

INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING



This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

<https://www.issmge.org/publications/online-library>

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

The paper was published in the proceedings of the 8th Australia New Zealand Conference on Geomechanics and was edited by Nihal Vitharana and Randal Colman. The conference was held in Hobart, Tasmania, Australia, 15 - 17 February 1999.

A Failure Criterion for Intact and Fissured Clays

M. D. Liu

B.E., M.Phil., Ph.D.

Research Assistant, University of Sydney, Australia

J. P. Carter

B.E., Ph.D., F.I.E.Aust, M.A.S.C.E., C.P.Eng.

Professor, University of Sydney, Australia

Summary A general failure criterion for clays is proposed. For intact clays the proposed formula unifies the three identifiable strengths of soils, *i.e.*, peak strength, critical state strength and residual strength, into a single isotropic failure criterion. By the introduction of two new parameters, describing the anisotropy and the degree of fissuring, the criterion may be extended to model the anisotropic strength of fissured clays. The capabilities of the proposed criterion are demonstrated and evaluated by comparing its predictions with experimental data. It is obvious that soils with different structures need different numbers of parameters to define their failure criteria. Indeed, the complexity and accuracy of the criterion can be selected to match the available data.

1. INTRODUCTION

The term "soil strength" is often used to define at least two different categories of soil behaviour and strength mobilisation: the peak strength and the post-peak strength. The post-peak strength of clays can be sub-divided into the critical state strength and the residual strength. The strengths of clays at these three distinct conditions are recognised as being of great importance in engineering practice, *e.g.* Morgenstern, 1967; Skempton, 1985. Stiff clays *in situ* often contain discontinuities such as fissures. The mechanical behaviour of a soil mass is generally affected by the presence of such fissures, and can even be dominated by them, especially with respect to its shearing resistance and its permeability (Skempton & Petley, 1967; Liu *et al.*, 1998). However, there appears to be no consistent theoretical framework to describe the strength variation of clay from the peak strength condition, to critical state strength and then to the residual strength condition, and indeed no formulation to describe the strength if fissures are present within a soil mass. This paper presents a study of the three different strengths of clays. An anisotropic failure criterion is proposed, which unifies the three strengths into one operational strength, the magnitude of which is controlled by the stress and strain states of the soil and the plastic distortional work, as well as the soil mineralogy. The proposed criterion may also be modified to account for the influence of fissures. Characteristics of the proposed strength criterion are demonstrated, and an evaluation of the criterion is made.

In this work it is assumed that the individual fissures are discontinuous and small relative to the scale of the problem so that a soil mass with fissures

can be treated as a continuum with "equivalent" anisotropic material parameters.

2. STRENGTH OF CLAYS

2.1 Critical State

The concept of a critical state of deformation was first introduced by Casagrande (1936), and was adopted in the establishment of the Critical State Soil Mechanics (Schofield & Wroth, 1968). It is a state at which soil behaves as a frictional fluid without further change in the stress state and voids ratio. By definition, soil has no structure at critical state. The concept of steady state deformation of soil was also introduced by Casagrande (1949). It is a state at which soil deforms at constant velocity, with the stress state and voids ratio remaining the same. The state exists only so long as the deformation continues, and soil in this state has a unique structure (Poulos, 1981). There has long been debate on whether the critical state and the steady state are actually the same state (Casagrande, 1975; Poulos, 1981). Recent comprehensive studies suggest that they are essentially the same (Been *et al.*, 1991; Ishihara, 1993; Chu, 1995), although the term "steady state" is often used in reference to the behaviour of sand and the term "critical state" is used in reference to clay. It has been demonstrated that a critical state exists for other geo-materials, such as naturally and artificially cemented sands, and soft and hard rocks (Carter & Airey, 1994; Novello *et al.*, 1995). The critical state of deformation may be defined as follows. All soils have critical states of deformation, at which they can be continuously distorted with their stress state and voids ratio remaining constant. There exists a unique relationship between the voids ratio and the effective mean stress at critical states, which is mainly controlled by soil mineralogy. At critical

state a soil behaves as a perfectly plastic material, and it has no special structure. The characteristics of the critical state of deformation are:

1. Soil at critical state has no further resistance to shearing, and no further volumetric deformation is produced.
2. At critical state the voids ratio of a given soil is dependent on the mean stress only. As a result, the critical state line in the $e-p'$ space is dependent on the soil mineralogy only, irrespective of the stress paths during loading and the original soil structure. The critical state line is a fundamental intrinsic property of a soil.
3. Soil deforming at critical state is continuously remoulded. Consequently, it does not exhibit any particular structure.

Experimental data overwhelmingly indicate that the mechanical properties of soil at the critical state are independent of the original structure that may have arisen from sample preparation or depositional history and the subsequent modification of the structure resulting from test loading.

2.2 Residual Strength

The concept of residual strength received careful study after investigations in the late 1950s on landslides showed that the shearing resistance of soil in a number of cases was much smaller than the "final" critical state strength measured in the laboratory (Gould, 1960; Skempton 1964). Residual strength is defined as the shear strength of a soil that can be mobilised on a polished sliding surface, after it has been formed through the soil due to the alignment of its platy particles. For any given soil it is the minimum strength attainable. There are four major aspects of the residual strength, viz.

1. Soil must have enough plate-like particles so that a smooth slickensided surface can be formed.
2. The sliding surface of well-aligned soil particles must exist for the residual strength to be mobilised.
3. The sliding surface of well-aligned platy particles can facilitate residual failure only along that surface.
4. The residual sliding surface once formed is usually not modified by subsequent deformations of relatively small magnitude.

The residual strength is primarily dependent on the mineralogy of the soil, which includes the chemistry of the pore fluid (Lupini *et al*, 1981). For most practical problems the pore fluid is water, and

the chemistry usually remains approximately the same for a given site. In this case the dominant factor influencing the residual strength will be the clay fraction, ω_c , as defined by Skempton (1964)

$$\omega_c = \frac{G_{0.002}}{G} \times 100\% \quad (1)$$

where $G_{0.002}$ is the weight of the clay particles less than 0.002 mm in size, and G is the total weight of the soil sample. The influence of stress level on the residual strength has also been studied, e.g., Sinclair *et al* (1967), and Lupini *et al* (1981). It was found that an influence of stress level does exist, but it is of secondary importance in comparison with the soil mineralogy.

2.3 Peak Strength

Unlike the critical state or residual state strength, the peak strength of a soil is not an intrinsic material property. For example, Been and Jefferies (1985) demonstrated the dependence of peak strength on the state parameter, Φ . Liu and Nakai (1992) carried out a series of tests on Toyoura sand and demonstrated that the peak friction angle for Toyoura sand at a given initial stress and strain state changed from 32°, the critical state strength, to 38.5°, depending on the stress path of the test.

3. ISOTROPIC SOIL STRENGTH

In the following formulation, the definitions of the terms, q , p' , η , M and the friction angle ϕ are the same as those used by Schofield and Wroth (1968). q is the deviator stress, p' is the effective mean stress, η is the stress ratio q/p' , and M is the value of η at critical state (or at the residual strength).

3.1 Post-Peak Strength of Clays

Based on the experimental data from researchers such as Lupini *et al*, (1981), and Skempton (1985), the residual strength of clays M_r is assumed to be

$$M_r = M_{rl} + (M_{cs} - M_{rl}) \prod_1 \left(\frac{5 - 0.1\omega_c}{3} \right) \quad (2)$$

where M_{rl} is the lower bound of the residual strength, ω_c is input as a percentage, M_{cs} is the critical state strength, and operation $\prod_1(x)$ with respect to a variable x is defined as

$$\prod_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \quad (3)$$

The platy particles are partially orientated for a soil state between the critical state and the residual state. A new parameter ω_o , the degree of particle orientation, is therefore introduced. It is postulated that soil strength varies linearly with particle orientation ω_o . Therefore, the strength of a soil at a state between the critical state and the residual state can be expressed as

$$\eta_p = M_{cs} - (M_{cs} - M_r) \prod_1(\omega_o) \quad (4)$$

It is suggested that the degree of particle orientation can be linked with the plastic distortional work, W_d^p , by the following equation

$$\omega_o = \frac{\ln W_d^p}{\ln W^p} \text{ for } W_d^p \leq W^p \quad (5)$$

where W^p is the magnitude of plastic distortional work needed to form the sliding surface for the residual state. The maximum value of ω_o is 1 when the residual sliding surface has been formed.

It should be pointed out that even though equation (4) describes a transition to the residual strength, the equation is not a rigorous anisotropic strength criterion, because it describes the shear strength of soil along the residual sliding surface only. A rigorous anisotropic failure criterion should be able to describe the variation of soil strength for shearing in all possible directions.

3.2 A General Isotropic Failure Criterion for Clays

Been and Jefferies (1985) successfully showed that the influence of stress level and voids ratio on soil peak strength can be unified into one quantity, the state parameter Φ , defined here as

$$\Phi = e + \lambda \ln p' - e_{cs} \quad (6)$$

where e is the voids ratio, and e_{cs} and λ are the standard soil parameters defining the position and slope of the critical state line in e - $\ln p'$ space.

After examining the available experimental data, Liu and Carter (1998) proposed the following expression for the variation with the state parameter of the peak strength, defined in terms of the ratio of the deviator stress to the effective mean stress at the peak, $\eta_p (=q/p')$, i.e.

$$\eta_p = (1 - \Phi) M_{cs} \quad (7)$$

Combining equations (2), (4) and (7), the following general isotropic failure criterion for clay is proposed

$$\begin{aligned} \eta_p = & (1 - \Phi) M_{csl} - (M_{csl} - M_{rl}) \prod_1(\omega_o) \\ & + (1 - \Phi) [(M_{csu} - M_{csl}) + (M_{csl} - \\ & M_{rl}) \prod_1(\omega_o)] \prod_1\left(\frac{5 - 0.1\omega_c}{3}\right) \end{aligned} \quad (8)$$

where M_{csl} and M_{csu} are the lower and upper bounds of the critical state strength with variation in the clay fraction. Equation (8) describes the peak strength of a clay, or its "operational" strength, as a function of clay fraction, state parameter, and the distortional plastic work applied to the soil sample.

4. INFLUENCE OF FISSURES

Liu *et al* (1998) observed the following features of the strength of a fissured soil mass.

1. The peak strength of a fissured clay varies between the peak strength of the intact parent clay and the post-peak strength of the clay, either the critical state strength or the residual strength.
2. With an increase in the degree of fissuring, the peak strength of a soil mass changes from the highest strength of the intact clay to the lowest strength of the completely fissured clay.
3. The peak strength (and stiffness) decreases with the increase in the ratio of the mobilised fissure surface to the overall failure surface. For example, in the case of a soil mass with one set of planar fissures, the highest resistance to shearing occurs when the failure surface is perpendicular to the fissures, and the lowest resistance occurs when the failure surface is coincident with a fissure surface.
4. Fissures with rough surfaces normally have a peak strength higher than their post-peak strength. Soil with non-planar fissures generally has shear strength greater than a soil with planar fissures.

The following factors influence the strength of a fissured soil mass. The first set of factors arises from features of the parent clay, which include: (a) soil mineralogy, (b) voids ratio, and (c) anisotropy associated with the parent material, or the structure of the clay. The second set arises from features of the fissures, which include: (a) intensity of fissuring defined in terms of the area of fissures per unit volume, (b) geometry of the fissure, (c) smoothness of the surface of fissures, and (d) anisotropy associated with the fissures.

The influence of these factors has been discussed in qualitative detail by Liu *et al* (1998). However, quantitative description of the influence of individual factors and the interaction among these

factors is extremely complicated. It is highly unlikely that appropriate measurements can or will be carried out for such a detailed description to be useful in practice. A rational way to model the effects on soil properties of fissures is to study the isotropic strength reduction through parameters describing isotropic states of the soil mass, and to study the distortion through parameters describing anisotropic states of the soil mass. Accordingly, in this study parameters describing the fissures are those that can be derived directly from the variation in mechanical properties caused by the presence of fissures and can be determined from laboratory tests, not those describing the geometric features of fissures. Of course, the former are dependent on the latter. Hence, it is proposed that the overall effect of the fissures should be represented by three factors.

1. The degree of fissuring, ω_f , describes the isotropic reduction of soil strength. If there is no fissuring, then $\omega_f = 0$. If the soil is completely fissured, then $\omega_f = 1$. A soil is completely fissured if the strength of the soil mass is not affected by any increase of fissures.

2. The anisotropy parameter a_l is a scalar quantity introduced to represent the effect of strength anisotropy for failure along a given plane l . Its value varies with the direction of shearing. For simplicity it is assumed that sliding in any direction along plane l involves mobilisation of the same shear strength. In general, the anisotropy of a fissured soil mass is dependent on both the anisotropy of the parent clay and the presence of fissures with preferred orientations. A major simplification is adopted in this paper, in that the combined effect of both the parent clay and the fissures can be represented by a single parameter a_l .

3. The degree of particle orientation along a sliding surface ω_o . This parameter has already been used to describe the residual strength.

Equation (8) can be modified to allow for the effect of anisotropy and fissuring. The following equation for the strength of fissured clay is proposed

$$\eta_p = \left[M_{rl} + (M_{csu} - M_{rl}) \prod_1 \left(\frac{5 - 0.1\omega_c}{3} \right) \right] + (M_{csl} - M_{rl})(1 - \omega_o) \left[1 - \prod_1 \left(\frac{5 - 0.1\omega_c}{3} \right) \right] - a_l \Phi (1 - \omega_f)(1 - \omega_o) \times \left[M_{csl} + (M_{csu} - M_{csl}) \prod_1 \left(\frac{5 - 0.1\omega_c}{3} \right) \right] \quad (9)$$

It is seen that equation (8) is a special case of equation (9), corresponding to $a_l = 1$ and $\omega_f = 0$. In equation (9), the first term represents the residual strength, which is the minimum strength attainable for any soil. The second term represents the reduction from the critical state strength to the residual strength, which is dependent on the degree of particle orientation only. For soil with insufficient platy particles to form a smooth sliding plane, M_r is equal to M_{cs} , and in this case the sum of the first two items is always equal to the critical state strength. The third term is contributed by the state parameter Φ , and it is this particular part only that is affected by the degree of fissuring and the anisotropy of the fissured soil mass.

4.1 Anisotropy Parameter

It is appropriate to provide a brief analysis of the scalar anisotropy parameter a_l . For simplicity, it is assumed in this paper that the strength of a fissured soil mass is cross-anisotropic. The three principal axes of the anisotropy can be denoted by vectors $A1$ and $A3$ and $A3$. Suppose also that the values of a_l , corresponding to failure by sliding in the three principal direction are $[1+a_1, 1+a_3, 1+a_3]$. Because the parameter a_l represents a purely distortional effect, the following constraint can be imposed

$$a_1 + 2a_3 = 0 \quad (10)$$

A plane may be represented by its normal and the value for a_l for sliding along the plane represented by the normal l can be calculated as follows (Fig. 1),

$$a_l = 1 + a_1 \cos^2 \langle l, A1 \rangle + a_3 \cos^2 \langle l, A3 \rangle \quad (11)$$

where $\langle \rangle$ denotes the angle between the normals of two planes.

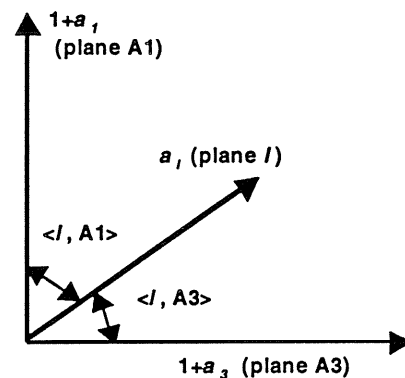


Figure 1. Parameter a_l for cross anisotropy.
(A1 and A3: the principal axes).

5. EVALUATION OF THE CRITERION

5.1 Critical State Strength and Residual Strength

At a critical state of deformation, $\Phi=0$, and $\omega_o=0$. At the residual strength state, $\omega_o=1$, and $\Phi=0$. According to the proposed equation (9), soil strengths at these two states are dependent on the clay fraction only. To describe the variation of critical state friction angle and residual state friction angle with clay fraction, as presented by Skempton (1985), the following values for the soil parameters are adopted: $M_{csf}=0.819$, $M_{csu}=1.221$, $M_{rl}=0.236$. A comparison between the experimental data of Skempton and the theoretical simulation described by equation (9) using the values listed above is shown in Fig. 2. The prediction of the proposed criterion matches the experimental data well.

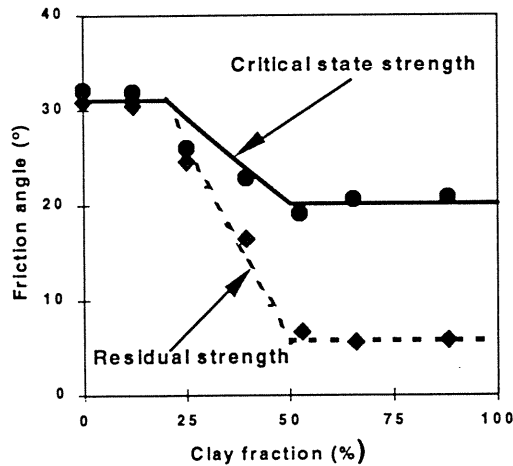


Figure 2. Variation of critical state and residual strength with clay fraction (test data after Skempton, 1985).

5.2 Peak Strength of Intact Clays

For intact clay, $\omega_o=0$, and $\omega_f=0$. In this case the peak strength criterion can be simplified from equation (9) as

$$\frac{\eta_p}{M_{cs}} = 1 - \Phi \quad (12)$$

In order to predict the boundaries of the experimental peak strength data summarised by Been *et al* (1985), two values for critical state friction angles were used, viz., $\phi_{cs}=34.6^\circ$, and $\phi_{cs}=30.5^\circ$. Predictions from equation (12) using these values are shown in Fig. 3. The proposed linear relationship represents satisfactorily the experimental variation of peak strength with the state parameter.

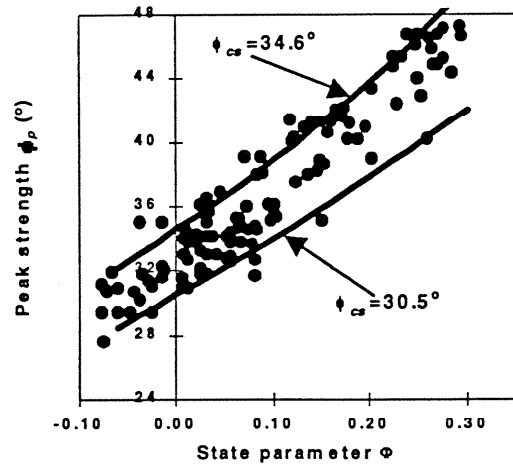


Figure 3. Peak strength variation with Φ (test data from Been *et al*, 1985).

5.3 Strength Variation with Particle Orientation and Degree of Fissuring

A qualitative illustration of the influence of fissuring on clay strength is shown in Fig. 4 for a soil that has positive values of the state parameter, Φ .

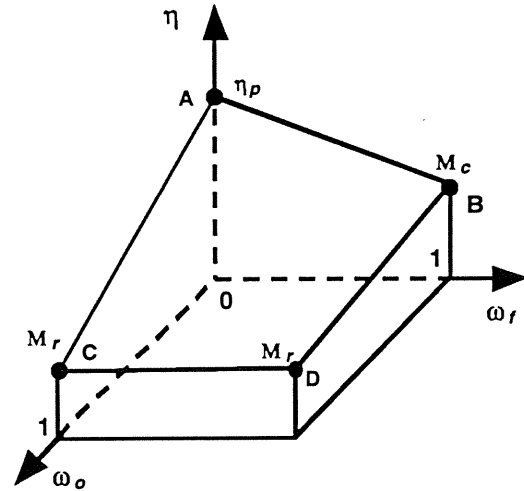


Figure 4. Variation of clay strength with degree of fissuring and particle orientation.

The peak strength, η , is plotted against the two variables ω_f , the degree of fissuring, and ω_o , the degree of particle orientation. Soil has its maximum resistance at point A when there is no fissuring and no special orientation for the particles, i.e. $\omega_f=\omega_o=0$. Soil has a critical state strength at point B when the soil mass is completely fissured and soil particles have no special orientation, i.e. $\omega_f=1$ and $\omega_o=0$. Soil reaches the residual strength at point D when the soil mass is completely fissured and its particles have formed a smooth sliding surface, i.e. $\omega_f=\omega_o=1$. Even though the whole soil mass is not fissured at point C, the resistance is also equal to the residual strength, as soil particles have

formed a smooth surface along the sliding plane and $\Phi=0$.

5.4 Effect of Anisotropy

The undrained shear strength of lightly overconsolidated Welland clay measured in a conventional triaxial apparatus is shown in Fig. 5. Details of the tests have been presented by Lo (1965). The undrained shear strengths of samples obtained from a given depth have been normalised by the maximum strength $s_{u,max}$ measured at that depth. The boundaries of the normalised strengths are indicated by the solid squares in the Fig. 5. θ represents the angle between the axis of the cylindrical soil specimens and the vertical direction. It is assumed that the angle between the normals of the failure planes of two samples is numerically equal to the angle between the axes of the two cylindrical specimens, as they were *in situ*. Samples with the maximum and minimum strengths are respectively the vertically cut sample, i.e., $\theta=0^\circ$, and the horizontally cut sample, i.e., $\theta=90^\circ$. It is assumed that the values of the anisotropy parameter a_i for the vertically and the horizontally cut samples are a_1 and a_3 respectively.

Natural Welland clay can be assumed to be a cross-anisotropic material. Lo (1965) reported no fissures in the clay, $\omega_p=0$, and specimens tested in the laboratory did not undergo any large shear deformation, $\omega_o=0$. Samples were obtained from different depths at the same location. The parameter a_3 for soil at different depths at the same location may be assumed to have the same value because of the similar geological history of all samples.

Considering equations (10) and (11), after some manipulation of the general equation (9), an undrained strength equation for Welland clay can be written as

$$q_u = \eta_p p' = p'(1 - \Phi)M_{cs} + a_3 \Phi p' M_{cs} \quad (13)$$

$$(2 \cos^2 \theta - \sin^2 \theta)$$

where q_u is the deviatoric shear stress at failure. It follows that the maximum shear strength $q_{u,max}$ can be written as

$$q_{u,max} = p'_m (1 - \Phi_m) M_{cs} - a_3 \Phi_m p'_m M_{cs} \quad (14)$$

In equation (14), the subscript m denotes the value corresponding to the test in which the maximum shear strength was measured. The initial values of Φ for specimens from the same depth are assumed to be the same. Because of anisotropy, the values of Φ at the peak state are usually different for samples with different θ . p' at the peak strength also varies

for different specimens. The normalised undrained shear strength is thus

$$\frac{q_u}{q_{u,max}} = \frac{(1 - \Phi)}{(1 - \Phi_m - a_3 \Phi_m)} \left(\frac{p'}{p'_m} \right) + \left(2 \cos^2 \theta - \sin^2 \theta \right) \frac{a_3 \Phi}{(1 - \Phi_m - a_3 \Phi_m)} \left(\frac{p'}{p'_m} \right) \quad (15)$$

For isotropic samples, $a_3=0$, $p'=p'_m$, and $\Phi=\Phi_m$, and therefore, as required, $q_u = q_{u,max}$

Based on the form of equation (15) and the experimental data, the following approximate empirical expression is proposed to describe the undrained strength of the anisotropic Welland clay

$$\frac{q_u}{q_{u,max}} = 0.787 + 0.107(2 \cos^2 \theta - \sin^2 \theta) \quad (16)$$

The simulation made using equation (16) is also shown in Fig. 5, and is marked by a solid line. It is seen that the anisotropic strength of the naturally structured soil can be described reasonably satisfactorily by the approximate equation. A similar pattern of anisotropic strength has been observed widely for other clays, sands, and fissured clays (Ladd *et al*, 1977, McGown *et al*, 1974).

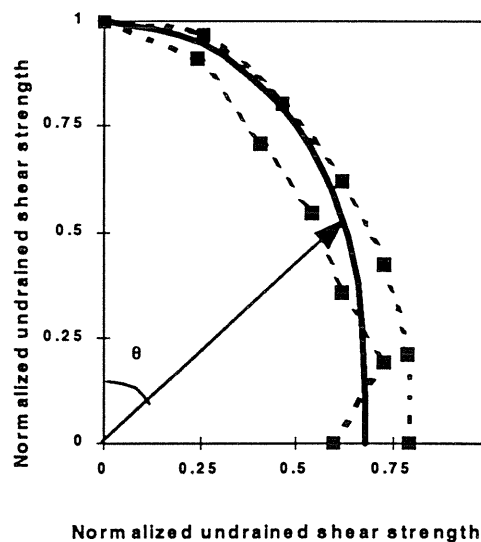


Figure 5. Anisotropic strength of Welland clay (test data from Lo, 1965)

6. CONCLUSIONS

A study of the shear strength of clays has been presented and an anisotropic failure criterion was proposed. This formula unifies the strength of these soils at the peak, critical state and residual conditions into one operational strength criterion. The shear strength of a clay is described as a function of soil mineralogy, the state parameter, soil

anisotropy, and particle orientation. A clay deposit with fissures has been treated as a continuum, and the overall effect of fissuring on soil strength is represented by the degree of fissuring, ω_f , the anisotropy of a fissured soil mass, a_i , and the degree of particle orientation along a sliding surface, ω_o . The proposed criterion has been extended to describe the strength of a fissured soil mass, as well as non-fissured soils.

Important characteristics of the proposed general failure criterion have been demonstrated. These include its ability to describe the variation of the critical state strength and residual strength with clay fraction, the variation of peak strength with state parameter and soil anisotropy, and the effect on soil strength of particle orientation and the degree of fissuring. Some of the predictions of the theoretical criterion have been compared with experimental data. It has been demonstrated that the proposed strength criterion has the capacity to describe accurately the operational strength of both intact clay and a soil mass with fissures.

7. ACKNOWLEDGEMENTS

A grant from the Australian Research Council in support of the research described in this paper is gratefully acknowledged.

8. REFERENCES

- Been K., Jefferies M. G. and Hachey J. (1991), "The critical state of sands", *Geotechnique*, Vol. 41(3), pp. 365-381.
- Been K. and Jefferies M. G. (1985), "A state parameter for sands", *Geotechnique*, Vol. 35(1), pp. 99-112.
- Bolton M. (1986), "The strength and dilatancy of sands", *Geotechnique*, Vol. 36(1), pp. 65-78.
- Carter J. P. and Airey D. W. (1994), "The engineering behaviour of cemented marine carbonate soils", *Geotechnical Engineering: Emerging Trends in Designs and Practice*, Saxena (ed), pp. 65-101.
- Casagrande A. (1975), "Liquefaction and cyclic deformation of sands", *Proc. 5th Pan-American Conference on Soil Mechanics*, Buenos Aires, Vol. 5, pp. 80-133.
- Casagrande A. (1949), "Discussion of 'Excavation of slopes in the Panama Canal' by Binger *et al*", *Trans. ASCE*, Vol. 114, pp. 870-874.
- Casagrande A. (1936), "Characteristics of cohesionless soils affecting the stability of slopes on earth fills", *J. Boston Society of Civil Engineers*, Vol. 23(1), pp. 13-32.
- Chu J. (1995), "An experimental examination of the critical state and other similar concepts for granular soils", *Canadian Geotechnical J.*, Vol. 32(5), pp. 1065-1075.
- Gould J. P. (1960), "A study of shear failure in certain Tertiary marine sediments", *Proc. Research Conference on Shear Strength of Cohesive Soils*, Boulder, pp. 615-642.
- Ishihara K. (1993), "Liquefaction and flow failure during earthquakes", *Geotechnique*, Vol. 43(3), pp. 351-415.
- Ladd C. C., Foott R., Ishihara K., Schlosser F., and Poulos H. G. (1977), "Stress-deformation and strength characteristics", *Proc. 9th Int. Conference Soil Mechanics and Foundation Engineering*, Vol. 2, pp. 421-494.
- Liu M. D. and Nakai T. (1992), "Softening of sand", *Proc. 27th Conference Japanese Society of Soil Mechanics and Foundation Engineering*, pp. 523-526.
- Liu M. D. and Carter J. P. (1998), "A general isotropic failure criterion for soils including peak, critical state, and residual strengths", Research Report, Sydney University.
- Liu M. D., Carter J. P., and Thorne C. P. (1998), "A failure criterion for fissured clay", Research Report, Sydney University.
- Lo K. Y. (1965), "Stability of slopes in anisotropic soils", *J. Soil Mechanics and Foundation Engineering Div.*, ASCE, Vol. 91(4), pp. 86-105.
- Lupini J. F., Skinner A. E. and Vaughan P. R. (1981), "The drained residual strength of cohesive soils", *Geotechnique*, Vol. 31(2), pp. 181-213.
- McGown A., Saldivar-Sali A., and Radwan A. M. (1974), "Fissure patterns and slope failures in till at Hurlford, Ayrshire", *Q. J. Engineering Geology*, Vol. 7, pp. 1-26.
- Morgenstern N. R. (1967), "Shear strength of stiff clay", *Proc. Geotechnical Conference*, Oslo, Vol. 2, pp. 59-69.
- Novello E. and Johnston I. W. (1995), "Geotechnical materials and the critical state", *Geotechnique*, Vol. 45(2), pp. 223-235.
- Oda M. (1988), "Introduction of inherent anisotropy of soils in the yield function", *Micromechanics of Granular materials*, Stake *et al* (ed), pp. 81-90.
- Poulos S. J. (1981), "The steady state of deformation", *J. Geotechnical Engineering*, ASCE, Vol. 107(5), pp. 16241-16233.
- Schofield A. N. and Wroth C. P. (1968), *Critical State Soil Mechanics*, MacGraw-Hill, London, 310p.
- Sinclair S. R. and Brooker E. W. (1967), "The shear strength of Edmonton Shale", *Proc. Geotechnical Conference*, Oslo, Vol. 1, pp. 295-299.
- Skempton A. W. (1985), "Residual strength of clays in landslides, folded strata and the laboratory", *Geotechnique*, Vol. 35(1), pp. 1-18.
- Skempton A. W. and Petley D. J. (1967), "The strength along structural discontinuities in stiff clays", *Proc. Geot. Conf.*, Oslo, Vol. 2, pp. 29-46.
- Skempton A. W. (1964), "Long term stability of clay slopes", *Geotechnique*, Vol. 14 (2), pp. 77-101.