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Application of Effective Stress Concept to Unsaturated Soils

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Summary The application of the effective stress concept to unsaturated soils is critically reviewed. The validity and the appropriateness of the relationship proposed by Khalili and Khabbaz (1996, 1998) for the determination of the effective stress parameter χ are examined using both shear strength and volumetric change data. Extremely good agreement is obtained between the measured and predicted values in all cases. It is shown that quantitative predictions of shear strength and deformation in unsaturated soils can be made using the effective stress concept. The model parameters will be exactly the same as those used in saturated soils, except for a single parameter which can be determined in any soil physics laboratory. This is in contrast to the current models of unsaturated soils, which require extensive laboratory testing.

1. INTRODUCTION

One of the most fundamental contributions to the development of the modern soil mechanics has been the introduction of the effective stress concept. According to this concept, all measurable effects of a change of stress, such as compression, distortion and a change of shearing resistance, are exclusively related to a change in the stress in the solid phase of the soil or the effective stress. The effective stress concept makes possible application of the principles of continuum mechanics to many soil engineering problems which otherwise could only be treated in an empirical manner (Blight, 1967).

For saturated soils, the effective stress concept is described as (Skempton, 1961):

$$\sigma'_{ij} = \sigma_{ij} - (1 - \frac{c_s}{c}) u_w \delta_{ij} \tag{1}$$

in which σ'_{ij} is the effective stress tensor, σ_{ij} is the total stress tensor, u_w is the pore water pressure, c_s is the compressibility of the solid grains, c is the drained compressibility of the solid skeleton, and δ_{ij} is the Kronecker's delta. For unsaturated soils, the effective stress is defined as (Bishop, 1959):

$$\sigma'_{ij} = (\sigma_{ij} - u_a \delta_{ij}) + \chi (u_a - u_w) \delta_{ij} \tag{2}$$

where u_a is the pore air pressure and χ is the effective stress parameter, attaining a value of 1 for saturated soils and a value of zero for dry soils.

Despite the widespread application of the effective stress concept in saturated soils, its use in unsaturated soils has been a controversial issue, particularly in certain situations such as volumetric deformation involving collapse. In fact, most investigators in this field have questioned the validity of the effective stress concept in unsaturated soils (e.g. Matyas and Radhakrishna, 1967; Fredlund and Morgenstern, 1977; Alonso et al, 1990). However, in recent years, in light of new theoretical developments, it is increasingly recognised that non-recoverable deformations such as collapse can be readily described within the framework of the effective stress concept by defining the yield surface as a function of suction (Khalili and Khabbaz, 1996, 1998; Kohgo, 1993; Bolzon et al, 1996). It should be stated that even in saturated soils irrecoverable deformations such as dilation and/or collapse (ie. in metastable structures) cannot be explained in terms of the effective stresses without invoking an appropriate plasticity model.

Another difficulty in the application of the effective stress concept to unsaturated soils has been the necessity to evaluate the effective stress parameter, χ. Considerable attempts have been made in the past to quantify χ both experimentally and theoretically. However, most of these attempts have been focused on finding a relationship between χ and a volumetric parameter such as the degree of saturation, S_r. As noted by Coleman (1962), χ is a parameter strongly related to the soil structure, and therefore it should not be surprising if no correlation is found between χ and a volumetric parameter such as the degree of saturation. Khalili and Khabbaz (1996, 1998) showed that, by plotting the values of χ

reported in the literature against a more appropriate parameter such as the "suction ratio" (defined as the ratio of matric suction over the air entry value[1]) a unique relationship may be obtained for most soils, Figure 1.

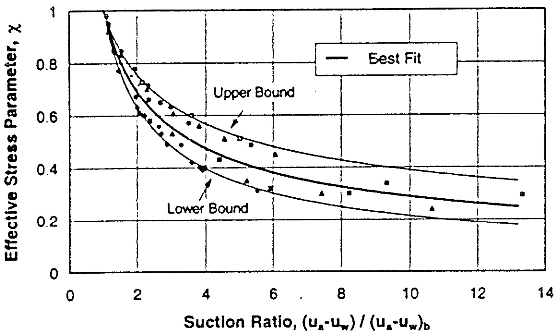


Figure 1. χ versus suction ratio. after Khalili and Khabbaz (1996,1998).

The best-fit in Figure 1 is expressed as:

$$\chi = \left[\frac{(u_a - u_w)}{(u_a - u_w)_b}\right]^{-0.55} \qquad (3)$$

in which $(u_a - u_w)_b$ is the air entry value.

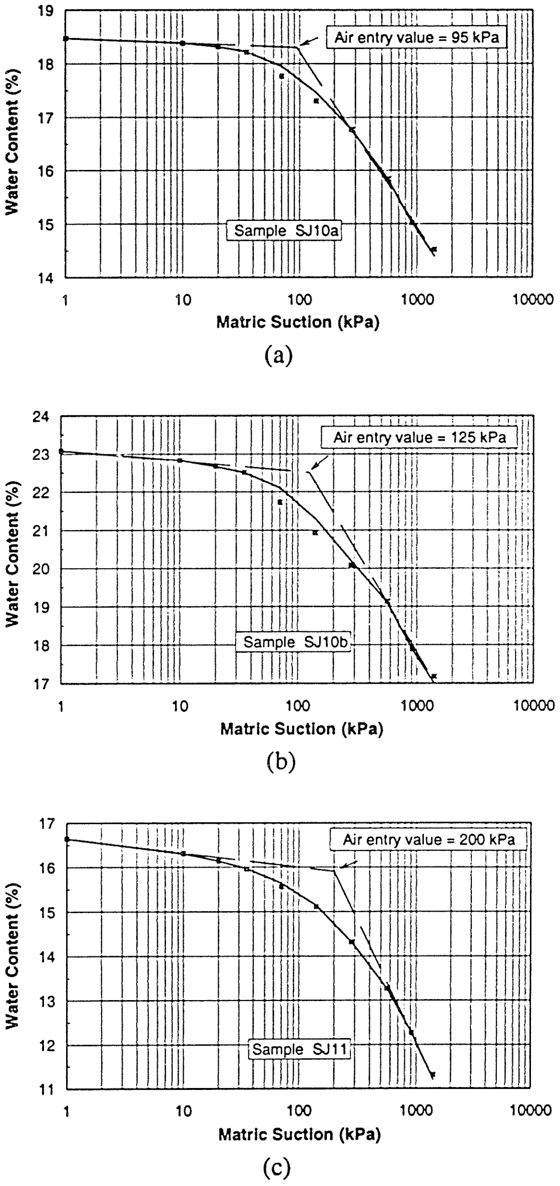
To examine the validity of the proposed relationship, Khalili and Khabbaz (1998) conducted 17 suction controlled shear strength determination tests on two laboratory compacted soils: (1) a compacted kaolin, and (2) a compacted sand (75%) and kaolin (25%) mixture. Extremely good agreement was obtained in all the cases between the calculated and measured results.

The main objective in this paper is to provide further experimental evidence as to the validity of the relationship given in Equation (3), using both shear strength and volumetric change test data. Notice that, to be valid, the effective stress relationship must be applicable to both shear strength and volumetric change data.

2. SHEAR STRENGTH TEST DATA

The shear strength test data presented herein were conducted as part of the overall structural integrity assessment of Hume Dam, located on Murray River in south-eastern Australia. Three undisturbed soil samples were taken from the southern Junction area of the dam, using 50 mm diameter thin wall tubes. The tubes were waxed on both ends to ensure the integrity of the samples and to retain the natural water content.

Once in the laboratory, the samples were carefully extruded and cut to size for triaxial testing. The basic properties of the soils, designated SJ10a, SJ10b and SJ1, are given in Table 1. The soil water characteristic curves are shown on Figure 2. The grain size distributions of the soils are given in Figure 3.



(a)

(b)

(c)

Figure 2. Soil-water characteristic curves.

Both saturated and unsaturated tests were conducted on the soil samples. The saturated tests were conducted using conventional triaxial testing equipment and the standard multistage shear strength determination procedure with pore pressure measurements. Prior to shearing, each sample was soaked through the base and then saturated using a back pressure of 300 kPa until a B (Skempton's pore pressure parameter) value of 0.95 was obtained.

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[1] The air entry value corresponds to the matric suction above which air recedes into the soil pores.

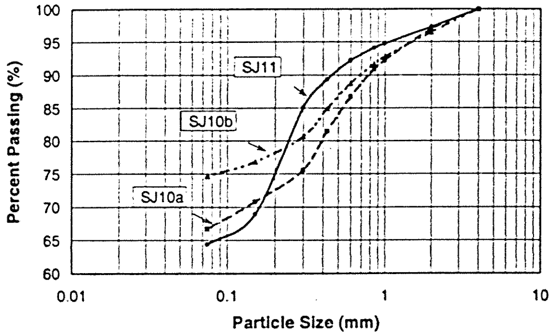


Figure 3. Grain size distribution.

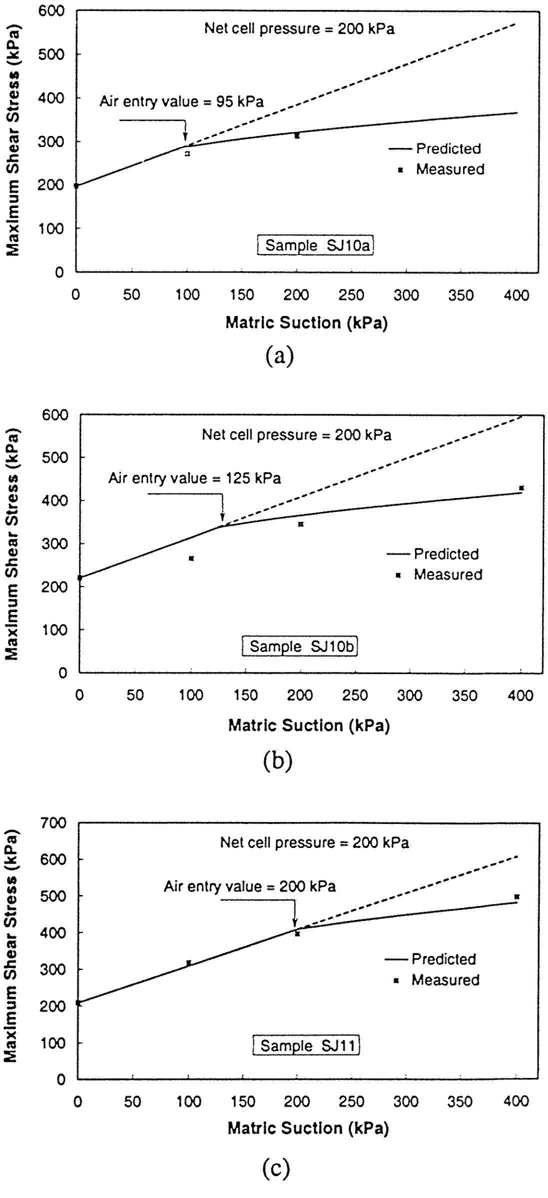
Table 1. Basic properties of soil samples used in shear strength experiments.

Sample No.	SJ10a	SJ10b	SJ11
Depth (m)	11.5-11.8	20.5-20.8	23.5-23.8
Initial Water Content (%)	17.0	17.9	13.4
Liquid Limit (%)	39.4	32.7	25.4
Plastic Limit (%)	18.4	20.5	18.6
Dry Density (Mg/m^3)	1.82	1.69	1.91
Fine Fraction (%)	66.7	74.65	64.4
USCS Symbol	CL	CL	CL
Effective Cohesion* (kPa)	5	19	5
Effective Friction Angle*	29°	29°	30°
Air Entry Value** (kPa)	95	125	200

* Obtained using the standard consolidated undrained shear strength tests, with pore pressure measurements, on saturated soil samples.
** Obtained using the pressure plate technique (ASTM, 1968, 1972) and the soil water characteristic curves shown on Figure 2.

The unsaturated tests were conducted using a Bishop-Wesley hydraulic triaxial cell, modified for testing unsaturated soils. The pore water pressure was measured at the base of the sample, whereas the pore air pressure was measured at the top. To allow independent measurement and control of pore-water pressures, a high air entry ceramic disk (1.5 MPa) was used at the base of the samples. Prior to each test the ceramic disk was saturated to resist the passage of free air at differential pressure less than the air entry value. A spiral grooved water compartment was located immediately below the high air entry disk for trapping and flushing air bubbles that may diffuse through the disk. A pressure transducer was installed below the compartment for measuring the applied water pressure. A coarse porous disk with a low air entry value was placed at the top of the sample and below the loading cap. This disk was connected to the pore air pressure control system through a hole drilled in the loading cap. The controlled air pressure was monitored during tests using a pressure transducer mounted into the loading cap.

A total of 8 suction controlled tests were conducted, two on SJ10a, three on SJ10b and three on SJ11. Following setting up in the triaxial cell, each sample was allowed to reach equilibrium under an applied net cell pressure (σ_3 - u_a) and suction (u_a - u_w), prior to shearing. The net cell pressure applied in the experiments was 200 kPa; the suction values were 100 and 200 kPa for SJ10a, and 100, 200 and 400 kPa for SJ10b and SJ11. All samples were sheared at a strain rate of 0.003 % per minute. The cell pressure, σ_3, the sample air pressure, u_a, and pore water pressure, u_w, were held constant during the shearing process.



(a)

(b)

(c)

Figure 4. Comparison between predicted and measured shear strength test results.

The results of the tests along with a comparison of the predicted and calculated shear strengths are given in Table 2. Graphical representations of the results are given in Figure 4. The air entry values on Figure 4 were obtained using the pressure plate technique (ASTM, 1968, 1972) and the soil water characteristic curves shown on Figure 2. The effective shear strength parameters (c' and ϕ') were obtained using the results of the conventional triaxial tests with the soil in a saturated state, as given in Table 1. The predicted shear strength values were calculated using the effective stress concept and the relationship,

$$q = c' \cos\phi' + [(p - u_a) + \chi(u_a - u_w)]\sin\phi' \quad (4)$$

in which $q = (\sigma_1 - \sigma_3)/2$ and $p = (\sigma_1 + \sigma_3)/2$. The effective stress parameter χ was obtained using the relationship given in Equation (3).

As can be observed from Figure 4 and Table 2, extremely good agreement exists between the calculated and the measured values, with less than 6% error in all cases, except for one case in which the error is around 19%.

Table 2. Shear strength test results.

Sample No.	Matric Suction (kPa)	Shear Strength Measured (kPa)	χ	Shear Strength Calculated (kPa)	% Error
SJ10a	100	271	0.97	288	6.3
	200	314	0.66	321	2.2
SJ10b	100	265	1	315	18.9
	200	345	0.77	365	5.8
	400	430	0.53	420	2.3
SJ11	100	318	1	309	2.8
	200	396	0.68	409	3.3
	400	498	0.68	481	3.4

3. VOLUMETRIC CHANGE ANALYSIS

According to the principle of effective stress, the equation describing volumetric changes in unsaturated soils may be described as,

$$\frac{\Delta V}{V} = c\Delta[(\sigma - u_a) + \chi(u_a - u_w)] \quad (5)$$

in which $\Delta V/V$ is the volumetric strain, and c is the drained compressibility of the soil matrix. If the applied stress is kept constant, the volumetric strain can be written as,

$$\frac{\Delta V}{V} = c\Delta[\chi(u_a - u_w)] \quad (6)$$

Similar to saturated soils, Equations (5) and (6) can only be used predictively if the stress path, the stress direction, and the stress type used in the laboratory in determination of c are the same as those expected in the field.

This aspect is particularly important in unsaturated soils as suction affects not only the effective stresses, but also the preconsolidation pressure or the yield stress of the soil skeleton. In fact, it has been this dual effect of suction on unsaturated soils behaviour, which appears to have caused so much confusion in the literature. Collapse, for instance, is a direct consequence of a shift in the preconsolidation pressure with a change in the value of suction. The shift in the preconsolidation pressure can also cause vastly different volumetric responses in identical soils, which are subject to the same effective stresses but different saturation conditions. In the literature, this is frequently interpreted as the breakdown of the effective stress concept as applied to unsaturated soils.

To avoid the complications arising from a shift in the preconsolidation pressure, all volume change test data presented in this paper were conducted in the elastic "recompression" range. By pre-loading the samples and working in the elastic range (i.e. at stresses below the preconsolidation pressure) the effects of any shift in the preconsolidation pressure on the deformation response are completely removed.

3.1 Test Programme

To investigate the application of the relationship given in Equation (3) to volumetric change data, a series of de-saturation tests were conducted on a laboratory prepared sample of kaolin. Kaolin powder was carefully mixed and wetted with sufficient distilled water using a spray gun to a high water content (close to liquid limit). The soil was then placed in a sealed plastic bag and allowed to cure for at least 2 days, to ensure uniform distribution of water throughout the soil and moisture equilibrium. The index properties of kaolin are given in Table 3. The soil water characteristics of the sample is given in Figure 5.

Table 3. Index Properties of kaolin used in volume change experiments.

Property	Value
Liquid Limit, LL (%)	63 %
Plastic Limit , PL (%)	30 %
Specific Gravity	2.61
Dry Density (Mg/m^3)	1.237
Percent finer than 10 µm	100 %
Percent finer than 3 µm	70 %
USCS Symbol	CH
Air Entry Value (kPa)	85

All volumetric change tests were conducted in a modified oedometer, developed at University of New South Wales, capable of independent measurement and control of pore air, pore water pressures, total deformation of the sample and water volume change within the sample. Similar to the modified triaxial cell, the pore air pressure in the modified oedometer was controlled through a coarse porous stone placed at the top of the sample, and the water pressure was controlled through a saturated high air entry value porous stone placed at the bottom of the sample. The suction within the sample was controlled using the axis translation technique, which is a common technique to prevent cavitation in the laboratory testing of unsaturated soils.

Following setting up in the equipment, the sample was consolidated to 600 kPa, in load increments to normal stresses 50, 100, 200, 400 and 600 kPa. The sample was then subjected to a series of loading and unloading cycles in the range of 100 kPa to 600 kPa, in order to obtain elastic response (Figure 6). Once an elastic response was achieved, the sample was unloaded to 100 kPa and suction was applied in 100 kPa increments.

The results of the tests along with a comparison of the predicted and calculated volumetric changes are given in Table 4. Graphical representations of the results are given in Figure 7. The air entry value on Figure 7 was obtained using the pressure plate technique and the soil water characteristic curve shown on Figure 5. The predicted volumetric changes values were calculated using the effective stress concept and the relationship given in (6). The effective stress parameter χ was obtained using the relationship given in Equation (3). The drained compressibility, *c*, was obtained using the saturated load response of the soil.

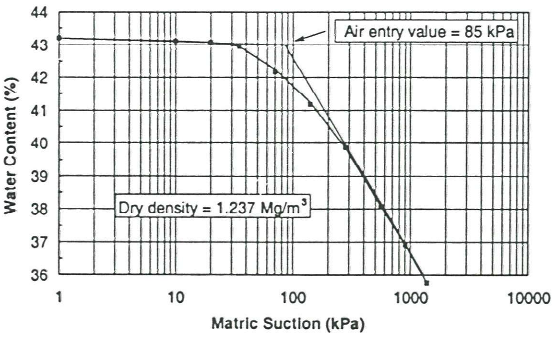


Figure 5. Soil water characteristic of kaolin.

As can be observed from Figure 7 and Table 4, extremely good agreement exists between the calculated and the measured values, with less than 9% error in all cases.

The implication of this is that quantitative predictions of deformation in unsaturated soils can be made using the effective stress concept. The model parameters will be exactly the same as those used in saturated soils, except for a single parameter (the air entry value) which can be determined in any soil physics laboratory. This is in contrast to the current models of unsaturated soils, which require extensive laboratory testing (Vanapalli et al, 1996).

For a complete characterisation of elasto-plasticity in unsaturated soils, also required will be a suction hardening function to define the evolution of the yield surface (or the shift in the preconsolidation pressure) due to a change in the suction. This latter aspect of unsaturated soils behaviour is currently the subject of an extensive programme of laboratory testing at the University of New South Wales.

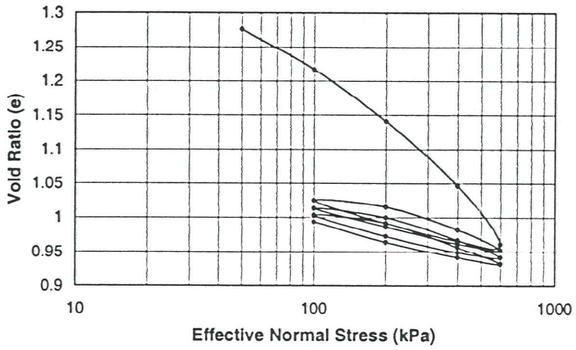


Figure 6. Loading and unloading consolidation tests on kaolin.

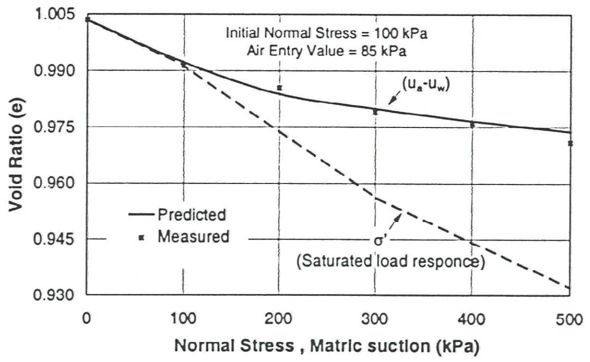


Figure 7. Comparison between predicted and measured volume change test results.

Table 4. Volume change test results.

Matric suction (kPa)	χ	Excess void ratio due to suction loading (Δe)		%
		Measured	Predicted	Error
100	0.914	0.0121	0.0112	7.4
200	0.625	0.0180	0.0196	8.9
300	0.500	0.0245	0.0232	4.1
400	0.427	0.0274	0.0268	2.2
500	0.377	0.0325	0.0296	8.9

4. CONCLUSIONS

The validity and the application of the relationship proposed by Khalili and Khabbaz (1996, 1998) between χ and the suction ratio is further examined through a new set of shear strength and volumetric change data. Extremely good agreement is obtained between the measured and predicted values in all cases.

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