The paper was published in the proceedings of the 8th Australia New Zealand Conference on Geomechanics and was edited by Nihal Vitharana and Randal Colman. The conference was held in Hobart, Tasmania, Australia, 15 – 17 February 1999.

Geotechnical Models of Failure in Unbound Pavements

M. Boulbibane
MA, Ph.D.
Research Fellow, School of Engineering, University of Auckland, New Zealand
I.F. Collins
MA, Ph.D., FIPENZ, MASCE, MASME
Associate Dean, School of Engineering, University of Auckland, New Zealand

Summary Current design procedures for unbound pavements are based on a number of highly questionable assertions and do not relate to actual failure mechanisms. The basic models used incorporate the elastic but not the plastic behaviour of the basecourse and subgrade. They also assume that the pavements deteriorate indefinitely, whereas in practice pavements are frequently observed to "shakedown" to an equilibrium state. This paper will review progress in developing a new approach to unbound pavement design, based on shakedown theory, which incorporates the plastic as well as elastic properties of the pavement layers, provides predictions of various failure modes, such as rut formation and subsurface slip and allows for the effects of moisture infiltration and soil suction to be gauged.

1. INTRODUCTION

The application of empirically established theoretical models of geomaterials to pavement design has lagged behind most other branches of geotechnical engineering. Perhaps this is mainly due to the fact that the loads are applied repeatedly and the resulting damage is built up over a long period of time, in marked contrast to problems involving foundations or slopes. The current design procedures for unbound pavements employed in Australia, New Zealand and elsewhere are essentially based on the knowledge of elastic properties of the basecourse(s) and subgrade. The conventional argument for this approach is that a successfully designed pavement should clearly not produce significant permanent strains. For example Croney (1977) states that:" The shear strength of soil is not of direct interest to the road engineer... the soil should operate at stress levels within the elastic range... The pavement engineer is, therefore, more concerned with the elastic modulus of soil and the behaviour under repeated loading". This philosophy is in marked contrast to most structural design procedures, such as the design of frame structures, foundations, slopes etc., which are designed against failure by using a plastic analysis to determines the critical failure loads and then ensuring that the structure operates below a specified safety margin of these critical conditions.

The new pavement design procedure AUSTROADS (1992) recently adopted in both Australia and New Zealand is frequently referred to as a "mechanistic" procedure. Like its predecessors however this procedure is based upon the assumption that "failure" occur when certain strains reach a critical level. In the case of unbound pavements the vertical compressive strain at the top of the subgrade layer is deemed to be the crucial factor. However, in the design model this critical strain is calculated from the layer stiffnesses and is hence elastic and fully recoverable. Thus whilst recent developments have improved the methodology of calculating the critical strains, and have given the designer greater flexibility by replacing charts by computer programs, the underlying physical model has not changed since it was introduced 37 years ago by Dorman (1962). The failure model is still essentially elastic.

In the extant pavement design procedures the situation is further confused by the methods used to determine the elastic moduli. The stiffness of the subgrade is normally determined by some power law relation between the stiffness and the CBR value (Ullidtz (1987)), despite the fact that many experimental and theoretical studies have shown that the CBR measurement depends on the plastic as well as elastic properties of the soil (Porter (1950), Hight and Stevens (1982) and Brown (1994)). Indeed for stiff soils the CBR value is a measure of undrained shear strength and is essentially unrelated to the elastic properties (Brown (1994), Turnbull (1950) and Black (1962)). The nonlinearity of the elastic properties of the basecourse is well recognised and described by the use of the resilient modulus (Seed *et al* (1962)), which depends on the mean effective stress, the deviator stress, accumulated plastic strain and suction. Brown (1996) defines the resilient modulus as the ratio of the repeated deviator stress to the recoverable (resilient) axial strain in a triaxal test. This definition acknowledges the existence of non-recoverable (plastic) strains, but nevertheless these are ignored in the development of the subsequent models. There is hence a clear need to develop a pavement model which includes permanent (plastic) as well as recoverable (elastic) strains and which predicts the various type of pavement failure, such as rut formation, surface and subsurface slip and crack formation, all of which are governed by the strength properties of the pavement material. This need has recently been eloquently argued by Brown (1996) in the 36th Rankine Lecture to the British Geotechnical Society.

2. SHAKEDOWN MODELS OF PAVEMENTS

In an unbound pavement, the temperature and loading rate dependent, viscous surface bituminous layer plays no structural role, so that the pavement can be regarded as consisting of a number of rate-independent, elastic-plastic layers. The theory of the response of such structures to repeated loads is well developed and generally known as "shakedown theory" c.f. Lubliner (1990) for example. It has been extensively applied to frame structures and pressure vessels and more recently to model the wear resistance of metal surfaces to repeated rolling and sliding loads by Johnson (1985) and Ponter *et al* (1985).

Four types of response of an elastic/plastic structure to cycles of loading are illustrated schematically here in Figure 1.

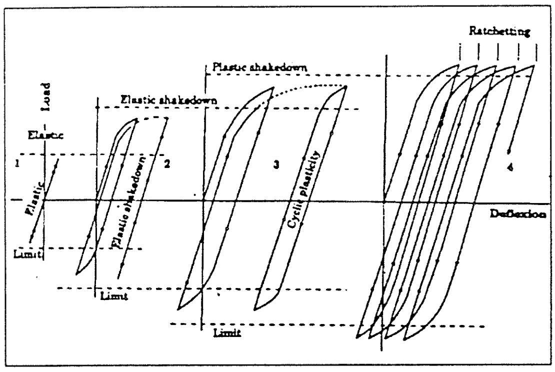

Figure 1. The four types of response of an elastic/ plastic structure to repeated loading cycles.

1) At sufficiently low loading levels the response is purely elastic and no permanent strains occur

2) At higher loading levels, the response is initially plastic but after a finite number of load application the response is purely elastic and no further permanent strain occur. When this happens the structure is said to have "shaken down".

3) At still higher load levels, the ultimate response cycle may be of the form of a closed loop, analogous to low cycle fatigue; a state known as "cyclic or alternating plasticity".

4) Alternatively the permanent strain may go on increasing indefinitely; a response known as "ratchetting". In a pavement such a response would be demonstrated by the formation of a rut or the generation of slip planes.

The critical load level below which shakedown occurs, but above which permanent strains continue to occur is called the "shakedown load". The appropriateness of the shakedown load as the key design parameter for pavements seems first to have been suggested by Sharp and Broker (1984), who estimated this load using a two-dimensional, plane strain, model in which the wheel is modelled by an infinitely long cylinder. These authors assumed that the material in the various pavement layers deformed plastically when the Mohr-Coulomb condition was achieved. These calculations have been extended by Raad *et al* (1988,1989), Hossain and Yu (1996), but in all cases a two dimensional model was assumed and the estimate of the shakedown load was obtained by consideration of the stresses – the so called lower bound approach of Melan.

Collins and Cliffe (1987) showed that by employing the alternative kinematic approach due to Koiter, it was possible to consider much more realistic three-dimensional models in which the wheel load is applied over a circular area and the deformation is localised beneath the wheel. This analysis was extended to include layered models and two-wheel loads by Collins and co-workers in (1993, 1994). In all these calculations however failure was assumed to occur by subsurface slip in the direction of travel, paralleling the analysis in Ponter *et al* (1985) for metal surfaces. More recently Collins and Boulbibane (1997, 1998) have presented the results of some preliminary calculations in which failure is assumed to be a result of rut formation. This development has made the analysis more realistic and is our principal concern here.

Upper bounds to the shakedown load can be obtained by postulating a kinematically admissible velocity field -i.e. failure mechanism- and calculating the ratio of the plastic dissipation to the virtual elastic work rate in the assumed mechanism:

$$\lambda = \frac{\int_V \sigma_{ij}^p e_{ij}^p dV}{\int_V \sigma_{ij}^e e_{ij}^p dV} \qquad (1)$$

where e_{ij}^p and σ_{ij}^p, are the time-independent strain-rate and plastic stress fields respectively and σ_{ij}^e is the elastic stress field produced by the applied load, P. The shakedown load is then definitely less than or equal to λP. Out of a family of competing mechanisms, that which minimises λ gives the best upper bound, and is the failure mode most likely to occur. In the calculations described here the optimum bound has been found using the method of " simulated annealing" (Goffe *et al* (1994)).

3. RUT FORMATION MODELS

Initially we will consider a one layer pavement. The load is assumed to travel from $-\infty$ to $+\infty$ in the X-direction, so that the residual stress field and the geometry of the failure mechanism is independent of X. The two basic types of possible mechanisms are illustrated in Figure 2. That in Figure 2(a) illustrates failure by slip in the travel direction in a shallow channel beneath the wheel, whilst that in Figure 2(b) represents the incipient formation of a rut.

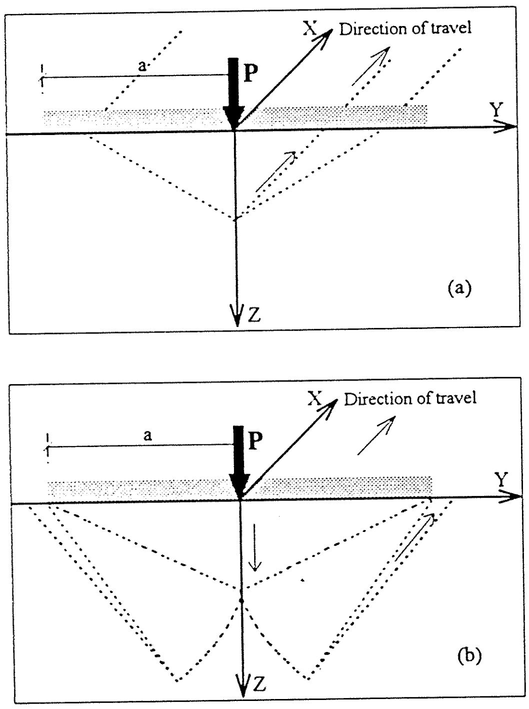

Figure 2. (a) Slip and (b) rut formation mechanisms.

In this mode the material beneath the wheel moves downwards and outwards and upwards as in the well know footing failure solutions. However, here it is found that in the optimum solution, the displaced materials moves upwards to the surface over a very narrow region adjacent to the wheel edge, in contrast to the corresponding optimum footing solutions, where the surface is disturbed over a region 2-4 times the radius of the loaded area (c.f. Atkinson (1981) or Bolton and Lau (1993)).
In both types of deformation the failure modes are essentially plane strain, so that the plastic dissipation rate can be worked out, just as in standard limit analysis calculations. Assuming an associated or normal flow rule, the dissipation rate per unit length of a velocity discontinuity is $C[v_t]$, where C is the cohesion and $[v_t]$ is the jump in the tangential velocity component. The ratio of the normal to the tangential velocity jumps is equal to $\tan\phi$, where ϕ is the angle of internal friction. The virtual elastic work rate in the denominator of (1) is calculated in the same way , except that C is replaced by the "elastic cohesion" defined by

$$C^e = \left|\sigma^e_{nt}\right| - \sigma^e_{nn} \tan\phi \tag{2}$$

where σ^e_{nt} and σ^e_{nn} are the elastic shear and normal stress components on the velocity discontinuity.
The results are most conveniently presented in terms of a non-dimensional load factor

$$\mu = \frac{P_s}{\pi C a^2} \tag{3}$$

where $P_s = \lambda P$ is the applied normal shakedown load, a is the radius of the circular loaded region and C is the cohesion of the basecourse.

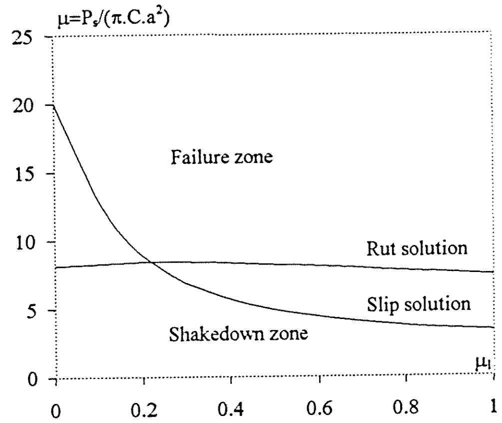

Figure 3. Variation of load factor with surface friction for slip and rut solutions.

In a uniform homogeneous pavement model, μ will be a function of μ_l the surface loading friction coefficient, ϕ the internal angle of friction of the basecourse material and the form of the loading distribution - here assumed uniform. However, it will not depend on the stiffness of the material. In a layered model, the dimensionless load factor will depend additionally on the *ratios* of the layer cohesions and stiffnesses, the values of the angle of internal friction and of Poisson's ratio in each layer and the ratio of the layer depth to the radius of the loaded area.
The variation of μ with μ_l for the two types of mechanisms is shown in Figure 3 for a single layer with $\phi = 35°$. It is seen that for $\mu_l < 0.2$ the rutting

mode is the preferred failure mechanism, whilst for larger values of μ_l the slip mode is predicted to occur.

4. SHAKEDOWN LOAD FOR NON-ASSOCIATIVE MATERIALS

In the above calculation the shakedown load is calculated assuming that the material obeys the normality rule. In reality soils and granular materials frequently exhibit non-associated behaviour. In this section an extension of the kinematic method of the shakedown analysis for non associated materials is given for translational failure mechanisms.

The straightforward generalisation of limit analysis techniques to non-associated materials does not work, since the expression for the plastic dissipation rate depends on the individual stress components, which of course are unknown. However, Drescher and Detournay (1993) have shown that for plane strain block sliding modes, estimates of failure loads can still be obtained by using a "comparison" associated material with cohesion $\tilde{C}$ and friction angle $\tilde{\phi}$, where

$$\tilde{C} = \omega C \quad \text{and} \quad \tan\tilde{\phi} = \omega \tan\phi \tag{4}$$

where

$$\omega = \frac{\cos\phi\cos\psi}{1 - \sin\phi\sin\psi} \tag{5}$$

where ψ is the dilatancy angle ($\psi = 0$ for incompressible material).

To illustrate the applicability of the procedure, a number of calculations are presented.

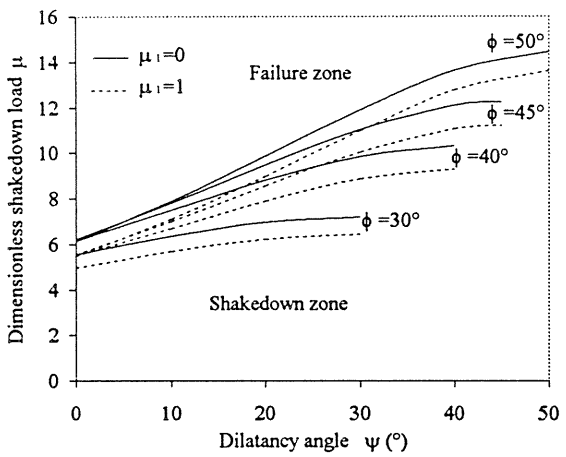

Figure 4. Variation of μ against dilatancy angle for homogeneous semi-infinite pavement.

The first example, shown in Figure 4, shows the variation of dimensionless shakedown load with dilatancy angle for four given friction angles.

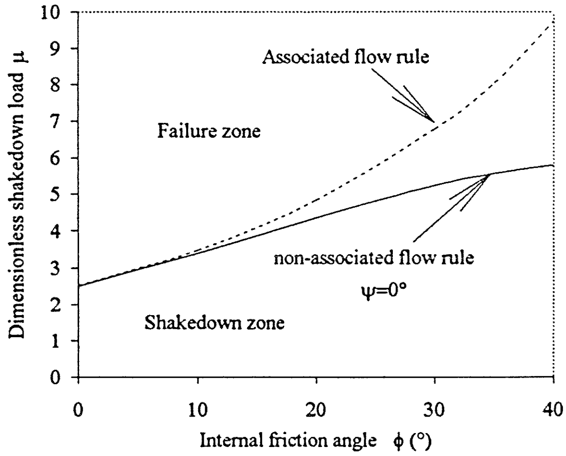

Figure 5. Associated and non-associated shakedown behaviour for one layer pavement system.

Figure 5 shows the difference in the values of the shakedown load by assuming firstly the associated flow rule and secondly assuming the material is incompressible. The results obtained for $\psi = 0$ can be considered as applying to a fully saturated pavement under undrained conditions. For small friction angles the shakedown load is relatively unaffected by the flow rule, but the drop in the value of the predicted shakedown load for friction angles in the range 30°- 40° is highly significant.

5. STRUCTURAL ANALYSIS OF MULTI-LAYER SYSTEM

Detailed calculations for two layered pavements have also been performed, some of the results have already been presented in Collins and Boulbibane (1997).

In reality rutting failures frequently occur when the pavement has a relatively weak subgrade. A possible failure mode is shown in Figure 6. Some preliminary results of calculations for this situation are shown in Figure 7 in a form suitable for design purposes.

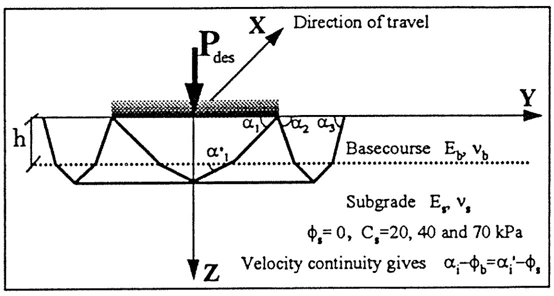

Figure 6. Proposed wedge mechanism for pavement analysis.

The subgrade is modelled as a purely cohesive material with $\phi = 0°$ and C=40 kPa. The radius of the loaded area is 0.14m, the design load is 40 kN and the ratio of the basecourse to subgrade stiffness is 3.

Figure 7 shows the critical basecourse thickness at which shakedown will just occur for values of basecourse cohesions up to 200 kPa and basecourse friction angles of 30° and 40°. If a pavement is designed with a depth less than these critical values it is predicted to eventually fail.

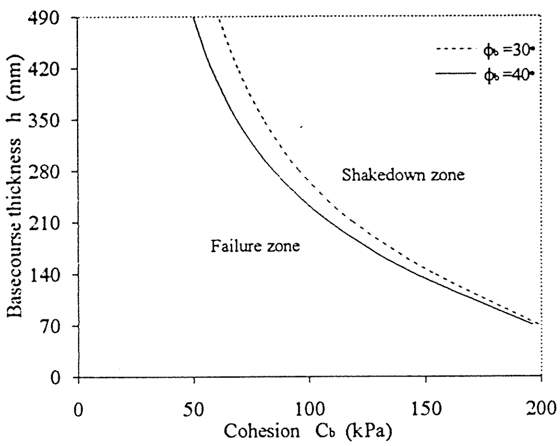

Figure 7. Variation of basecourse thickness with basecourse cohesion for various friction angles.

Figure 8 illustrates the interrelationship between the basecourse cohesion and basecourse thickness h at failure for a two layer pavement system. The graphs giving the critical basecourse thicknesses for specified basecourse cohesions are given for subgrade cohesions of 20, 40 and 70 kPa. The friction angles in the basecourse and subgrade are 30° and 0° respectively. For pavements with basecourses thickness greater than 300 mm, the subgrade cohesion has no effect, since the optimum failure mode is contained entirely in the basecourse.

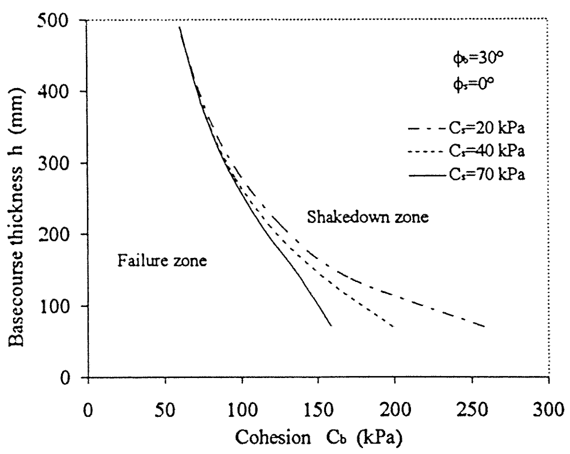

Figure 8. Variation of basecourse thickness with basecourse cohesion for various subgrade cohesions.

6. DISCUSSION AND CONCLUSIONS

The models described above assume a Coulomb model for the various pavement layers determined by their cohesions and friction angles. Values of these have been determined experimentally by G.C. Duske in a triaxal apparatus under monotonic loading for a variety of New Zealand aggregate materials and are given in Appendix B to Collins and Wang (1994). The subgrade can perhaps best be modelled as a perfectly cohesive material. The undrained shear strength can be estimated from its CBR value or preferably by direct field tests. The computation performed to date has enabled some preliminary design charts to be constructed. However, before some sample test tracks are constructed it is necessary to consider a number of other aspects of the model.

A number of investigations have already been done by including the effect of dual wheel loads and the self-weight of the pavement material. Other model modifications currently being investigated include :

- Three-dimensional failure modes in which the basecourse material is displaced both forwards and sideways.
- The effects of lateral constraints or unsupported edges of the pavement, and of lateral friction, as occurs when vehicles take bends.
- The presence of pore pressures or suctions in the pavement by using the principle of effective stress.
- Improving the material description of the basecourse aggregate and subgrade material, by incorporating volumetric hardening/softening effects and employing more modern models such as those of critical state soil mechanics.
- The effect of compaction.
- Elastic anisotropy.

7. ACKNOWLEDGEMENTS

The authors wish to express their gratitude to the PGSF and Transit New Zealand for financial support, and to Mr F. Bartley of Bartley Consultants, Auckland, for many helpful discussions.

8. REFERENCES

Atkinson, J.H. (1981). *Foundations and slopes*, University series in civil engineering, Oxford.

AUSTROADS Guide (1992). *Pavement Design* - A Guide to the Structural design of Road Pavements.

Black, W.P.M. (1962). A method of estimating the California Bearing Ratio of cohesive soils from plasticity data, *Géotechnique*, Vol 11, pp. 14-21.

Bolton, M.D. and Lau, C.K. (1993). Vertical bearing capacity factors for circular and strip footings on Mohr-Coulomb soil, *Can. Geotech. J.*, No. 30, pp. 1024-1033.

Brown, S.F. (1994). Behaviour of layered systems under repetitive loading, *XIII ICSMFE, New Delhi, India*, pp. 321-325.

Brown, S.F. (1996). Soil mechanics in pavement engineering, *Géotechnique* 46, No. 3, pp. 383-426.

Collins, I.F. and Cliffe, P.F. (1987). Shakedown in frictional materials under moving surface loads, *Int. J. Num. and Anal. Methods in Geomechanics*, No. 11, pp. 409-420.

Collins, I.F. and Wang, A.P. (1992). *Shakedown analysis of layered pavements*, Report No. 505, School of Engineering, University of Auckland.

Collins, I.F., Wang, A.P. and Saunders, L.R. (1993). Shakedown in layered pavements under moving surface loads, *Int: J. Num. and Anal. Methods in Geomechanics*, 17, pp. 165-174.

Collins, I.F., Wang, A.P. and Saunders, L.R. (1993). Shakedown theory and the design of unbound pavements, *Road and Transport Research*, No. 2, pp. 29-38.

Collins, I.F. and Wang, A.P. (1994). kinematic theory and the shakedown of frictional materials, *Proc. of the First Asia-Oceania Int. Symposium on Plasticity*, University Press, Beijing, China.

Collins, I.F. and Wang, A.P. (1994). Shakedown theory and pavement design, *Proceedings of the 8th International Conference on Computer Methods and Advances in Geomechanics*, Morgantown, West Virginia, USA, pp. 1465-1470.

Collins, I.F. and Boulbibane, M. (1997). Pavements as structures subjected to repeated loadings, *The Mechanics of Structures and Materials, Grzebieter, Al-Mahaidi and Wilson (eds) Balkema, Rotterdam*, pp. 511-516.

Collins, I.F. and Boulbibane, M. (1998). The application of shakedown theory to pavement design, *METALS AND MATERIALS*, Vol. 4, No. 4, pp. 832-837.

Croney M. (1977). *The design and performance of road pavements*, London, HMSO.

Dorman, G.M. (1962). The extension to practice of a fundamental procedure for the design of flexible pavements, *Proc. Int. Conf. On Struct. Design of Asphalt Pavements*, Ann Arbor, Mich, pp. 785-793.

Drescher, A. And Detournay, E. (1993). Limit load in translational failure mechanisms for associative and non-associative materials, *Géotechnique*, Vol. 43, No. 3, pp. 443-456.

Goffe, B., Ferri, G.D. and Rogers, J. (1994). Global optimization of statical functions with simulated annealing, *Journal of Econometrics*, vol. 60, no. ½, pp. 65-99.

Hight, D.W. and Stevens, M.G.H. (1982). An analysis of the California Bearing Ratio test in saturated clays, *Géotechnique* Vol. 32, No. 4, pp. 315-322.

Hossain, M.Z. and Yu, H.S. (1996). Shakedown analysis of mult-ilayer pavements using finite element and linear programming, *Porc. 7th Australia- New Zealand conference on Geomechanics*, Adelaide, pp. 512-520.

Johnson, K.L. (1985). *Contact mechanics*, Cambridge University Press.

Lubliner, J. (1990). *Plasticity theory*, Macmillan, New York.

Raad, L., Weichert, D. and Najm, W. (1988). Stability of multilayer systems under repeated loads, *Transportation Research Record*, No. 1207, pp. 181-186.

Raad, L., Weichert, D. and Haidar, A. (1989). Analysis of full-depth asphalt concrete pavements using shakedown theory, *Transportation Research Record*, No. 1227, pp. 53-65.

Raad, L., Weichert, D. and Haidar, A. (1989). Shakedown and fatigue of pavements with granular bases, *Transportation Research Record*, No. 1227, pp. 159-172.

Ponter, A.R., Hearle, A.D. and Johnson, K.L. (1985). Application of the kinematic shakedown theorem to rolling and sliding contact points, *J. Mech. Phys. Solids*, No. 33, pp. 339-362.

Porter, O.J. (1950). Development of the original method of pavement design, Development of CBR flexible pavement design method for airfields - *a symposium. Trans. ASCE.* 115, pp.461-467.

Seed, H.B., Chan, C.K. and Lee, C.E. (1962). Resilience characteristics of subgrade soils and their relation to fatigue failures, *Proc. Int. Conf. Structural Design of Asphaltic Pavements*, Ann Arbor, pp. 611-636.

Sharp, R.W. and Booker, J.R. (1984). Shakedown of pavements under moving surface loads, American Society of Civil Engineers, *J. of Transport Engineering* 110, pp. 1-14.

Turnbull, W.J. (1950). Appraisal of the CBR method, Development of CBR flexible pavement design method for airfields - *a symposium Trans. ASCE* 115, pp. 547-554.

Ullidtz, P. (1987). *Pavement Analysis*, Elsevier, Amsterdam.