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Consolidation Around a Heat Source

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SUMMARY When a heat source, such as a cannister of nuclear waste, is buried deep in a saturated soil the soil undergoes an increase in temperature. The rise in temperature causes he pore water and soil skeleton to expand. The coefficient of expansion of the pore water is usually much greater than that of the soil skeleton and so initially this differential volume increase leads to an increase in pore pressure and a reduction in effective stress near the heat source. If the increase in temperature is too great the effective stress may decrease to such an extent that cracking could occur. The excess pore pressures generated by the temperature increase will however tend to dissipate because of the consolidation process. Thus the increase in pore pressure may not be as severe as first anticipated.

In this paper the distribution of temperature and pore pressure around spherical and cylindrical sources is examined. The effect of consolidation of the surrounding soil significantly reduces the increase in pore pressure due to temperature increases and thus reduces the risk of cracking.

1 INTRODUCTION

If a heat source such as a canister of radioactive waste is buried in a saturated soil the source will cause a temperature rise in the soil. This temperature rise will cause both the soil pore water and the soil skeleton to expend. In general the volume increase of the pore water is greater than that of the voids in the soil skeleton and so the differential volume change leads to an increase in pore water pressure and a consequent reduction in effective stress. If this reduction in effective stress is too great the soil may fracture, leading to an increased rate of migration of pore water or even to a progressive failure, which ultimately reaches the surface. If the deposit is sufficiently permeable consolidation will occur and the excess pore water pressure, generated by the increase in temperature, will dissipate and reduce the severity of these effects.

In this paper an analytic solution is developed for a spherical heat source buried deep in a saturated thermo-elastic soil. The solution for a point heat source is found by letting the radius approach zero. The point source solution is then integrated to obtain the solution for a cylindrical canister.

2 BASIC EQUATIONS

The equations for the one dimensional consolidation of a two phase elastic soil were first developed by Terzaghi (1923). These equations were generalised to include three dimensional effects by Biot (1941) and were subsequently generalised to include the effects of anisotropy and visco-elasticity (Biot, 1956, 1962). Biot´s equations can be modified to incorporate thermal effects as follows:

2.1 Development of Equations

The equations governing the consolidation of a saturated thermoelastic soil are:

2.1.1 The equations of equilibrium

Three equations of the form:

$$\partial\sigma_{xx}/\partial x + \partial\sigma_{xy}/\partial y + \partial\sigma_{xz}/\partial z = 0 \quad \text{(typically)} \qquad (1)$$

where $\sigma_{xx}, \sigma_{yy}, \cdots, \sigma_{xy}$ denote the increase in total stress components over the initial equilibrium state (compressive stresses being reckoned positive) and where changes to the density of soil, due to thermal expansion, have been neglected.

2.1.2 Effective stress strain temperature relation

The effective stress strain relations for an isotropic thermo-elastic soil have the form:

$$\varepsilon_{xx} + a'\theta/3 = [\sigma'_{xx} - \nu'(\sigma'_{yy} + \sigma'_{zz})]/E'$$

$$\gamma_{xy} = \sigma_{xy}/G \qquad (2)$$

where E', ν' are the drained values of Young's modulus and Poisson's ratio, $G = E'/2(1 + \nu')$ is the shear modulus of the soil, θ is the increase in temperature (over an initial equilibrium state) and a' is the coefficient of volume expansion of the soil. Clearly if no structural changes occur during expansion a' will be identical to a_s the coefficient of volume expansion of the skeletal material.

The effective stress increases are given by relations of the type:

$$\sigma'_{xx} = \sigma_{xx} - p$$

where p is the excess pore water pressure.

2.1.3 Volume constraint equation

It is assumed that the skeleton is incompressible (to stress) and that the pore water is incompressible (to pressure) then it is easy to establish that:

$$\nabla . v = \partial\varepsilon_v/\partial t + a_u\partial\theta/\partial t \qquad (3)$$

where $a_u = a_s(1-n) + a_w n$

and a_s, a_w are the coefficients of expansion of the skeletal material and pore water respectively, $\underset{\sim}{v}$ is the apparent velocity vector, n is the porosity and ε_v is the volume strain.

2.1.4 Darcy's law

The flow of pore water in the soil is governed by Darcy's Law:

$$\underset{\sim}{v} = - k \underset{\sim}{\nabla} p/\gamma_w \tag{4}$$

where k is the coefficient of permeability of the soil and γ_w is the unit weight of water.

2.1.5 Thermal energy balance

In many applications mechanical contributions to energy balance may be neglected when compared to thermal contributions. In such cases the net rate of inflow of energy into an element of the material will be just balanced by increases in the internal energy of the pore water and the soil skeleton. Thus, neglecting convective terms,

$$-\underset{\sim}{\nabla}.\underset{\sim}{h} = m\partial\theta/\partial t \tag{5}$$

where $m = n\gamma_w c_w + (1-n)\gamma_s c_s$

and γ_w, γ_s are the densities of the pore water and the skeletal material, c_w, c_s are their specific heats and $\underset{\sim}{h}$ is the heat flux vector.

2.1.6 Fourier's law of heat conduction

The flow of heat in the soil is assumed to be governed by Fourier's Law:

$$\underset{\sim}{h} = - K \underset{\sim}{\nabla}\theta \tag{6}$$

where K is the coefficient of heat conduction.

3 SOLUTIONS FOR A SPHERICAL SOURCE

Consider a rigid impermeable heat source, which has been placed at great depth below the surface of a homogeneous, saturated, thermoelastic soil. Clearly because of the great depth of burial the problem exhibits spherical symmetry and so the field quantities depend only on R, t.

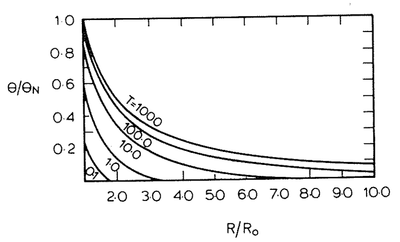

Fig 1 Temperate Isochrones for a Spherical Source

3.1 Temperature Distribution

Under conditions of radial symmetry Equations (5,6) may be combined to give:

$$\kappa\partial^2(R\theta)/\partial R^2 = \partial(R\theta)/\partial t \tag{7}$$

where $\kappa = K/m$.

For a constant heat source of strength Q, it may be shown that this equation has the solution

$$\theta = \theta_N R_o/R \quad f[\kappa t/R_o^2, R/R_o] \tag{8}$$

where

$$f = \text{erfc}(b) - e^{a^2 + R/R_o}\text{erfc}(b+a) \tag{9}$$

where $\Delta R = R - R_o$

$a^2 = \kappa t/R_o^2$

$b^2 = \Delta R^2/4\kappa t$

$d^2 = \kappa t/R^2$

and $\theta_N = Q/4\pi K R_o^2$ = final surface temperature.

The temperature distribution is shown in Fig. 1. Where $T = \kappa t/R_o^2$ is the dimensionless time.

3.2 Solution for a Completely Impermeable Soil

If the soil is relatively impermeable then the excess pore pressures generated by the increase in temperature will dissipate very slowly. It is consequently of interest to examine the limiting case of a completely impermeable soil (k = 0).

It is fairly simple to show that the pore pressure is a simple multiple of the temperature, viz:

$$P = X\theta \tag{10}$$

where $X = (\lambda+2G)a_u - (\lambda+2G/3)a'$

and $\lambda = 2G\nu'/(1-2\nu')$

Equations (1,2) may then be integrated to show that the only non-zero displacement is:

$$u_R/R = a_u Qg/(4\pi KR) \tag{11}$$

The only non zero stress components are

$$\sigma_{RR} = 4 a_u G Qg/(4\pi KR)$$

$$\sigma_{\phi\phi} = 2 a_u G Q(f-g)/(4\pi KR) \tag{12}$$

$$\sigma_{\psi\psi} = 2 a_u G Q(f-g)/(4\pi KR)$$

the function f was defined by equation 9 and the function g is defined by:

$$g = d^2 - (\Delta R^2/2R^2 - d^2)\text{erfc}(b) + fR_o\Delta R/R^2$$

$$-\Delta R d e^{-b^2}/(R\sqrt{\pi}) \tag{13}$$

Thus in this case all field quantities may be expressed in terms of the two functions f, g.

3.3 Solution for a Permeable Soil

The functions f, g described in the previous sub-section can be used to obtain expressions for the field quantities in the case when the soil has a finite permeability and it is found on integrating equations (1-6) with the simplification of spherical symmetry that:

$$\theta = Q/4\pi KR \; f(ct/R_o^2, R/R_o)$$

$$p = \frac{Q/4\pi KR}{(1-c/\kappa)}\left[f(ct/R_o^2, R/R_o) - f(ct/R_o^2, R/R_o)\right]$$

$$u_R/R = a_u Qg^*/4\pi KR \tag{14}$$

$$\sigma_{RR}/G = 4a_u Qg^*/4\pi KR$$

$$\sigma_{\phi\phi}/G = \sigma_{\psi\psi} = 2 a_u Q(f^*-g^*)/4\pi KR$$

where the functions f, g are defined by equations (9, 13) respectively and

$$f^* = Y\, f(\kappa t/R_o^2, R/R_o) - Z f(ct/R_o^2, R/R_o)$$

$$g^* = Y\, g(\kappa t/R_o^2, R/R_o) - Z g(ct/R_o^2, R/R_o) \qquad (15)$$

with $Z = X/[a_u(1-c/\kappa)(\lambda+2G)]$

$$Y = Z + b'/[a_u(\lambda+2G)]$$

If the soil were impermeable, it follows from equation 14:

$$p_N = [(\lambda+2G)a - b']\, Q/4\pi R = X\theta_N \qquad (16)$$

This normalised pore pressure is independent of the elastic properties of the soil depending only upon position, time and the ratio of the coefficient of consolidation to the diffusivity, c/κ.

The variation of normalised pore pressure with time is shown in Fig. 2 for the values $c/\kappa = 1/3$, 1, 3. The behaviour is as would be expected; at first the rise in temperature causes a corresponding rise in excess pore pressure, at the same time the pore pressure starts to dissipate, and so the rate of increase of pore pressure decreases with time. After some time a maximum excess pore pressure is achieved, and thereafter the rate of dissipation of pore pressure exceeds the rate of generation due to temperature change and so ultimately all the excess pore pressures dissipate. Fig. 2 shows that the consolidation process is most effective in reducing the severity of the rise of excess pore pressure. Thus if $c/\kappa = 3$ the excess pore pressure rises to less than eight per cent of the value it would reach if no dissipation occurred, if $c/\kappa = 1$ the rise is a little under fourteen percent while even if $c/\kappa = 1/3$, the rise is no more than twenty three percent.

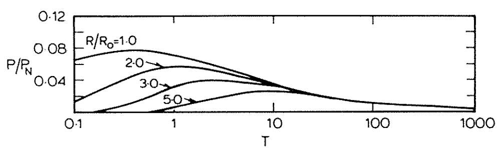

Fig 2a Variation of Pore Pressure With Time (c/κ=3)

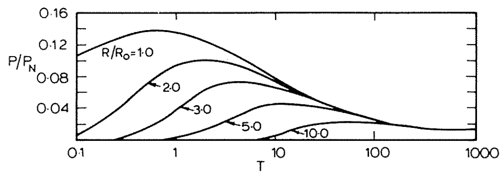

Fig 2b Variation of Pore Pressure With Time (c/κ=1)

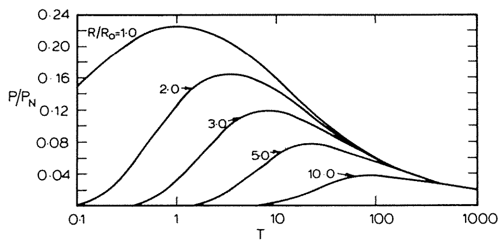

Fig 2c Variation of Pore Pressure With Time (c/κ=1/3)

Equations 14 may be used to evaluate the stresses and it is found that the increase in radial stress is relatively small and compressive and its maximum value increases as the soil becomes more impermeable. Second, the increase in circumferential stress is rather larger but it is also compressive. This means that the major reduction in effective stress is likely to occur in the radial direction.

3.4 Solution for a Point Source

In later applications we require the solution for a point source of heat. This may be obtained from equations (9,13) by allowing $R_o \to 0$ and we find that f, g are now given by:

$$f = f_{ps} = \mathrm{erfc}(b)$$

$$g = g_{ps} = d^2 + (1/2 - d^2)f_{ps} - de^{-b^2}/\sqrt{\pi} \qquad (17)$$

where $b^2 = R^2/4\kappa t$

4 APPROXIMATE SOLUTION FOR CYLINDRICAL SOURCE

The solution of the problem of a cylindrical radiating source involves the solution of an extremely difficult mixed boundary value problem. A simple approximate solution may be obtained by assuming that the cylindrical source can be simulated by the integration of the point source solution throughout the cylindrical volume, that is, to consider the heat source to consist of soil impregnated with a heat radiating substance.

To illustrate this approach we consider the case of a cylinder length 2h and diameter $2r_o$ as shown in Fig. 3 for the particular case in which: $a'/a = 1/4$, $\nu' = 0.4$, $c/\kappa = 2$.

It is found that $\theta_N = 1.45qr_o^2/K$ at the midpoint on the surface of the cylinder. If the soil was impermeable the pore pressure would reach a value of $p_N = X\theta_N$ at this point.

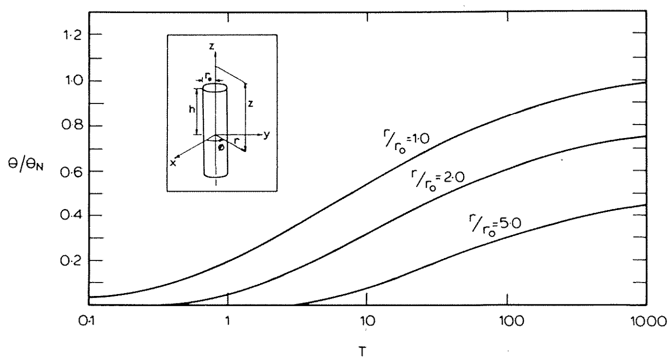

Fig 3 Variation of Temperature With Time for a Cylindrical Source

The variation of temperature and pore pressure on the plane z = 0 is shown in Figs. 3 and 4, while the variation of stress normalised with respect to the pore pressure p_N is shown in Figs. 5, 6 & 7.

$T = \kappa t/r_o^2$ is the dimensionless time.

The cylinder is a relatively long one and so there is little drainage in the vertical direction. Thus, dissipation of pore pressure and heat diffusion proceeds more slowly than for the spherical source. The effect of the increase in temperature is to generate excess pore pressures but the consolidation process ensures that the excess pore pressure only reaches a small fraction (approximately twelve percent) of the value it would achieve if no consolidation occurred. Changes in the direct stresses are again small and compressive, the smallest being that in the radial stress component.

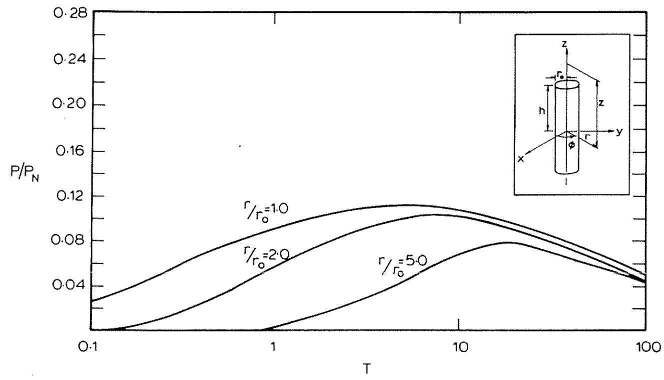

Fig 4 Variation of Pore Pressure With Time for a Cylindrical Source ($c/\kappa=2$)

5 CONCLUSIONS

A theory of the consolidation of soil for non isothermal conditions which takes account of the differential thermal expansion of the pore water and soil skeleton and which is based on simple concepts of volume constraint and the effective stress principle has been developed.

This theory has been used to develop an analytic solution for an impermeable rigid, spherical, heat source and a point source surrounded by a thermoelastic permeable soil. Examination of the solution shows that the rise in temperature causes the pore pressure to rise but that the excess pore pressures generated in this way are dissipated quickly and rise to only a small fraction of the value that they would achieve if the soil were completely impermeable. The stress changes around the sphere are in general small and compressive, the radial stress undergoing the smallest increase.

An approximate solution to the problem of a cylindrical heat source has been found by integrating the solution for a point source over the cylindrical volume. Much the same conclusions can be drawn for this case as for the sphere.

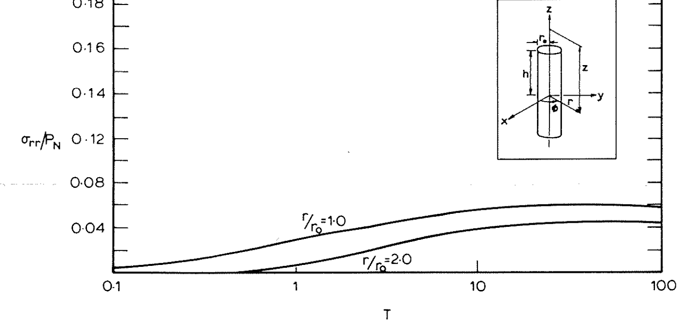

Fig 5 Variation of σrr With Time for a Cylindrical Source

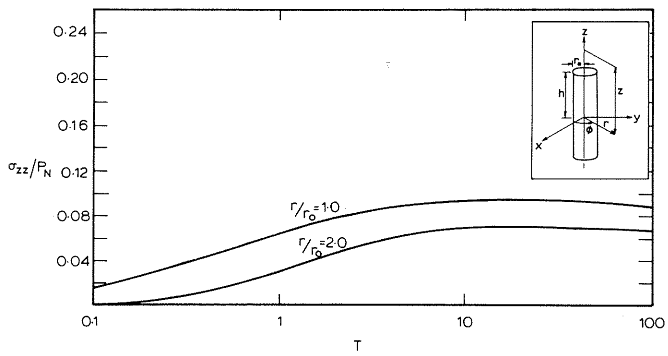

Fig 6 Variation of σ_{zz} With Time For a Cylindrical Source

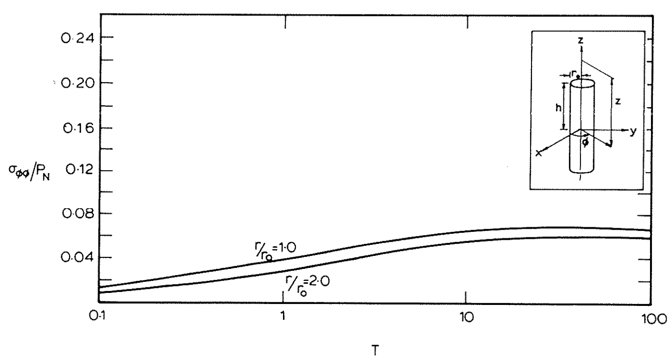

Fig 7 Variation of $\sigma_{\theta\theta}$ With Time For a Cylindrical Source

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