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Response of a Two-dimensional Wedge by Taking the Effects of Shear and Bending Moment into Account

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SUMMARY The present work consists of a theoretical investigation of the response of a two dimensional elastic wedge subject to an arbitrary disturbance. Expressions are derived for the deflections and rotations together with shears and bending moments which develop in the wedge owing to an imposed time dependent disturbance. The frequencies of the wedge are for the two first modes of oscillation, the moduli of elasticity are expressed by the n-th order of depth from the apex where $0 \leq n < 1$.

1 INTRODUCTION

Remarkable progress has been made towards understanding the behavior of earth and rock fill dams subjected to earthquake shaking, and many researchers are enumarated in this field such as Hatanaka 1955; Ambraseys 1960; Ishizaki et al. 1962; Chopra 1969; Minami 1969; Makdisi et al. 1978; Seed 1967; Mori et al. 1980; Gazetas 1981; Abdel-Ghaffer et al. 1981; Ohmachi et al. 1983.

However, the conventional analytical procedures for potential deformations of such dams caused by horizontal ground excitation, in the upstream-downstream (i. e. lateral) direction have been handled by shear beam theory and static finite element analyses also suggest that the shear beam theory is of practical use for the flat dam even if its shear modulus of material increases as the n-th power of the distance from the crest. This means that the same natural frequency occurs in the flat dam disregard of its wedge slope. However, it may be imagined that the steeper slope should occur the stronger bending effect, and so it is urgent to learn the relation between flatness and adequancy of shear beam theory.

2 SIMPLIFYING ASSUMPTIONS

With reference to the system of coordinate axis shown in Fig. 1, the assumptions are simplified to derive the governing equations of motion as follows;

1) The dam consist of uniform material having a constant mass density ρ, and a constant Poisson's ratio ν, but elastic moduli in shear and compression, G and E, reflecting the dependance of material stiffness, on effective normal stress. Gazetas insists that

$$G = G(z) = Gm\ (z/H)^{2/3} \qquad (1)$$

where Gm equals maximum value of G at the base of the dam (z=H). We shall introduce the equations to represent the variation of soil stiffness with depth from the crest:

$$G = Gm\ (z/H)^{n} \qquad (2)$$

where $0 \leq n < 2/3$.

2) The soil exhibits linear stress-strain behavior on assumption which is acceptable for small level of strain. This often extend to large deformation by performing so called strain compatible viscoelastic model.

3) The normal and shear stresses, σz and τyz, are of linear and constant distributions in the y direction. This is guaranteed by the results of two dimensional stress problem in wedge under a lateral body force. Let us put the coordinates to a semi-infinite elastic medium subjected to the body force Y in the y direction, as shown in Fig. 2. Then, the body force yields the stresses in the symmetric wedge AOB, as follows:

$$\sigma r = - C\ r\ \sin 3\theta + D\ r\ \sin\theta + \frac{Y}{4}\ r\ \sin\theta \qquad (3)$$

$$\sigma\theta = C\ r\ \sin 3\theta + 3\ D\ r\ \sin\theta + \frac{3Y}{4}\ r\ \sin\theta \qquad (4)$$

$$\tau r\theta = C\ r\ \sin 3\theta + D\ r\ \sin\theta - Y\ r\ \cos\theta \qquad (5)$$

in which C and D can be determined to satisfy the boundary conditions

$$\sigma\theta = 0, \quad \tau r\theta = 0 \qquad \text{for } \theta = \pm\ \phi \qquad (6)$$

Thus obtained stress distributions are reverted into the y, z coordinates to the following forms:

$$\sigma z = \frac{Y}{2}\ y\ \cot^2\phi \qquad (7)$$

$$\sigma y = \frac{Y}{2}\ y \qquad (8)$$

$$\tau xy = - \frac{Y}{2}\ z \qquad (9)$$

which means the stresses linearly or constantly distribute for the corresponding body force, and it may be allowed that shear coefficient κ is one in this case.

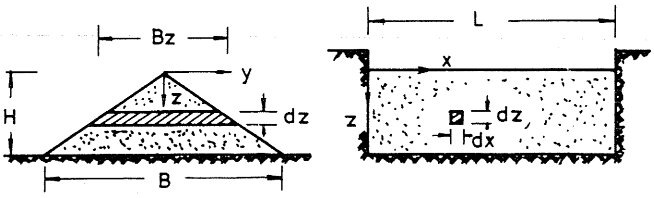


Fig. 1. General view of a dam and coordinate system.

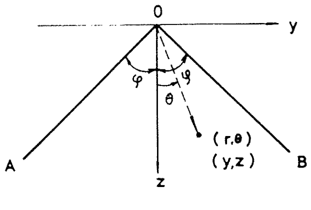


Fig. 2.
The coordinates to a semi-infinite elastic medium.

3 EQUATION OF MOTION

Let x, y and z be the orthogonal coordinates of any point in a two-dimensional symmetrical wedge Fig. 3. The motion of the base and vertical sides of the wedge under the influence of the distributing boundary motion expresses by g(t), will be transmitted by shearing force and bending moment to any element in the oscillating medium. We will, however, proceed our discussion disregard of the bending moment about the axis z. Thus, we have only to take two displacements v and w, in the directions y and z, respectively. Taking linear distribution of stress in the lateral direction, we assume that

$$w = y\,\psi(x,z), \qquad v = v(x,z) \qquad (10)$$

Denoting r, Bz, Iz as density, lateral breadth, moment of inertia about the axis x, and Qz, Qx as shearing forces at the sections z = const., x = const., we can express the equation of motions,

$$\frac{\partial Qx}{\partial x} + \frac{\partial Qz}{\partial z} = \rho\ Bz\ \left(\frac{\partial^2 v}{\partial t^2} + \frac{\partial^2 g}{\partial t^2} \right) \qquad (11)$$

$$\frac{\partial Mzx}{\partial x} - Qz + \frac{\partial Mz}{\partial z} = \rho\ Iz\ \frac{\partial^2 \psi}{\partial t^2} \qquad (12)$$

where (refering to Fig. 3),

$$Bz = \frac{B}{H} z , \qquad Iz = \frac{1}{12} \left(\frac{B}{H} \right)^3 z^3$$
$$Gz = Gm \left(\frac{z}{H} \right)^n , \qquad Ez = Em \left(\frac{z}{H} \right)^n \qquad (13)$$

and each sectional forces is related to the displacements with:

$$Mz = Ez\ Iz \frac{\partial \psi}{\partial z} , \qquad Qz = Gz\ Bz\ \left(\psi + \frac{\partial v}{\partial z} \right)$$
$$Mzx = Gz\ Iz \frac{\partial \psi}{\partial x} , \qquad Qx = Gz\ Bz \frac{\partial v}{\partial x} . \qquad (14)$$

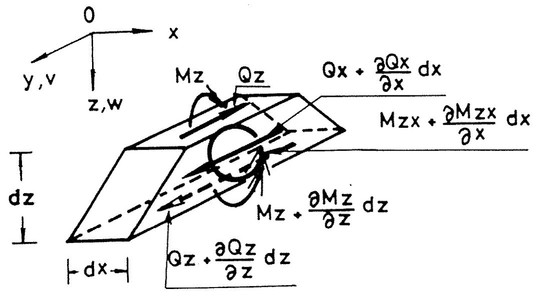


Fig. 3. All forces acting on infinitesimal body.

4 EQUATION OF NATURAL FREQUENCY

The natural frequencies of this system will be found by letting g(t) = 0, and putting,

$$\psi = \frac{\theta}{z^2} \sin \frac{m\pi}{L} x \sin pt, \qquad \Psi = \frac{\theta}{z^2}$$
$$v = V \sin \frac{m\pi}{L} x \sin pt$$
$$Qx = Qx \cos \frac{m\pi}{L} x \sin pt$$
$$Qz = Qz \sin \frac{m\pi}{L} x \sin pt$$
$$Mz = Mz \sin \frac{m\pi}{L} x \sin pt \qquad (15)$$

$$Mzx = Mzx \cos \frac{m\pi}{L} x \sin pt$$

where p is circular frequency, into eq.(14), we have the equations governing the amplitude of each quantity, and again by using one more abbreviation:

$$\eta = \frac{z}{H} , \quad \beta = \frac{pH}{Cs} , \quad Vp^2 = \frac{Em}{Gm} , \quad \Theta = Gm \frac{\theta}{H} ,$$
$$Rb = \frac{H}{B} , \quad RL = \frac{H}{L} , \quad \mu = 12\ Rb^3/Vp^2 ,$$

we finally come to the equations of natural frequency as follows:

$$\frac{d^2 Qz}{d\eta^2} - \frac{1}{\eta} \frac{dQz}{d\eta} + \beta^2 \frac{1}{\eta^n} Qz =$$
$$\beta^2 \frac{1}{Rb\eta} \Theta + m\ \pi\ RL \left\{ \frac{dQx}{d\eta} - \frac{Qx}{\eta} \right\} \qquad (16)$$

$$\frac{d^2\Theta}{d\eta^2} - (1-n) \frac{1}{\eta} \frac{d\Theta}{d\eta} +$$
$$\left\{ \beta^2 \frac{1}{Vp^2 \eta^n} - \frac{2n}{\eta^2} - (m\ \pi\ RL)^2 \frac{1}{Vp^2} \right\} \Theta =$$
$$\frac{\mu}{\eta^{(1+n)}} Qz \qquad (17)$$

$$m\ \pi\ RL\ Qz =$$
$$m\ \pi\ RL \frac{\eta^{(n-1)}}{Rb} \Theta - (1+n) \frac{Qx}{\eta} + \frac{dQx}{d\eta} \qquad (18)$$

In particular, in case of n = 0, we have,

$$\frac{d^2 Qz}{d\eta^2} - \frac{1}{\eta} \frac{dQz}{d\eta} + \bar{\beta}^2\ Qz = \bar{\beta}^2 \frac{\Theta}{Rb\eta} \qquad (19)$$

$$\frac{d^2\Theta}{d\eta^2} - \frac{1}{\eta} \frac{d\Theta}{d\eta} + \bar{\beta}^2 \frac{\Theta}{Vp^2} = \frac{\mu}{\eta} Qz \qquad (20)$$

where $\bar{\beta}^2 = \beta^2 - (m\ \pi\ RL)^2$.

5 BOUNDARY CONDITIONS

Firstly we assume natural boundary in the x direction. The top of the wedge, η = 0, should be satisfied by

$$\left. \begin{array}{l} Qz0 = 0 \\ Mz0 = 0 \end{array} \right\} \qquad (21)$$

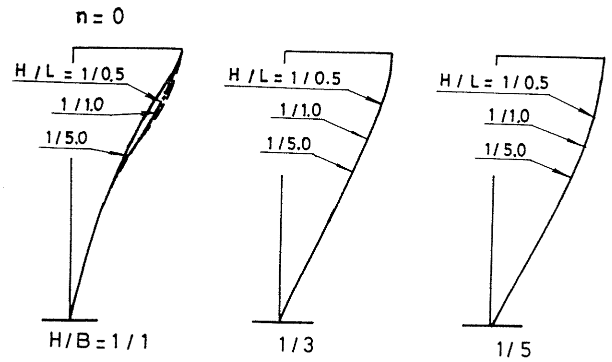
and the bottom of it, η = 1,

$$\left. \begin{array}{l} \Theta = 0 \\ u = 0 \end{array} \right\} \qquad (22)$$

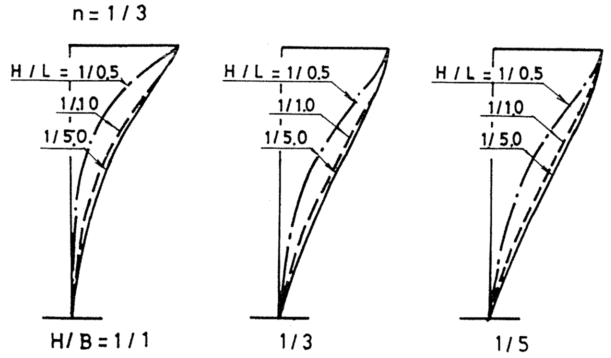
Eigen-value connected with the natural frequency is analyzed by means of the finit difference method which can give fairly accurate results with number of divisions 20. And the elastic constant is assumed as Poisson's ratio ν = 0.45.

6 NUMERICAL CALCULATION

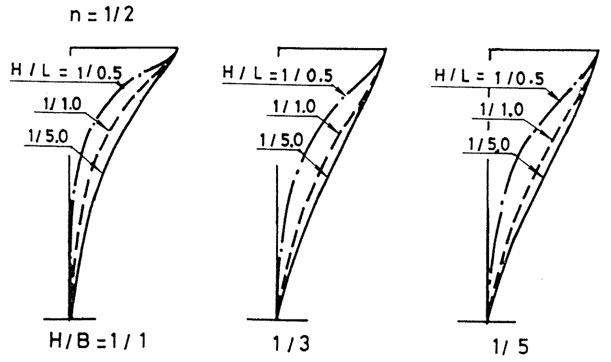
The first modes of three kinds of wedge aspect, $H/B = 1/1,\ 1/3,\ 1/5$, are shown in Fig. 4(a)~(d) for $n = 0,\ 1/3,\ 1/2,\ 2/3$, respectively. Each figure has the three curves with $H/L = 1/0.5,\ 1/1.0,\ 1/5.0$.



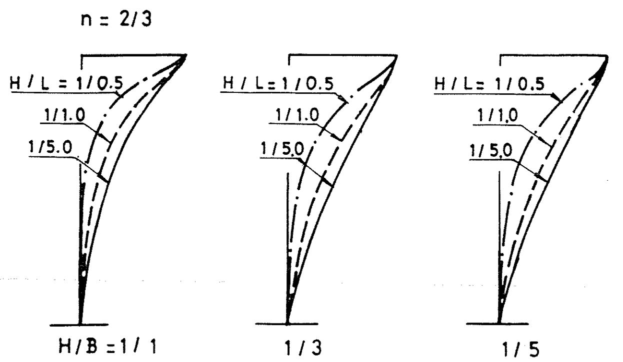
(a) In case of $n = 0$.



(b) In case of $n = 1/3$.



(c) In case of $n = 1/2$.



(d) In case of $n = 2/3$.

Fig. 4. Comparison of mode shapes along depth of wedge with aspects H/B and H/L.

The modes for $n = 0$ take almost the same shape in each aspect. However, in case except $n = 0$, the mode takes quite different shape from each other for the different aspects and the canyon widths.

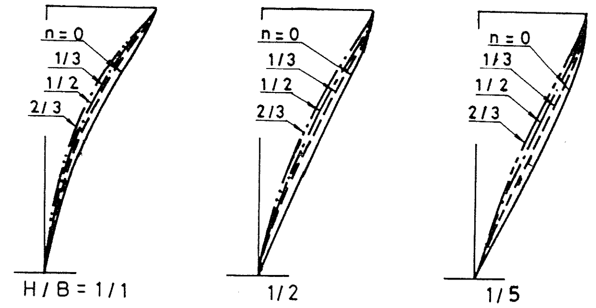


Fig. 5. First mode shapes along depth of wedge in case of $H/L = 1/3$.

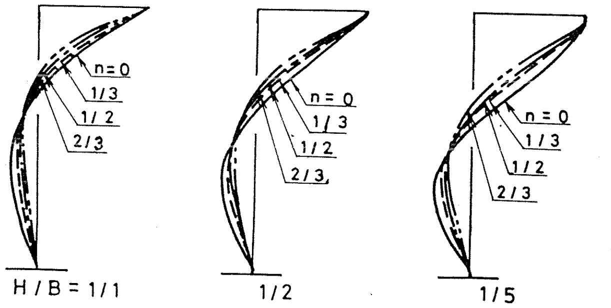


Fig. 6. Second mode shapes along depth of wedge in case of $H/L = 1/3$.

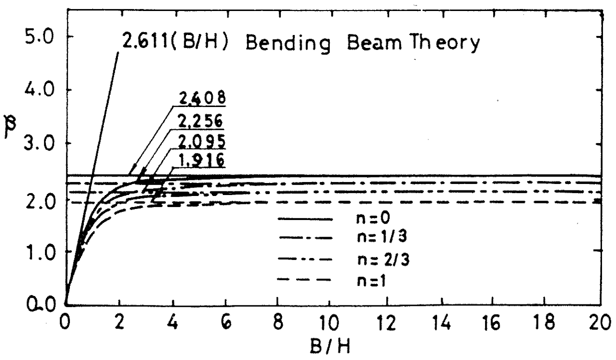


Fig. 7. The first eigen-value of wedge taking shear and bending moment into account with infinite length.

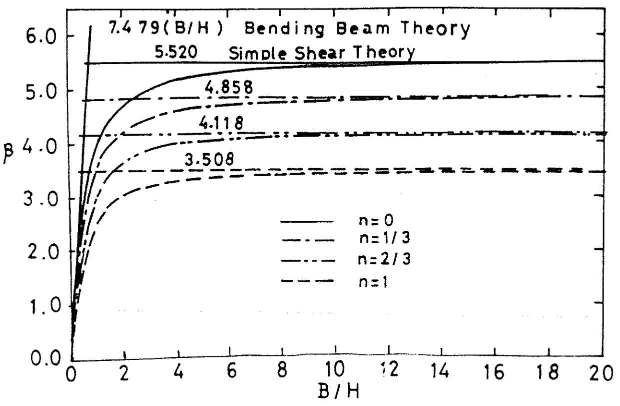
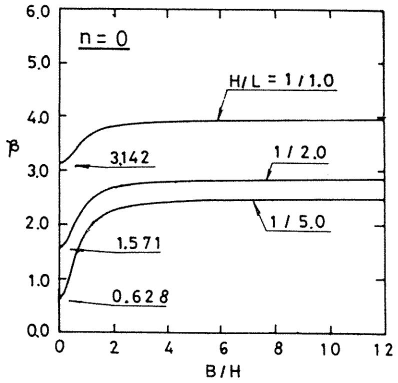
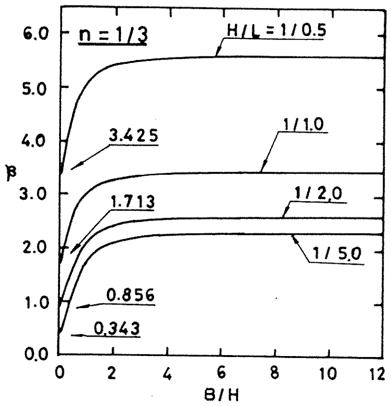


Fig. 8. The second eigen-value of wedge taking shear and bending moment into account with infinite length.

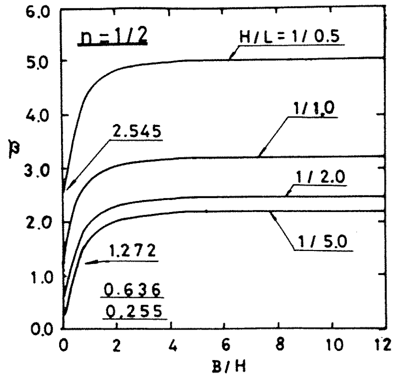
In case of n = 2/3 that, Gazetas insists, the modes strongly change with the variation of the canyon with H/L. The 1st and 2nd modes are shown in Fig. 5 and 6, for n = 0, 1/3, 1/2, 2/3, with three aspects H/B = 1, 1/2, 1/5 in each figure. The eigen-value of the wedge taking shear and bending moment into account are drawn with the variation of B/H in Fig. 7 and 8, together with the ones obtaind by the shear beam theory, from which we can see the limit of the flatness indicating the adequancy of the simple shear theory. That is about B/H = 5. For the 2nd mode, the limit seems to be around B/H =8, as shown in Fig.8. The discussion seems to go in similar way even when we take the canyon width into account as shown in Fig. 9(a)~(d).



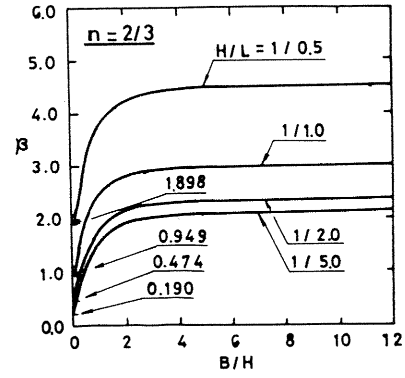
(a) In case of n = 0.



(b) In case of n = 1/3.



(c) In case of n = 1/2.



(d) In case of n = 2/3.

Fig. 9. Variation of eigen-value β with B/H.

7 FINAL REMARKS

1 The paper has presented a simple analytical method of horizontal oscillation in the upstream-downstream direction which considers both the in-homogeneity of the dam, coming from the dependance of soil stiffness on the confining pressure and existance of the rectangular canyon, together with bending and shearing effects. In particular, the method proves quite successful in finding the limit of ratio between the dam height and the bottom width, less than which the conventional simple shear theory can work, that is B/H = 5.

2 The soil stiffness index n effects on a shape of the vertical modes, while the rectangular canyon width has nothing to do with the modes shape in the simple shear theory. However, not only the soil stiffness index n, but also the canyon width have much to do with the vertical mode in the theory taking the bending and shear into account.

3 When the height of the dam is twice as much as the canyon width, the vertical modes take quite different shape from each other except n = 0, between the bending and shear theory and simple shear theory. However, the case of the ratio between dam height and the canyon width is smaller than 1/5.0, the shapes of vertical modes are almost equal to each other both theories.

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