Settlement and Permeability of Clays

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Summary A numerical method of analysis of one-dimensional consolidation tests is described which unites the primary and secondary phases, includes soil weight, allows permeability to be a function of porosity, and is not limited to small strains. The method identifies by iteration the soil constants which give the best fit to settlement/time records, with results from all load increments analysed as a continuous entity. In addition to predicting field settlements, the analysis yields a permeability/porosity equation which allows permeabilities to be gauged subsequently from measurements of porosity alone. The method is likely to predict field consolidation more accurately than methods which match only the primary phase, or do not include soil weight, or hold permeability constant during each increment, or are limited to small strains, or are applied only to each increment separately. Predictions of field consolidation are generally different from those given by the t/h^2 rule.

1. INTRODUCTION

1.1 Amounts and Rates

Final field settlements are, in most situations, 'amounts which occur before rates become negligible'. This means that amounts and rates cannot be separated completely.

Conventional void ratio vs effective stress (e–σ') relationships do not include rates of reduction of void ratio, but tend to imply that such rates are negligible at all points on the line relating e to σ'. It will be shown below that they show void ratios at stages of laboratory tests which may have no relevance to field conditions, and that they may not therefore yield reliable predictions of field settlements.

1.2 Computer Power

The availability of powerful computers allows numerical methods of solution to be adopted. These remove the need for simplifying assumptions to be made, in particular –

(a) the division into primary and secondary phases
(b) the ignoring of self-weight
(c) the assumption that permeability remains constant (with time and depth) during each increment of loading
(d) the adoption of a relationship between void ratio and effective stress (alone).

1.3 Primary and Secondary

In conventional tests, settlement is divided into primary and secondary phases using a construction such as that advocated by Taylor (1942).

This procedure is adopted in many standards, eg New Zealand Standard NZS 4402, Test 7.1. Common practice is to use the deduced "100% primary" settlement as the basis for an e–σ' plot, from which field settlements are calculated.

1.4 Gravity Effects

At laboratory scale, gravity effects within the so (due to the submerged weight of the soil particles) are usually negligible compared with applied loads. However, at field scale this becomes less true with distance (depth) below the ground surface.

Gravity effects should therefore be included if the analysis is to be used to predict field consolidation.

Conceptual division of a thick field stratum into several layers with different stress levels is not an adequate approach because it can be applied only to *amounts* and not to *rates* of consolidation.

1.5 Effects of Load Increment Ratio

Newland and Allely (1960) reported that in laboratory tests, the ratio of secondary to primary settlement increases as load increment ratio – l.i.r. – is decreased.

At field scale l.i.r. decreases with distance below the ground surface. For this reason alone secondary settlements can be expected to be more significant at field scale than at laboratory scale.

1.6 Permeability "Strangles" Consolidation

Because consolidation proceeds from the drainage face(s) to the undrained "mid-plane" reductions in porosity and permeability near the drainage face at small times after load applications must inhibit the consolidation process.

Typically, permeability decreases by 35% during individual (l.i.r. = 1) load increments: this is not negligible, and the conventional assumption – Terzaghi (1943) – of permeability remaining constant with time and with depth during each increment should not therefore be retained in numerical analysis.

1.7 Better Analysis Deserved – and Possible

The settlement–time information obtained from conventional tests is of a quality which can withstand more detailed analysis than just the derivation of 100% primary compression values and coefficients of consolidation c_{vc}.

The numerical method described below, (i) avoids all of the above difficulties, and can therefore be expected to give better field predictions, and (ii) provides a method for deriving permeability/ porosity (k/n) relationships for clayey soils which is simpler and faster, and much less prone to major error than permeameter testing.

2. A UNIFIED THEORY

2.1 Primary and Secondary on an e – σ' Plot

The conventional assumption that primary consolidation is associated with increasing effective stress, while secondary consolidation occurs at constant stress (with zero excess pore pressure) is illustrated in Figure 1.

Hawley (1971) showed that whatever criterion is used to distinguish between primary and secondary consolidation processes, elements of soil near a drainage face tend to pass into the secondary phase before elements deeper within the soil mass, see Figure 1.

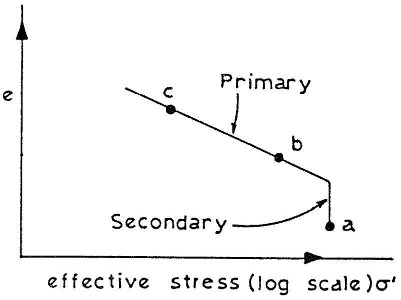

Figure 1. Paths in e – σ' space for conventional primary and secondary processes.

Figure 1 shows secondary settlement occurring at constant stress (ie small excess pore pressure). Some simultaneous states of elements of soil (a) at the drainage face (b) half way between the drainage face and mid-plane, and (c) at the mid-plane, are shown.

Bjerrum (1967) summarised many of the commonly observed facts of consolidation tests in a single e – σ' plot, Figure 2.

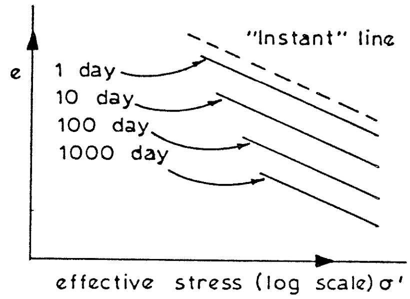

Figure 2. Laboratory primary and secondary settlements (after Bjerrum 1967).

In Figure 2 the equal spacing between the 1, 10, and 100 day lines reflects the observation that secondary settlement is approximately linear when plotted against log time, a pattern reported by Buisman (1936).

Because the 100% primary line must include some secondary settlement an "Instant" line must exist somewhat above the 100% primary line, Bjerrum (1967).

2.2 Limited Relevance of Pore Pressure

Pore pressure (u) **of itself** is not relevant to consolidation, as the use of back pressures in some testing illustrates. Likewise, pore pressure gradients, ($\delta u/\delta z$) **of themselves** are not relevant to consolidation, as steady state seepage illustrates, Hawley (1973).

Consolidation is the result of **spatial changes in pore pressure gradient** ($\delta^2 u/\delta z^2$)

The fact that, in conventional consolidation tests, all three of the above (u, $\delta u/\delta z$, and $\delta^2 u/\delta z^2$) become small at the same time does not mean that pore pressure can be used as a surrogate for the other two.

Spatial rate of change of pore pressure gradient – more precisely $\delta/\delta z\ [k.\delta u/\delta z]$ – is a measure of strain rate, ie of consolidation rate.

2.3 The Secondary Process

During the 'secondary' phase pore water must flow out of each element of the soil faster than it flows in. For such flows to occur, pore pressure, pore pressure gradients, and spatial changes in pore pressure gradients must all exist.

All consolidation must involve spatial changes in pore pressure gradients. That is what all consolidation is, whether it be labelled primary, secondary or creep.

Indeed, in the absence of evidence to the contrary, all relationships employed in consolidation analysis may be assumed to apply to the secondary as well as the primary phase with the exception of the stress/strain relationship. As shown in Figure 1, in laboratory tests the stress/strain relationship undergoes a very major change, to a 'creep' relationship where appreciable strain occurs without appreciable change in stress.

In the analysis presented in this paper, such a division into primary and secondary phases is avoided by adopting a stress/strain/strain rate ($e - \sigma' - \delta e/\delta t$) relationship rather than a stress/strain ($e - \sigma'$) relationship such as that illustrated in Figure 1.

The justification for this is that the behaviour predicted by the theory then models observed laboratory behaviour much more completely.

2.4 Stress/Strain/Strain Rate

Hawley (1973) showed that if the lines in Figure 2 are regarded as representing a surface in $e - \sigma' - \delta e/\delta t$ space, this surface may be used to replace the lines shown in Figure 1 as the 'constitutive relationship' for the soil. This is the last of six relationships required for an analysis of the consolidation process.

2.5 The Six Relationships Governing Consolidation

(a) **Continuity**
This states that (in a saturated soil) the reduction in thickness of an element of soil in any time interval is equal to the amount of water which flowed out of it less the amount which flowed into it in that time interval.

(b) **Darcy's law**
McNabb (1960) showed that when the equations of continuity and Darcy's law are written for elements of *soil* rather than elements of *space*, they are simpler. They are also more easily applied in numerical computation. Inconsistency in this matter cannot be saved by appeal to 'small strains'.

The velocity term in Darcy's law is a *discharge* velocity, ie what the velocity would have been had the flow occupied the full cross section of soil.

Continuity and *Darcy's law* combine to give:

$$\delta e/\delta t = (1 + e) \,.\, \delta/\delta z \,[\, k/\gamma_w \,.\, \delta u/\delta z \,] \qquad (1)$$

where $\delta u/\delta z$ is pore pressure gradient across an element of soil of (current) thickness δz, k is the permeability of the soil within that element at that time, δe is the change in void ratio in time interval δt, and γ_ω is the density of the pore water.

(c) **The effective stress equation**

$$\sigma' = \sigma - u \qquad (2)$$

ie the effective stress is equal to the total stress minus the pore water pressure.

(d) **Equilibrium**
Equilibrium in the Earth's gravitational field. It is here that the difference between the densities of the solid soil particles and the pore water is taken into account.

Only two constants are needed for this – gravitational acceleration (9.81 m/s^2) and the solids density of the soil particles, eg 2.65 t/m^3.

This aspect of the theory, together with the consistent use of elements of *soil* advocated by McNabb (see above) was included in the theory presented by Gibson, England and Hussey (1967).

The above four relationships involve few assumptions or approximations. Errors introduced by such matters are regarded as negligible compared with those which can be introduced by the fifth and sixth relationships, see below.

The relationships, Continuity, the Effective Stress Principle and Equilibrium are not 'soil dependant'. They involve no 'soil constants' other than 'solids density' and have the same algebraic form for all soils. The form of Darcy's law may, in the absence of evidence to the contrary, be taken as being the same for all soils, and the constant in it is given by the fifth relationship, the permeability/porosity relationship, see (e) below.

Characteristics of **particular** soils are therefore expressed entirely in the algebraic form of, and constants in, the final two relationships.

(e) **Permeability – porosity relationship**
The authors make the assumption that permeability may be expressed as a function of porosity **alone.** The possibility of layering at the microscopic scale is therefore not catered for.

The relationship adopted in the computations described below is:

$$k = k_1 \,.\, \exp\,[\, A_k\,(n - n_1)\,] \qquad (3)$$

where k_1 is permeability at a porosity n_1 and A_k is a constant for a particular soil.

An advantage of numerical computation is that different algebraic forms of, and constants in, this equation may be adopted in order to achieve better correspondence between computed and observed laboratory settlement/time curves for particular soils, see below.

(f) **The stress/strain relationship**
The relationship used by the authors is:

$$\delta e/\delta t = B\,[\,1 - (e_L - e_u)/(e_L - e)] \qquad (4)$$

where B, e_L and e_u are soil constants defining the position and shape of the surface in $e - \sigma' - \delta e/\delta t$ space which constitutes the 'stress/strain/strain rate' relationship for the particular soil.

At any (current) effective stress, e_L and e_u are the void ratios on (respectively) the "Instant" line (Figure 2) and a lower line where (for computation purposes) $\delta e/\delta t$ is taken to be zero. This aspect of the theory is set out in Hawley (1973).

2.6 Computational Sequence

For computation, the soil (first the laboratory sample, and then the field stratum) is regarded as comprising several horizontal layers, eg ten.

Initial void ratios [e_0 (1–11)] and pore pressures [u (1–11)] are assigned to the boundaries between the ten layers, and values of total and effective stresses at these points calculated.

Pore pressure gradients are then computed for each of the ten layers, together with values of permeability – from the k/n relationship – and values of $\delta e/\delta t$ calculated from (1) for each of the nine lower surfaces of layers.

An initial time increment is adopted for the first cycle of the computation through the ten layers. In subsequent cycles this time increment is increased to give constant increments of 'square root of time'.

Multiplying the $\delta e/\delta t$ values by the time increment gives changes in e for each layer, and hence new values of layer thickness, new total thickness and settlement.

The stress/strain/strain rate relationship (4) is then used to give the new effective stress and, by comparison with total stress, the new excess pore pressure, via (2).

Some values of some variables are chosen for plotting, eg settlement, mid-plane pore pressure, and spatial arrays (isochrones) of void ratio and permeability.

The progress of the computation may be observed by displaying isochrones of pore pressure, ie by plotting u(1) to u(11) vs depth at selected intervals of root time.

Pairs of e and σ' values at selected times are stored for plotting the $e - \sigma'$ paths followed by selected elements of the soil, eg the elements at the drainage face, half way between the drainage face and the mid-plane, and at the mid-plane.

2.7 Typical Results of Computations

Figures 3a, b and c, show the results of computations for a typical standard l.i.r. = 1 laboratory test, and Figures 4a and b, for a reduced l.i.r. test.

It can be seen that many features found in real soils and not modelled by the Terzaghi theory appear; specifically –
- a secondary phase,
- a sudden reduction in pore pressure at very small times, Taylor (1942)
- mid-plane pore pressures are very small (but non zero) during the secondary phase,
- the ratio of secondary to primary settlement increases with reduction in l.i.r. .

Although the $e - \sigma'$ paths followed in the l.i.r. = 1 test (Figure 3b) are very close to the 'ideal' paths shown in Figure 1, in the reduced l.i.r. computation some divergence is apparent, with the soil near the mid-plane in particular following a 'lower' and 'smoother' path.

The above computations were re-run with (only) the thickness of the soil increased – by a factor of 30, from 20 mm to 600 mm.

The results are shown in Figures 5a and b with the time scale stretched by a factor of 900. Had the t/h^2 scaling rule been obeyed these two Figures would have been identical to Figures 3a and b. This shows that predictions of field consolidation given by this method are generally different from those given by classical theory.

At the greater thickness the process cannot be easily divided into primary and secondary phases.

2.8 Successive Increments

The programme allows the computations for successive increments to follow each other, with the final array of e and u values in one increment being picked up as initial values for the next increment.

The computation results for the first increment are usually best ignored, and the increment regarded as useful for 'bedding in' the computation as well as for physically bedding in the sample. This reduces the influence of errors which must arise from the lack of information available on the initial state of the sample, the initial (small) stresses within the sample in particular.

2.9 Drainage Options

The programme has been extended to model drainage at the upper face only, the lower face only, or both.

2.10 Obtaining Soil Constants from Laboratory Tests

One begins with laboratory settlement/time data for a series of load increments, and the soil constants must be deduced from these.

Laboratory values of settlements at 16 mins, 100 mins, 400 mins and 1440 mins (24 hrs) are entered into the computation and lines of programming inserted which cause the soil constants to be 'corrected' after each complete 'run', in directions which will bring the computed line closer to them.

In Figure 3a the four laboratory settlement points appear as crosses, and two successive lines of computed settlement can be seen, the second (solid) one giving a better 'fit' to the four points.

The later (*secondary*) stages of a test reveal the 'low strain rate' characteristics of the soil most clearly. Conventional 24 hr increment tests lead therefore to better definition of the soil constants which influence the low strain rate part of the stress/strain/strain rate surface, and should therefore lead to more accurate field predictions than tests which follow the recent fashion for 20 min increments.

Similarly, the inclusion of one (or two) reduced l.i.r. increments improves the accuracy of predictions by being more dependent on the stress/strain/strain rate relationship,

Because both the stress/strain/strain rate and k/n relationships for any given soil can be assumed to be 'smooth and continuous' across all increments, the soil constants in them, after being determined for each increment, can be compared with values from other increments, adjusted for smoothness and the computations re-run.

In Figure 3a the following may be noted:

- secondary as well as primary settlement,

- rapid initial dissipation of mid-plane pore pressure observed by Taylor (1942). This is due to the distance between the Instant and 24 hr lines shown in Figure 2, ie to secondary consolidation.

- very small pore pressure during the secondary phase,

- laboratory settlements at times 16, 100, 400 and 1440 mins shown as crosses,

- three successive iterations of computed settlement showing improved 'fit' with laboratory settlements.

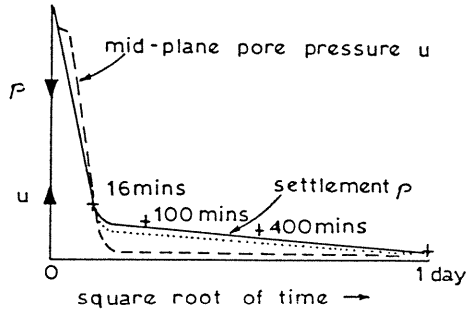

Figure 3a. Settlement and mid-plane excess pore pressure vs root time for l.i.r. = 1.

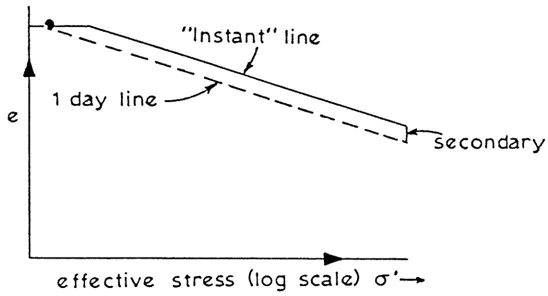

Figure 3b. Paths in e – σ' space for the computation shown in Figure 3a, cf Figure 1.

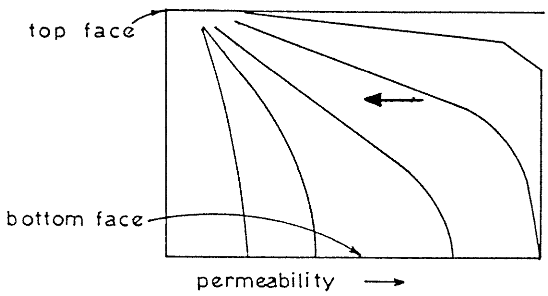

Figure 3c. Isochrones of permeability for the computation shown in Figures 3a and 3b.

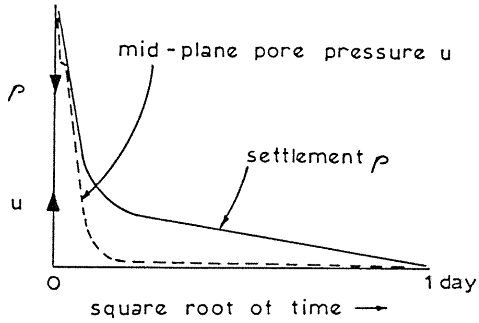

Figure 4a. For a reduced l.i.r. computation on the same soil used for computation shown in Figure 3a, b, and c.

In Figure 4a the increased ratio of secondary/primary settlement observed by Newland and Allely (1960) can be seen to be modelled.

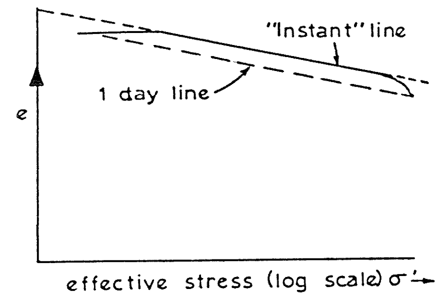

Figure 4b. Paths in e – σ' space followed during the computation shown in Figure 4a.

In Figure 4b the e – σ' paths begin to diverge from the conventional paths shown in Figure 1 and Figure 3b.

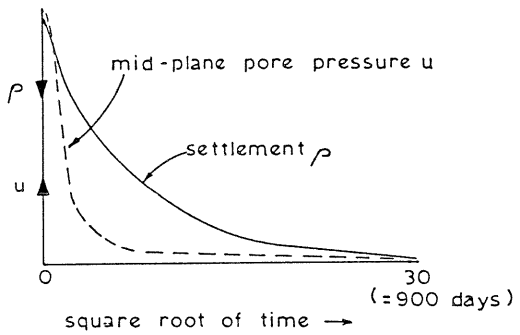

Figure 5a. Results of computation for soil shown in Figure 3a, b, and c but with sample thickness increased 30x.

In Figure 5a the effects of increasing the sample thickness by 30x (from 20 mm to 600 mm) may be seen. Had the t/h^2 rule been obeyed, this Figure would have been identical to Figure 3a.

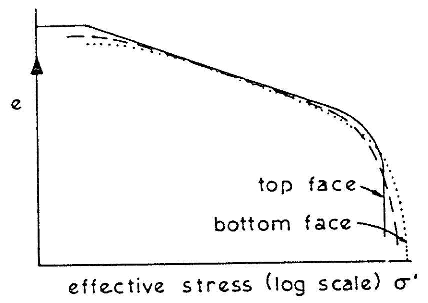

Figure 5b. Paths in e – σ' space for thick stratum computation shown in Figure 5a.

In Figure 5b major divergence from the Figure 1 "ideal" is apparent. At the chosen low stress level the effects of self-weight appear as higher values of σ' at the bottom than at the top of the stratum.

In Figure 5c the effects of the (submerged) weight of the soil particles are discernible as lower k values (from lower e values) at the base of the sample at the beginning and end of consolidation.

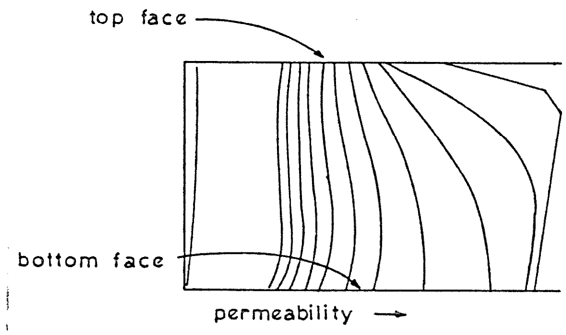

Figure 5c. Isochrones of permeability for the computation shown in Figures 5a and 5b.

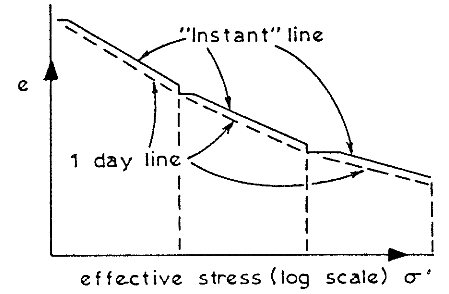

Figure 6a. Computed e – σ' paths for three increments of an l.i.r. = 1 test on a normally consolidated sample.

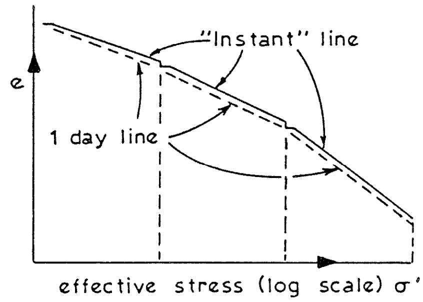

Figure 6b. Computed e – σ' paths for three increments of an l.i.r. = 1 test on an overconsolidated sample.

In Figures 6a and 6b the concave and convex overall stress paths are discernible, together with the near vertical 'secondary' paths at the end of each increment.

3. CONSOLIDOMETERS AS PERMEAMETERS

In achieving a close fit (in all increments) between the four laboratory settlement points and the computed values, the two constants in (2) are determined. This then becomes an 'observed k/n relationship', obtained from a test which is better controlled than permeameter tests can be.

Permeameter tests on clayey soils are unsatisfactory in that a head difference large enough to cause measurable flow will cause consolidation, which means that a non-uniform pattern of porosity appears, with smallest values at the downstream face of the sample and almost zero induced consolidation at the upstream face.

In addition to this, the likelihood of leakage down the sides of the sample is higher in a permeameter than in a one-dimensional consolidation test.

Once k/n relationships are known for a given clay they can be applied to that soil at different densities: measuring permeability then becomes a simple matter of measuring (saturated) dry density, deducing n and applying the k/n equation.

4. LAYERED FIELD STRATA

At field scale some layering of soils is commonly found. This means that after the soils in each layer have been sampled and tested, and the constants for each derived, the programme must be re-run using the different soil constants for the various layers and at the full scale.

5. FUTURE DEVELOPMENTS

Because the analysis can handle load increments of any size applied at any time intervals, extension to cope with 'as built' loading patterns is possible.

Extension to cope with situations where soil weight is the **only** load is also possible. This occurs during the construction of embankments and in sedimentation over geological time.

6. CONCLUSIONS

A numerical method for analysing standard one-dimensional consolidation tests is described which, by avoiding all of the simplifying assumptions which Terzaghi was obliged to make, and including strain rate in the stress/strain equation, models the observed laboratory settlement/time behaviour much more closely.

In particular it models the secondary as well as the primary phase, and the effects of reduced load increment ratio.

It may therefore be expected to give more reliable predictions of field behaviour – settlements and pore pressure dissipation. Field predictions are generally different from those given by the t/h^2 similarity rule.

The method yields a permeability/porosity relationship which allows permeability to be deduced in the field from comparatively easily measured values of porosity and soil solids density.

7. ACKNOWLEDGEMENTS

The New Zealand Government "Foundation for Research Science and Technology" provided financial support for transforming a theory which modelled soil-like behaviour using arbitrarily chosen soil constants, into one which begins with test results and deduces the soils constants. Also for extension to cope with drainage at the top, the bottom, or both, and to handle the analysis of successive increments continuously.

Professor J B Burland of Imperial College London gave timely encouragement for further work to be done on the theory whose basic features were developed in a PhD study in the 1960's.

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