying approach, as the location of the critical failure surface is primarily governed by the stress system in the soil. In this way, a slope can be analysed for its critical slip surface correctly and convincingly. The scheme presented here has proved to be a systematic and efficient approach and it involves no derivatives or elaborate computation. Once the stress distribution of a system is known, this search scheme can be used and thus it can be applied in conjunction with a wide range of numerical methods. The approach is currently being extended to include slopes with pore water pressures.

6. REFERENCES

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