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Radical Re-examination of Stiffened Raft Footing Technology

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Summary A radical re-examination of fundamental principles of analysis and design of stiffened raft footings is presented. A major flaw is exposed in existing theory of two-dimensional linearly elastic beams supported on linearly elastic foundation material, in respect of the treatment of interactive soil/structure response to foundation movements induced by moisture changes in reactive clay soils. Correct two-dimensional linearly elastic theory is presented. Results for a typical practical example indicate that residential stiffened raft beam design solutions published in Australian Standard AS-2870, and routinely applied in Australian practice, fail to satisfy traditionally accepted performance criteria. Ultimate strength analysis is presented as an introduction to two-dimensional non-linear theory. A three-dimensional ultimate strength design model based on the conventional yield line method is presented and recommended to obviate the mandatory design and construction selection process currently practised in Australia.

NOTATION

D	= effective depth of raft section. [mm]
EI	= beam bending stiffness. [kNm^2/m]
f	= external loading function.
k	= Winkler soil modulus. [kPa/m]
L	= beam length. [m]
M	= bending moment function. [kNm/m]
M_0	= bending moment at $x = 0$. [kNm/m]
M_{cr}	= cracking moment. [kNm/m]
M_u	= ultimate moment. [kNm/m]
p	= soil pressure function. [kPa]
p_0	= pressure at $x = 0$. [kPa]
p_E	= pressure at $x = (L/2)$. [kPa]
w_0	= uniformly distributed load. [kPa]
W_E	= concentrated edge load. [kN/m]
x	= independent variable. [m]
y	= relative deflection function. [mm]
y_0	= internal soil movement function. [mm]
y_m	= relative soil movement. [mm]
y_s	= characteristic surface movement. [mm]
λ	= interactive system stiffness. [kPa/m]
Δ	= relative deflection. [mm]

Other symbols are defined when first used.

1. INTRODUCTION

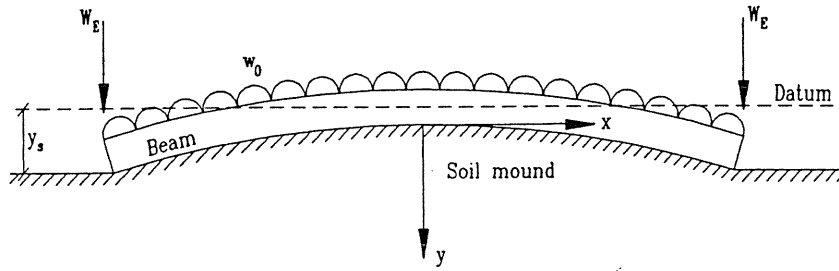
Stiffened raft footings have been increasingly used in residential construction on reactive clay sites in Australia and overseas for about a quarter of a century. Australian researchers can rightfully claim to have pioneered much of the relevant engineering technology, and advanced it to a stage where it was formally adopted by Standards Australia, and published as Australian Standard AS-2870, Residential slabs and footings (1986). The current edition of AS-2870 was published in 1996, and, by virtue of adoption by regulating authorities, it is now a mandatory design and construction document in

most Australian States. Although not party to the research team responsible for the initial preparation and on-going development of AS-2870, the author has conducted extensive private research on the subject over the last decade or so, concurrently practising in this specialised field of engineering. The author has published only one paper on the subject, (Van der Woude, 1997), which deals with philosophical issues. The present paper deals with technical issues. The author's ultimate conclusion is that several past research publications on the subject, including AS-2870, contain a serious fundamental technical flaw in respect of stiffened raft footings. This issue must be addressed professionally and openly, and therefore it is exposed in this paper. Without it this paper would not make much sense to the majority of the audience in general, and to the professional practitioners in this field in particular.

This paper has been divided into three parts: Part (1) identifies the fundamental flaws in existing linearly elastic theory of the 2-D beam-on-mound problem. Part (2) presents the fundamentally correct 2-D linear theory. Part (3) introduces 2-D non-linear theory, leading into a very simple 3-D ultimate strength model suitable for routine design office application. For reasons of space restrictions and intellectual property Copyright protection, many details have been omitted from this paper.

2. PART (1): EXISTING 2-D THEORY

The authoritative publication on the subject of stiffened raft footings is Walsh and Cameron (1997). Published as a handbook for the design of residential slabs and footings, it is an essential companion to Australian Standard AS-2870 (1996), especially for practitioners in this specialised field of engineering. The handbook also contains a list of references on



Notes: 1) Datum = Prepared site level. 2) Drying soil condition, i.e. edge settlement.

Figure 1. Problem description.

the subject that extends far beyond what most practitioners would ever need, helpfully shortening the author's list to two. Conference delegates are assured that the author has studied the relevant references in depth.

With reference to Figure 1, the stiffened raft beam design solutions published in AS-2870 have evidently been derived from solutions of the following 4th-order linear differential equation:

$$EI \left(\frac{d^4 y}{dx^4} \right) + k(y) = f + k(y_0) \quad (1)$$

Much discussion in available literature on the subject of soil-structure interaction is devoted to the establishment of the function (y_0) , commonly referred to as the "mound profile". In practice the mound profile (y_0) is established by field measurements on experimental sites covered with edge down-turned impervious membranes, and/or solution of the moisture diffusion equation. That is, the mound profile (y_0) is the shape the surface adopts when the underlying soil is subjected to moisture change. It is extremely important to note that the mound profile is nothing more than an empirical or theoretical function, representing movement of the soil surface when it is *free of pressure*. For the remainder of this part of the paper, the subscript zero in (y_0) should therefore importantly serve as a constant reminder that the mound profile is a *zero pressure* soil movement that occurs *only* when there is *no interaction* between the soil and the structure it supports.

A tacit assumption in the derivation of Equation (1) is that when there is soil-structure interaction, the soil pressure (p) equals $k(y-y_0)$. This is fundamentally incorrect, for several reasons:

- Since soil cannot sustain negative pressure, the identity $p = k(y-y_0)$ means that the soil must be compressed an amount (y_0) before it develops pressure, which is obviously incorrect.

- Another tacit assumption is the use of the principle of superposition, i.e. the function (y) is superimposed on the function (y_0) . Since it is always legitimate to add one or more mathematical functions, the identity $p = k(y-y_0)$ has nothing whatsoever to do with this fundamental principle of mathematics. The correct interpretation of the principle of superposition is that valid solutions of a linear system can be superimposed, but (y_0) is not a valid solution of the linear interactive beam-soil system.

- In private discussions, some colleagues try to juggle the sign convention, inevitably concluding that mound theory will work when centre heave is equated to edge settlement, again tacitly invoking the principle of superposition. Hence the "mound" concept. Centre heave is a philosophical misnomer that should be banished. In physical reality, soil moisture change, and hence movement, propagates from the edges inwards. Thus the real problem is either edge heave or edge settlement, excepting the rare incidence of accidental centre heave due to plumbing leaks.

Mathematically correct as the solution of Equation (1) might be, physically it incorrectly corresponds to superimposing a loaded pre-fabricated beam, raft, or house, on a building platform which has been pre-distorted into the assumed mound profile (y_0) . In other words, *mound theory correctly solves a hypothetical problem.*

Another condemnation of Equation (1) is the recognition of the term $k(y_0)$ on the R.H.S. as an *artificial* load function. By definition of soil modulus, $k(y_0)$ is the pressure that would develop if the soil swells an amount (y_0) against an infinitely rigid and immovable beam. This is the *only*, but again *hypothetical*, situation where Equation (1) yields a valid solution, viz. $y = y_0$.

Further evidence of flaws in mound theory may be gathered by examining the solution in detail:

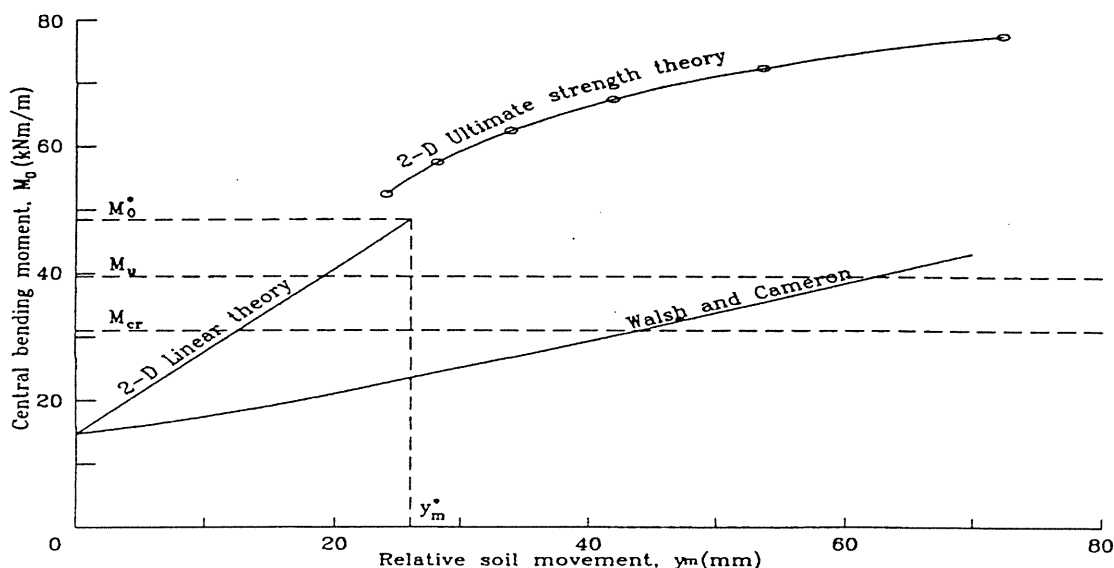


Figure 2. Bending moment (M_0) vs. relative soil movement (y_m).

- For the supposedly linear interactive beam–soil system, mound theory solutions are not linear with respect to soil movement (refer Figures 2 to 4), contradicting the basic assumption of linearly elastic behaviour of both beam and soil individually.
- Mound theory solutions do not satisfy all the boundary conditions of the problem. Correct application of the principle of superposition to the problem at hand is that the complete solution function (y) can be separated into components, one component (u) due to external loads, and another component (v) due to internal soil movement. The boundary conditions for the component functions (u) and (v) are not the same. Mound theory solutions satisfy the boundary conditions for (u), but not those for (v).
- Under soil drying conditions, i.e. edge settlement, mound theory solutions exhibit an increase in soil pressure in the edge zone (refer Figure 4), which is unexpected.
- When the edge distance (e), as defined by Walsh and Cameron (1997), is set at a fixed value, mound theory solutions are independent of soil movement, which is unacceptable. It would appear these authors have contrived a formula for edge distance as a function of (y_m) in order to achieve a likewise functional relation in mound theory solutions.

To conclude Part (1) of this paper, the author suggests that mound theory was probably conceived as a plausible method, strangely evolved into a seemingly complex problem, and it was progressively infected with somewhat illogical physics and mathematics.

3. PART (2): CORRECT 2-D LINEAR THEORY

3.1 Theory

To derive a valid linear solution of the 2-D beam-on-elastic-foundation problem, the soil pressure function (p) must be expressed in the form:

$$p = p_0 + \lambda(y) \tag{2}$$

The variable (λ) is defined as the true interactive beam-soil *system* stiffness. It has the same dimension as soil stiffness (k), viz. kPa/m, but the distinct difference is that (k) is a positive *soil constant*, whereas (λ) is a *system variable*, which may be positive or negative. The magnitude of (λ) cannot be set *a priori*, it can only be determined *after* the solution has been found, involving all the parameters of the problem.

The fundamentally correct differential equation for the 2-D representation of the problem at hand is:

$$EI \left(\frac{d^4 y}{dx^4} \right) + p_0 + \lambda(y) = f \tag{3}$$

The form of the solution of Equation (3) depends on whether (λ) is positive, negative, or zero. Positive values normally apply to static external load effects, and negative values to propagating internal soil movement effects. The form of the complete solution therefore depends on which of these two effects dominates the behaviour of the system. For symmetrical problems, i.e. symmetrical soil movement, equal edge loads (W_E), and uniformly distributed load (w_0), the solution of Equation (3) can be summarised in terms of the following identities:

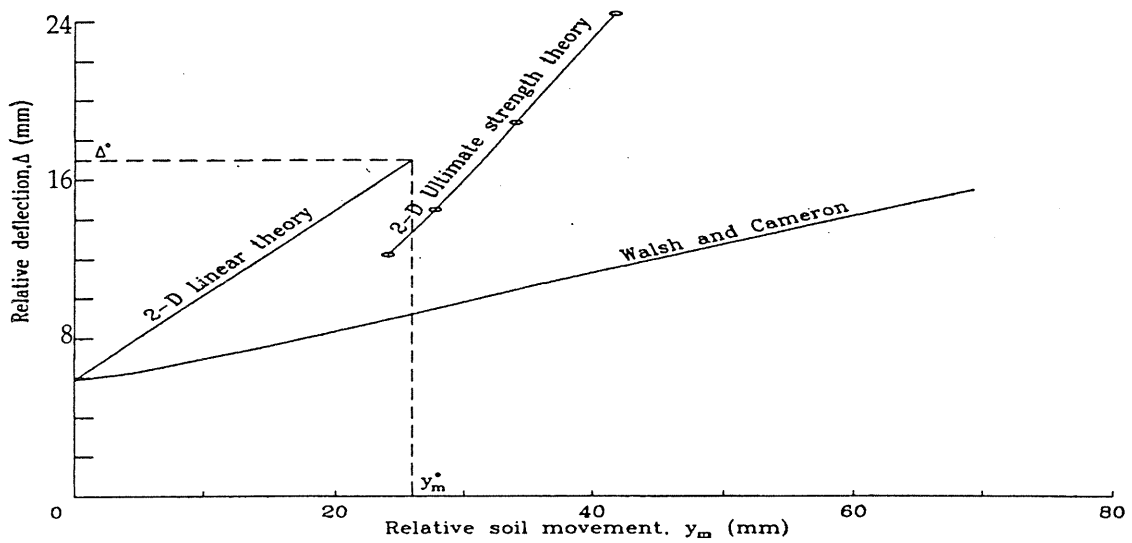


Figure 3. Relative deflection (Δ) vs. relative soil movement (y_m).

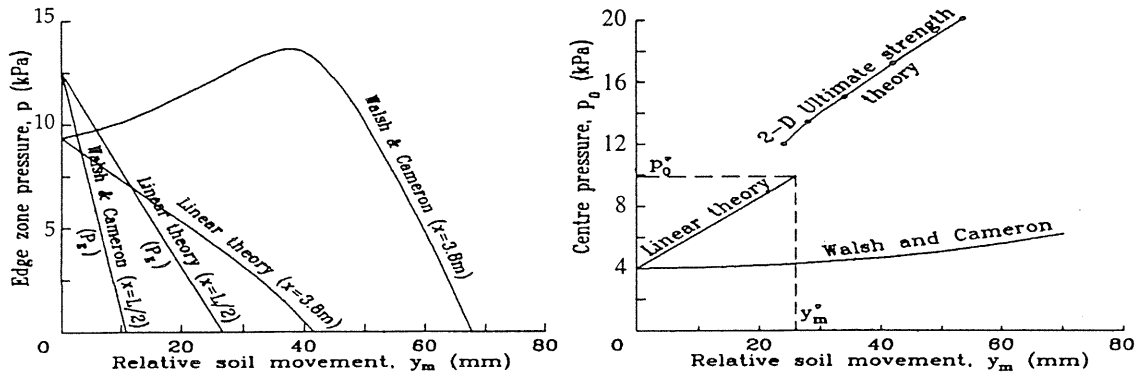


Figure 4. Soil pressure (p) vs. relative soil movement (y_m).

Relative deflection,
$$\Delta = \left(\frac{W_E L^3}{EI} \right) F_1$$

Central bending moment,
$$M_0 = (W_E L) F_2$$

Central pressure,
$$p_0 = w_0 + \left(\frac{W_E}{L} \right) F_3$$

Edge pressure,
$$p_E = w_0 + \left(\frac{W_E}{L} \right) F_4$$

Relative soil movement,
$$y_m = \Delta + \frac{p_0 - p_E}{k}$$

The dimensionless multiplying factors F_1 to F_4 are functions of the dimensionless variable $\left(\frac{\lambda L^4}{EI} \right)$.

When $\lambda = k$, Equation (3) yields the initial state solution, i.e. $y_m = 0$. This is the u-component of the complete solution discussed in Part (1) of this paper.

The range ($\lambda > k$) yields edge heave solutions, and the range ($\lambda < k$), including negative values, yields edge settlement solutions. A special case arises when $\lambda = 0$, which involves division by zero, but the results are nevertheless finite. In practice this case occurs at relatively low values of (y_m). As noted earlier, this special case marks the transition in the behaviour of the system from dominance of load effects to soil movement effects. It also marks a transition in the form of the pressure distribution, from a concave profile to a convex profile.

Plotting the solution in the form of performance graphs, Figures 2 to 4, it is found that the behaviour of interactive beam-soil systems is, as expected, truly linear.

3.2 Validity Limit

Since the soil pressure cannot be negative, the linear solution is valid until either (p_E) or (p_0) becomes zero. The former occurs in edge settlement problems, the latter in edge heave problems. The

common design condition for stiffened raft footings is symmetrical edge settlement, and the validity limit for this condition leads to the equation:

$$\frac{\tan \beta^* + \tanh \beta^*}{2\beta^*} = \frac{-2W_E}{w_0 L} \quad (4)$$

In Equation (4), $(\beta^*)^4 = \frac{\lambda^* L^4}{16EI}$, λ^* representing the limiting value.

Equation (4) has multiple roots, but only the lowest root is of interest. It can be demonstrated graphically that the lowest root lies between $\pi/2$ and 2.365. At $\beta^* = \pi/2$ the equation first changes sign from +ve to -ve infinity. At $\beta^* = 2.365$ the equation again changes sign. The validity limit design parameters are obtained by substituting λ^* into the relevant identities.

3.3 Observations

- The most striking feature of the mathematical model presented in this paper is the absence of the mound profile (y_0). To repeat a comment in Part (1) of this paper, the mound profile (y_0) is of no relevance in the linearly elastic behaviour of interactive beam-soil systems, because *the soil profile must be identical to the beam profile in the valid range of linearly elastic behaviour.*
- Soil modulus (k) plays a minor role in the linear solutions, appearing only in the initial state solution (u), and in the equation to determine (y_m) for solutions up to and including the validity limit (y_m^*).
- Uniformly distributed load (w_0) appears only in the identities for soil pressure, it has no effect on relative deflection and bending moment in the linear range of behaviour. Surprising as this may seem at first sight, it is correct, because the particular integral of Equation (3) corresponding to a distributed loading of constant intensity (w_0) is simply another constant (w_0/λ).
- Relative deflection is proportional to $(W_E L^3/EI)$, maximum bending moment is proportional to $(W_E L)$, and maximum soil pressure is proportional to (w_0) and (W_E/L) . In the validity limit solution for edge settlement the maximum bending moment, and maximum soil pressure, always occur at the centre of the beam.

3.4 Example

The beam details for a typical practical example are listed in Appendix A. Results of the correct linear solution are superimposed on those obtained by the Walsh and Cameron (1997) method in Figures 2 to 4. It is obvious that this example fails to satisfy traditionally accepted performance criteria.

4. PART (3): INTRODUCTION TO NON-LINEAR THEORY

In practice the behaviour of interactive beam-soil systems is non-linear, for many reasons. Even if the beam and soil remain linearly elastic, non-linear behaviour commences at the validity limit, i.e. at the onset of beam-soil separation. The initial non-linearly elastic behaviour is largely governed by asymptotic limits, i.e. at infinite soil movement. In the edge settlement limit the beam is fully cantilevered from the centre outwards. The asymptotic solution is statically determinate:

$$M_0^{**} = \frac{W_E L}{2} + \frac{w_0 L^2}{8} \quad (5)$$

$$\Delta^{**} = \frac{1}{EI} \left(\frac{W_E L^3}{24} + \frac{w_0 L^4}{128} \right) \quad (6)$$

In the edge heave limit the beam is simply supported, and the asymptotic solution is:

$$M_0^{**} = \frac{-w_0 L^2}{8}; \quad \Delta^{**} = \frac{1}{EI} \left(\frac{-5w_0 L^4}{384} \right) \quad (7)$$

Obviously the soil pressures in these asymptotic solutions are infinite, and meaningless relative to soil bearing capacity. Nevertheless, the asymptotic limits are useful for design purposes, and therefore need to be established. Soil bearing capacity rarely presents a problem with stiffened raft footings in practice.

Next in importance, in the non-linear behaviour of stiffened raft footings, is the non-linear aspect of structural behaviour, which ultimately determines failure. Although “failure” is used here in the context of strength, the following analysis links it to deflection.

The first step in ultimate strength analysis is to establish the failure mechanism. A two-dimensional stiffened raft beam fails when it develops what is sometimes called a “plastic hinge” at the centre. At this stage the beam deformations are highly localised, across one or more cracks, over a short distance at the centre. The outer halves of the hinged beam may reasonably be approximated to straight-line segments, i.e. the relative deflection function (y) is linear. Assuming the soil pressures are still within the linearly elastic range, it follows that the pressure distribution at failure is also approximately linear, varying from zero, at some distance from the edges, to a peak value at the centre of the beam.

Whilst the assumption of linearly elastic soil behaviour appears to be a bold one at first sight, it is in fact quite reasonable. In Part (2) of this paper it was shown that soil modulus plays a minor role,

appearing only in the equation to determine (y_m). The same applies in ultimate strength analysis. A further justification of this assumption is that non-linearity in soil behaviour primarily influences the form of the pressure distribution. The pressure resultant depends entirely on the total external load, and the end result is relatively insensitive to changes in the form of the pressure distribution. For example, in bending moment evaluation, the lever arm of the half-beam pressure resultant for a linear distribution is ($L/6$), and for a parabolic distribution it is ($3L/16$), a variation of only 11%.

The results of 2-D ultimate strength analysis are best illustrated graphically. The recommended procedure is to calculate in succession, for a range of values of beam-soil contact length (h), the following variables:

$$\begin{aligned} \text{Peak pressure,} & \quad p^* = \frac{2(2W_E + w_0L)}{h} \\ \text{Rotation of half-} & \quad \phi = \frac{2p^*}{kh} \\ \text{beam,} & \\ \text{Relative} & \quad \Delta = \frac{L}{2} \phi \\ \text{deflection,} & \\ \text{Relative soil} & \quad y_m = \Delta + \frac{p^*}{k} \\ \text{movement,} & \\ \text{Maximum bending} & \quad M_0 = \frac{W_E L}{2} + \frac{w_0 L^2}{8} - \frac{p^* h^2}{24} \\ \text{moment,} & \end{aligned}$$

Results for the beam in Appendix A are shown in Figures 2 to 4, confirming the conclusion in Part (2) of this paper that this example fails to satisfy traditionally accepted performance criteria.

5. 3-D ULTIMATE STRENGTH ANALYSIS

5.1 2-D or 3-D?

One questionable popular belief is that the 2-D beam-on-mound model is a reliable representation of the 3-D stiffened raft problem. A cursory comparison between beam and plate theories reveals that the difference in results may be substantial, in many practical cases of stiffened raft slabs, irrespective of which of the two dimensions is used for the beam length. The fact that existing methods have been shown in Part (1) of this paper to be fundamentally incorrect exacerbates the comparison. The author suggests there is a definite need to introduce 3-D theory into stiffened raft footing technology.

5.2 Stiffness or Strength?

Another questionable popular belief is that the design of residential stiffened raft footings is primarily governed by stiffness. It would appear this belief stems largely from theoretical results derived from the fundamentally incorrect 2-D model

exposed in Part (1) of this paper. The correct model in Part (2) of this paper indicates that increasing beam stiffness (EI) flattens the (Δ vs. y_m) graph, but it steepens the (M_0 vs. y_m) graph. In the limit, as (EI) tends to infinity, relative deflection tends to zero, but maximum bending moment tends to the asymptotic limit, Equation (5) or (7), albeit via a non-linear path. Moreover, as (EI) tends to infinity, and whilst the corresponding validity limit values (y_m^*) and (Δ^*) tend to zero, the gradient of the (M_0 vs. y_m) graph remains finite.

Bearing in mind that structural design must always consider both stiffness *and* strength, this popular belief also either dismisses or ignores the tried and proven traditional structural design parameter (L/D). The fundamentally correct 2-D model in Part (2) of this paper could be sensibly used to establish appropriate (L/D) ratios, at least as a guide to initial beam proportions on which to base a more detailed analysis or design.

5.3 Cracked or Uncracked Section Properties?

It would appear that stiffened raft beam design solutions published in AS-2870 have been derived primarily from theoretical studies using uncracked section properties. The author suggests this is a questionable, and potentially dangerous, departure from traditional practice in structural design of steel-reinforced concrete elements.

5.4 Proposed 3-D Strength Design Model

The author's main concerns, believed shared by many practitioners, are that the mandatory stiffened raft beam design solutions published in AS-2870 make no allowance for the size or shape of the house, and that the dead load of the house superstructure is not adequately taken into account. Part (2) of this paper unequivocally establishes that beam length and edge load are the major design input parameters. The author suggests that the following 3-D strength design model eliminates the main concerns, and that it obviates the current Australian design practice of a mandatory selection from a set of solutions published in AS-2870.

It would be frivolous to suggest that the proposed 3-D strength design model is a stand-alone design method, but it does provide a simple tool for routine design office application in the first instance. In the author's practical experience, stiffened raft designs based on a sensible initial (L/D) choice, followed by detailed 3-D strength analysis, are often adequate. Ultimately, a detailed 3-D deflection analysis cannot be ignored, but for the time being the author suggests that the 2-D linear theory in Part (2) of this paper will suffice in most cases.

The proposed 3-D strength design model is in accordance with accepted limit state design philosophy, using ultimate strength as a pre-defined

limit state. Some departure from accepted load factors may be warranted for stiffened raft footings, and the current AS-2870 practice of down-factoring the characteristic surface movement (y_s) to a design value (y_m), may also need to be reviewed. In the interim, the author recommends the following factors:

- Dead load: 1.25
- Live load: 1.5
- Characteristic surface movement: 1.0

The ultimate strength analysis of two-dimensional beams is readily extended to three dimensions. A 3-D stiffened raft slab fails by cracking in a pattern called the "yield line pattern". The 3-D failure mechanism consists of a number of essentially flat plane segments, rotating relative to one another

about the cracks (yield lines). The bending moment across a failure crack equals the ultimate moment capacity of the raft section, which, provided the stiffening ribs are properly spaced, is uniform over the whole raft. It is difficult to generalise house shapes, but most shapes can be divided into component rectangles, so the analysis of a single equivalent rectangle, or the worst component rectangle, should provide a reasonable starting point, at least for preliminary assessment purposes. The author has successfully used the conventional yield line method to determine the ultimate moment for rectangular rafts. For reasons given earlier, details of the author's analysis have been omitted from this paper, but results for the example in Appendix A are included in Figure 5, once more confirming that the typical practical example fails to satisfy traditionally accepted performance criteria.

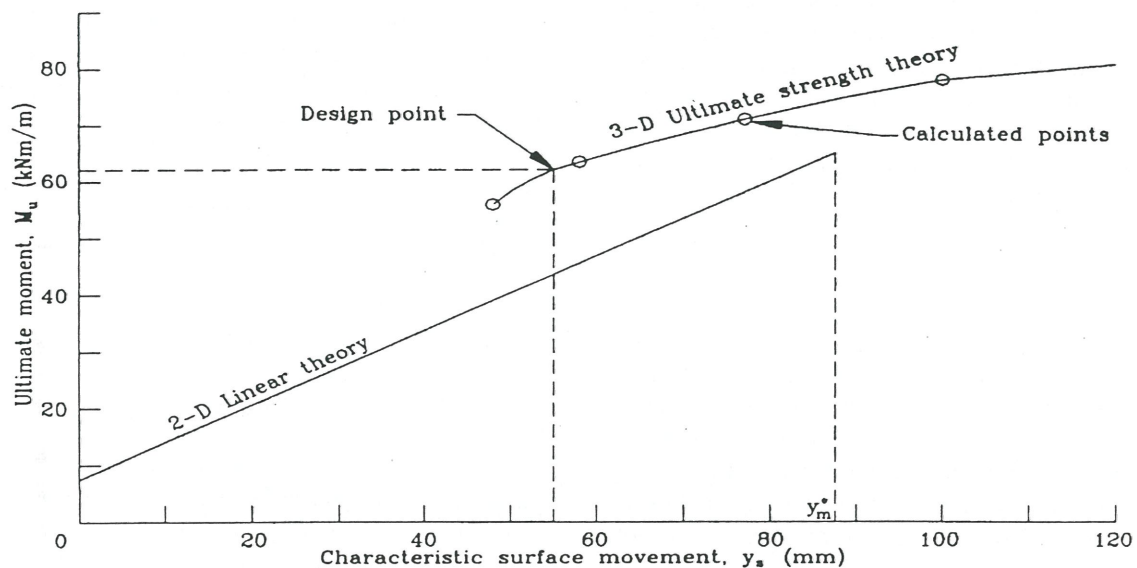


Figure 5. Three-dimensional ultimate strength design example.

6. CONCLUSION

Part (1) of this paper casts a shadow over a 25-year old theory and associated 13-year old practical experience with design and construction of residential stiffened raft footings in Australia. This is likely to inflict some pain on professionals practising in this area, which only time will heal. The public at large is entitled to expect reasonable performance of the practical end product of any theory, in this case residential stiffened raft footings. Part (1) therefore raises the serious philosophical question: has the unwitting public been deprived somewhat? The author's answer to this question is contained in the next paragraph. It is one of several philosophical issues raised by the author in an earlier paper, (Van der Woude, 1997), and it is suggested that the profession would do well to address these issues sooner rather than later.

Part (2) of this paper should lessen the pain from Part (1), by steering theory in the right direction to re-examine an important part of foundation design

and construction practice in Australia. Part (3) throws a little more light on the subject, and it offers a practicable design method to re-align stiffened raft footing technology with traditionally accepted performance criteria. By the same token it should help to restore reasonable community expectations.

7. REFERENCES

Standards Australia (1996). *AS-2870: Residential slabs and footings – Construction, and Supplement 1: Residential slabs and footings - Construction - Commentary.*

Van der Woude, F (1997). Australian Standard AS-2870: Past, present and future? *Australian Geomechanics* Number 32, December 1997. pp. 15, 16.

Walsh, P.F and Cameron, D.A (1997). *Handbook HB28: The design of residential slabs and footings.* Standards Australia and the Institution of Engineers, Australia.

APPENDIX A

Details of typical practical example:

The stiffened raft beam section chosen for the 2-D example in Parts (1) and (2) is prescribed in AS-2870 for articulated masonry veneer construction on a Class "H" residential building site. Uncracked section properties (M_{cr}) and (EI), evaluated in accordance with AS-2870, have been used in the analysis of a 10 m long beam of these proportions.

The 3-D rectangular stiffened raft slab in Part (3) is 15m x 10m, with an overall depth of 500 mm, reinforced with F102 slab fabric. The ultimate

moment (M_u) has been evaluated in accordance with Australian Standard AS-3600: Concrete structures. In Figure 5, the superimposed results of 2-D analysis of a 10 m beam of these proportions correspond to cracked section properties and factored loads. (Refer Section 5.4.)

Analysis and design input parameters:

Soil modulus, $k = 1000$ kPa/m.

Concrete modulus, $E_c = 15$ GPa.

Steel yield strength, $F_{sy} = 450$ MPa.

Concrete tensile strength, $F_{ct} = 1.8$ MPa.

Uniformly distributed load, $w_0 = 3.85$ kPa.

Equal concentrated loads at edges, $W_E = 10.9$ kN/m.