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Impact Strength of Sandstone

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SUMMARY The results of impact split-cylinder tests and impact uniaxial compression tests on sandstone carried out for the purpose of clarifying the behavior of rocks by impact loading are reported. For the analysis of the test results, bold assumptions were made, but the appropriate analytical results have been obtained.

1 INTRODUCTION

The impact strength and the breaking mechanism due to impact of such materials as rocks and concrete have not yet been clarified completely because there has been little need to do so. To enable a unified interpretation regarding the diversity of the breaking mode due to impact loading, the establishment of general breaking criteria is necessary. The objectives of this research were the macroscopic analysis of the breaking behavior by impact loading of hard sandstone, which is a brittle material, and the determination of the impact strength of rocks by the energy balance. For the tests, the cylindrical test pieces were used in all cases, and the axial impact load corresponding to uniaxial compression test and the impact load in diametral direction, corresponding to split-cylinder test, were applied. In the impact split-cylinder test, the splitting arose in the central part along the diameter of the columns, and crushing occurred outside. Consequently, separate test pieces were used for each test. The extent of this crushing region depended on not only the intensity of impact energy but also the breaking strength of rocks. Considering that the sandstone used for the tests was close to isotropic and homogeneous, the failure criterion of Drucker-Prager with two parameters, which have been widely used for rocks, were adopted.

2 EXPERIMENTAL METHOD AND RESULTS

The specimens used for this experiment were Kawazu tertiary sandstone, produced in Shizuoka Prefecture and greyish white in colour. The dry weight per unit volume was 18.62 Ncm^{-3}, and the specific weight of rock particles was 2.77. Both impact breaking tests and static breaking tests were carried out. For the impact test, a drop weight type testing apparatus was used, whereas for the static test, a 98.07 kN universal compression testing machine was used.

2.1 Impact test

In the impact test, impact split-cylinder test and impact uniaxial compression test were carried out. For each test piece, the velocity of elastic waves was measured, the dynamic modulus of rigidity G_D was determined, and the dynamic modulus of elasticity was obtained by assuming the Poisson's ratio to be 0.2. The dimensions of the test pieces used for the impact split-cylinder test and impact uniaxial compression test were ϕ5cm x L5cm and ϕ3cm x L6cm, respectively. Prior to the tests, the weight of a falling weight with which the test pieces were broken by one drop and the lower limit of its falling height were determined by trial. For the axial splitting, the weight of 29.4N and the falling height of 50 cm, while for the uniaxial compression the weight of 49N and the falling height of 40 cm were obtained. For comparison's sake, the split-cylinder test with a weight of 49N was also carried out and the impact testing apparatus used is shown in Figure 1. The number of the test pieces was 10 each. After the impact splitting, the distance from the centre of the test pieces to the tip of the crushing region (splitting length) γ_0 was measured, and the form of splitting was recorded (Figure 2). The mean values of the measued quantities are shown in Table I.

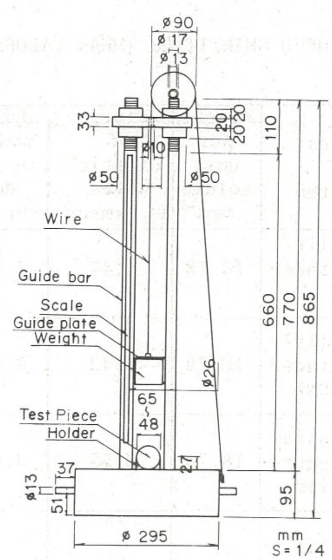

Figure 1. Impact testing appratus

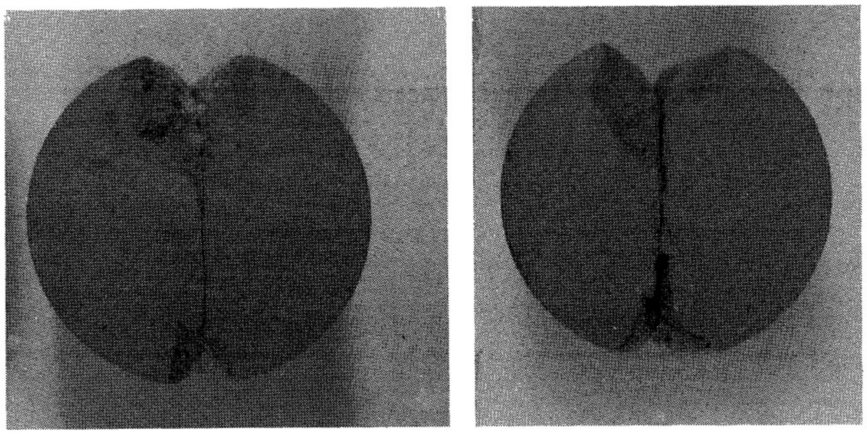

Figure 2-a Impact test

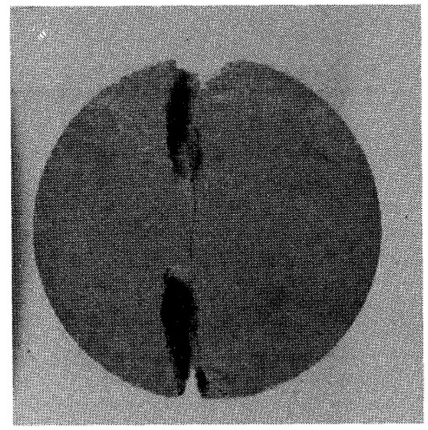

Figure 2-b Static test

2.2 Static test

The static test was performed for the purpose of obtaining the reference data on the impact strength characteristics of the material. For each test piece, the velocity of elastic waves was measured similarly to the case of impact test. Uniaxial compression tests were carried out on 31 test pieces of dimensions ϕ5cm x L10cm. At the same time, the strain measurement was performed. The mean values of the modulus of elasticity, Poisson's ratio and uniaxial compression strength of the rocks obtained as the results of test are shown in Table I. The dimensions of the test pieces used for the split-cylinder test were ϕ5cm x L5cm. The number of the test pieces was 39. From the average of the maximum load at the time of splitting, the mean splitting strength was 3.7 MPa. After breaking, the splitting length γ_0 was determined and the form of splitting was recorded (Figure 2). The mean value gained from the 39 test results are shown in Table I.

3 APPLICATION OF FAILURE CRITERION OF DRUCKER-PRAGER

Yield criteria proposed by Drucker and Prager (1952) is widely used at present as the failure criterion for soil, rocks and concrete. The criterion of Drucker-Prager is expressed as

$$\alpha I_1 + \sqrt{J_2} = k \tag{1}$$

Here, α and k are the strength constants determined by the properties of rocks. I_1 and J_2 in Equation (1) as are as follows.

$$\begin{aligned} I_1 &= \sigma x + \sigma y + \sigma z \\ J_2 &= \frac{1}{2}\left(\sigma x^2 + \sigma y^2 + \sigma z^2\right) + \tau yz^2 + \tau zx^2 \\ &\quad + \tau xy^2 - \frac{1}{6}\left(\sigma x + \sigma y + \sigma z\right)^2 \end{aligned} \tag{2}$$

TABLE I

MEASURED QUANTITIES (MEAN VALUES) IN SPLIT-CYLINDER TEST AND UNIAXIAL COMPRESSION TEST

Kind	Test item	Weight per unit volume γ Ncm^{-3}	Velocity of elastic waves V_p $kmsec^{-1}$	Dynamic modulus of rigidity G_D MPa	Dynamic modulus of elasticity E_D MPa	Static modulus of elasticity E MPa	Static splitting load P_f N	Splitting Strength MPa	Splitting length γ_0 cm	Remarks
Impact test	Split-cylinder (29.4N)	18.78	3.47	$\times 10^3$ 8.80	$\times 10^3$ 17.2	—	—	—	1.85	Height of fall of weight 50 cm
	Split-cylinder (49N)	18.78	3.42	8.45	21.7	—	—	—	1.32	" 50 cm
	Uniaxial compres-sion	18.70	3.55	9.03	19.7	—	—	—		" 40 cm
Static test	Split-cylinder	18.71	3.16	7.14	17.2	—	14122 (*2822)	** 3.70	1.68	* per unit length ** Determined by P/πR
	Uniaxial compres-sion	18.64	3.38	8.20	19.7	$\times 10^3$ 5.47	73755	37.5	—	

In Section 2, the results of split-cylinder and uniaxial compression tests were shown. The stress condition in a test piece by split-cylinder test is determined by the following equations as the stresses in a cross section when a force P is applied across a diameter of a cylinder of unit length.

$$\sigma x = -\frac{2P}{\pi}\left\{\frac{\cos\theta_1 \sin^2\theta_1}{\gamma_1} + \frac{\cos\theta_2 \sin^2\theta_2}{\gamma_2}\right\} + \frac{P}{\pi R}$$

$$\sigma y = -\frac{2P}{\pi}\left\{\frac{\cos^3\theta_1}{\gamma_1} + \frac{\cos^3\theta_2}{\gamma_2}\right\} + \frac{P}{\pi R} \qquad (3)$$

$$\tau xy = \frac{2P}{\pi}\left\{\frac{\cos^2\theta_1 \sin\theta_1}{\gamma_1} - \frac{\cos^2\theta_2 \sin\theta_2}{\gamma_2}\right\}$$

Here, θ_1, θ_2, γ_1 and γ_2 are as shown in Figure 3. By directly using the results of the static tests Equations (1) and (3), α and k can be obtained immediately. The procedure of this analysis is to be mentioned later, and first, the outline of the procedure of analysis of the impact strength α_D and k_D is shown.

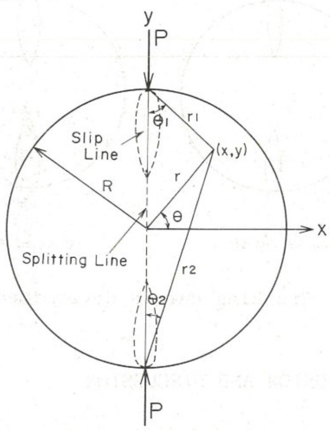

Figure 3. Loading point and arbitrary point in column split-cylinder test

3.1 Analysis of impact strength

In an impact split-cylinder test, a part of a rock was crushed, and another part was split by pressure. Figure 4 is the figure schematically showing the state of breaking in a cross section of a test piece.

① is the crushing region. The strain energy arisen up to the breaking in this region is the sum of elastic strain energy and inelastic strain energy. When it is assumed that the total sum W_{PC} of the total strain energy in crushing region and dissipated energy corresponds to β times the elastic strain energy, the following Equation (4) is obtained.

$$W_{PC} = 4\int_0^L dZ\int_{\theta c}^{\frac{\pi}{2}}\int_{\gamma_0}^{R} V_c r\,dr\,d\theta$$

$$= \frac{2\beta k^2{}_D L}{G}\int_{\gamma_0}^{R}\frac{\left(\frac{\pi}{2}-\theta_c\right)\left[2\left(\frac{1}{1+\nu}\right)\left(R^2+\gamma^2\right)+\left(R^2-\gamma^2\right)\left(3R^2+\gamma^2\right)\right]\gamma dr}{\left\{-2\alpha_D\left(R^2+\gamma^2\right)+\sqrt{\frac{4}{3}\left(R^2+\gamma^2\right)^2+\left(R^2-\gamma^2\right)\left(3R^2+\gamma^2\right)}\right\}^2} \qquad (4)$$

The angle θ_c as seen in the lower limit of integration in the equation is the opening angle from a point on a horizontal diameter to a point on the boundary line of a crushing region (crushing curves).

② is the splitting region. The energy consumed for separating a rock along a diameter is assumed to be small compared with the elastic strain energy in this region. When the distance from the centre of a cylinder to the intersection of a diameter connecting the loading points with crushing curves is denoted by r_o, the elastic strain energy W_e in this region becomes

$$|W_e|_{\gamma_0 \geqq \gamma_1} = 2L\int_0^{\gamma_0}\gamma d\gamma\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V_e d\theta$$

$$= \frac{P^2L}{2\pi R^2 G}\left[2\left(\frac{1}{1+\nu}\right)\left\{R^2\log\frac{R^2+\gamma_0{}^2}{R^2-\gamma_0{}^2} - \gamma_0{}^2\right\}\right.$$

$$\left. + \gamma_0{}^2 + \frac{2R^4}{R^2+\gamma_0{}^2} - \frac{2R^6}{(R^2+\gamma_0{}^2)^2}\right] \qquad (5)$$

③ is a non-crushing region similarly to ②, and it is regarded as being almost in elastic state. The elastic strain energy $|W_e|_{R\geqq\gamma\geqq\gamma_0}$ in this region is determined as follows by the integration for the annular region except the crushing region.

$$|W_e|_{R\geqq\gamma\geqq\gamma_0} = 4L\int_{\gamma_0}^{R}\int_0^{\theta_c} V_e d\theta d\gamma$$

$$= \frac{2P^2L}{\pi^2 R^2 G}\int_{\gamma_0}^{R}\left[2\left(\frac{1}{1+\nu}\right)\left\{\frac{R^4+\gamma^4}{R^4-\gamma^4}\tan^{-1}\left(\frac{R^2-\gamma^2}{R^2+\gamma^2}\cdot\frac{b}{\sqrt{1-b^2}}\right)\right.\right.$$

$$\left. - \frac{2b\sqrt{1-b^2}\cdot R^2\gamma^2}{(R^2+\gamma^2)^2-4R^2\gamma^2 b}\right\} + \frac{(3R^2+\gamma^2)(R^4+\gamma^4)}{(R^2+\gamma^2)^3}\tan^{-1}\left(\frac{R^2-\gamma^2}{R^2+\gamma^2}\cdot\frac{b}{\sqrt{1-b^2}}\right)$$

$$\left. - \frac{2b\sqrt{1-b^2}\cdot R^2\gamma^2(3R^2+\gamma^2)(R^2-\gamma^2)}{\{(R^2+\gamma^2)^2-4R^2\gamma^2b^2\}(R^2+\gamma^2)^2}\right]\gamma d\gamma \qquad (6)$$

For the derivation of the above equations (4), (5) and (6), Euqation (3) was used. On the crushing curves, Equation (1) is certainly satisfied. Accordingly, for the determination of θc, Equation (1) was used besides Euqation (3). At the time of the breaking the columns by the impact of a falling weight Q, when it is assumed that the change of the potential energy of the weight due to falling is totally converted to the sum of the energies of Equations (4), (5) and (6), the following equation for energy balance holds.

$$QH = W_{pc} + |W_e|_{r_0 \geqq r} + |W_e|_{R \geqq r \geqq r_0} \qquad (7)$$

H is the height of fall.

In the impact uniaxial compression test, when the axial stress in a cylinder is assumed to be uniform, σ_{DC} cab be determined immediately from the equation for energy balance. Applying this σ_{DC} to the failure criterion, and using it for Equation (1),

$$\alpha_D\sigma_{DC} + \frac{1}{\sqrt{3}}|\sigma_{DC}| = k_D \qquad (8)$$

is obtained.

In the above procedure, P, the equivalent splitting load in Euqations (5) and (6) is still an unknown force. Putting $\gamma = \gamma_0$ in the equation for crushing curves, (see Figure 4), the equation for the relation of P to α_D, k_D and γ_0 is obtained. γ_0 is the length determined by experiment. By using this equation of relation and Equations (7) and (8) simultaneously, the parameters α_D and k_D of impact breaking are obtained.

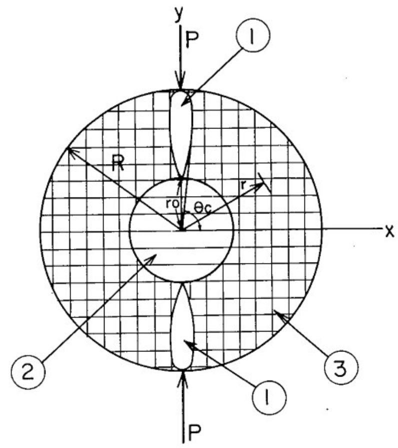

Figure 4. Division of breaking state figure

3.2 Analysis of static strength

Also in the static split-cylinder test, the test pieces break in a complex mode and both a splitting region and a shear slip region arise. The tensile failure at both ends of a splitting line and the shear failure arising along the boundary line of slipping region occurred simultaneously as shown in Figure 3. Immediately thereafter, the splitting region and the slipping region were separated, and a test piece was broken into four large fragments.

On the basis of such observation, the stress condition at both ends of a splitting line determined as the result of split-cylinder test can be regarded as statisfying the failure criterion (1). At both ends of a splitting line, $\Theta_1 = \Theta_2 = 0$ and $\gamma_1 = R - \gamma_0$, $\gamma_2 = R + \gamma_0$. When the stress obtained by using these relations in Equation (3) is put into Euqation (1):

$$\alpha \frac{2P}{\pi D}\left(2 - \frac{D^2}{R^2 - \gamma_0^2}\right) + \frac{2P}{\sqrt{6}\,\pi D}\sqrt{1 - \frac{2D^2}{R^2 - \gamma_0^2} + \frac{2D^4}{(R^2 - \gamma_0^2)^2}} = k \qquad (9)$$

Here, D = 2R, and P is a breaking load. Moreover, for uniaxial compression strength σ_c,

$$\alpha \sigma_c + \frac{1}{\sqrt{3}}|\sigma_c| = k \qquad (10)$$

hold. From above two equations, the static breaking parameters α and k are obtained.

4 ANALYSIS OF EXPERIMENTAL RESULTS

The experimental results in Section 2 were analyzed by the method mentioned in the previous chapter, and the material constants as shown in TABLE II were obtained. For the Poisson's ratio $\nu = 0.2$ and the dynamic modulus of rigidity $G_D = 8810$ MPa obtained by material testing, the dynamic material constants were $\alpha_D = 0.3 - 0.305$ and $k_D = 37.8 - 38.8$ MPa. The other columns in TABLE II show the values when Poisson's ratio was assumed to be $\nu = 0.18$, 0.22, 0.24, 0.26 and 0.28. Since only velocity V_p was measured in the material experiment, by assuming the value of Poisson's ratio ν, the dynamic modulus of rigidity G different for each value of ν. From TABLE II, it can be kown how the material constants change for $\nu = 0.18 - 0.28$. From the results of static breaking test, the material constants $\alpha = 0.405$ and $k = 6.5$ MPa were obtained. Figure 5 shows the crushing curves due to impact, determined analytically. It can be said that these are very similar as compared with the state of crushing in Figure 2.

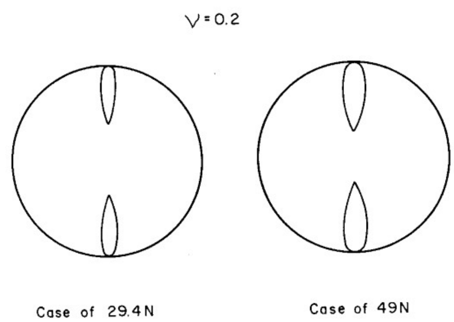

Figure 5. Crushing curves determined by analysis

5 CONCLUSION AND DISCUSSION

An analytical method for the impact cylinder splitting test is presented. The following conclusions are obtained from the analytical and experimental results.

i) The ratio of k_D to k determined in Chapter 4 was about 6. The fact that k_D was as large as this seems to be because the energy of the recoil of a weight, the energy of the deformation of the bearing plate, the energy required for splitting separation and so on were neglected in this analysis.

ii) The difference corresponding to the weight was observed in the equivalent splitting load P, but the material constants α_D and k_D hardly.

iii) From the value of the coefficient in Equation (4) for the same Poisson's ratio it can be said that the breaking by 49N weight was brittle fracture, whereas the breaking by a 29.4N weight was ductile fracture.

iv) As for the relation between Poisson's ratio ν and α_D and k_D, α_D decreased and k_D increased corresponding to the increase of ν. Accordingly, in this experiment, only V_p was measured, but it is necessary to obtain the values of E and ν independently by measuring V_s together with V_p.

v) There was not much difference between the value of impact equivalent splitting load P and that of static splitting load P_f.

TABLE II

BREAKING PARAMETERS α_D and k_D DETERMINED BY ANALYSIS AND EQUIVALENT SPLITTING LOAD P

Poisson's ratio ν	Dynamic modulus of rigidity G_D MPa	29.4 N Weight				49N Weight			
		Coefficient β	α_D	k_D	Equivalent splitting load P	β	α_D	k_D	P
0.18	8928	12.1	0.313	36.8	2136 N	8.5	0.31	37.1	3136 N
0.20	8810	11.9	0.305	37.8	2136	8.1	0.30	38.8	3214
0.22	8634	11.6	0.298	38.8	3176	8.0	0.291	39.9	3234
0.24	8497	11.2	0.285	40.8	2176	7.6	0.283	40.9	3292
0.26	8359	10.7	0.265	43.5	2234	7.4	0.277	41.7	3332
0.28	8232	10.4	0.252	45.3	2274	7.0	0.261	44.0	3410