Experimentally Determined Distribution of Stress Around a Horizontally Loaded Model Pile in Dense Sand

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SUMMARY A model pile instrumented with miniature load cells has been jacked into a bed of dense sand and loaded horizontally in two directions. From the load cell readings taken and a consideration of force equilibrium, the approximate distribution of stress around the circumference of the pile on horizontal loading has been obtained. The stresses induced on horizontal loading were found to be confined essentially to the leading face of the pile and comprised components in the horizontal plane and a longitudinal shear component resulting from flexing of the pile. The resultant stress in the horizontal plane was non-uniform and directed approximately radially towards the pile around much of its circumference.

In the analysis of a pile under horizontal loading, it is generally assumed that the horizontal load is resisted solely by a stress uniform across the width of the pile and directed parallel to the horizontal load. Taking no account of stresses not aligned with the horizontal load is found to underestimate the resistance of the pile/sand system to horizontal loading.

1 INTRODUCTION

The response of piles to horizontal loading has been measured by numerous researchers, at various scales ranging from small models to prototype sizes, using a variety of measuring techniques. In most cases, due to limited funding, available expertise and restrictions imposed by the purpose of the testing, only overall pile response has been measured, providing plots of applied horizontal load versus pile deflection at the ground surface. These results allow the available analytical methods to be calibrated for the particular test conditions, but do not allow the basic assumptions of the analytical methods to be checked.

This paper describes the results obtained on horizontal loading of an instrumented model pile which had been jacked into a bed of dense sand. The instrumentation comprised nine miniature load cells located down the embedded length of the model pile at a number of orientations designed to provide the maximum amount of information on the response of the pile to horizontal loading. The results enabled the approximate distribution of stress around the circumference of the pile on horizontal loading to be obtained. This stress distribution is compared with that generally assumed for analysis and the effect of using the assumed stress distribution is illustrated.

2 EXPERIMENTAL ARRANGEMENT

2.1 Sand Bed Preparation

The material used in the test bed was a medium to coarse grained, uniformly graded Leighton Buzzard sand nominally within the size range 0.6 to 1.2mm. This sand comprises sub-angular quartz grains and has a uniformity coefficient of 1.12 and specific gravity of 2.65. It was used air dry (moisture content 0.1%).

A dense sand bed was prepared by pluviation at a slow rate from a fixed hopper. The base of the hopper was located a minimum 600 mm above the surface of the sand bed and the sand rained from a grid or holes in the hopper base before passing through two fixed sieve meshes. The hole grid comprised 6 mm diameter punched holes on an equilateral triangular grid of side 80 mm, giving an open area of 0.5% of the total plan area. The sieve comprised a 3.02 mm aperture upper mesh and a 2.26 mm aperture lower mesh, separated vertically by 100 mm and with the lower mesh located a minimum 300 mm above the surface of the sand bed. To ensure adequate dispersion of the sand jets from the hopper and an even rain of sand below the sieve, the lower mesh was 45° out-of-plane with the upper mesh and the sieve was fixed in a horizontal plane. A perspex dust screen was located between the hopper base and the top of the testing tank.

2.2 Testing System

The steel testing tank in which the sand bed was prepared had an internal height of 1150 mm and an internal diameter of 850 mm. It was supported 500 mm above the laboratory floor on a frame to allow emptying of the tank through a shuttered hole in its base. Loading frames were bolted to the stiffened top flange of the testing tank and pressurised by liquid nitrogen. The pressure applied to the jacks was measured using a Budenberg Standard Test Gauge and from the pressure readings the load applied to the pile was calculated. Horizontal deflections of the pile on horizontal loading were measured using displacement transducers at or above the surface of the sand bed and dial gauges below the surface of the sand bed connected to the pile by thin rods. The rods passed through guides in the wall of the testing tank to the pile within the sand bed and were installed during sample preparation.

2.3 Model Pile and Instrumentation

The model pile was fabricated from untreated mild steel tubing of 30 mm outside diameter and with a wall thickness of 2.5 mm. The toe of the pile was closed with a flat mild steel end cap. The overall length of the pile was 1125 mm.
The model pile was instrumented with nine miniature load cells over the lower 750 mm of its length which was to be embedded in the sand bed. The eight load cells installed in the pile shaft were located in 12.7 by 12.7 mm square holes through the wall of the pile. Two U-shaped mile steel brackets served to reinforce each hole and it was onto these that the base of the load cell was attached using screws. The remaining load cell was installed in the end cap, with an active face measuring 5 mm in diameter. The locations and orientations of the load cells down the embedded length of the pile are shown on Figure 1. The shear webs of cells 1 to 6 and 8 were aligned longitudinally to the pile, while the shear webs of cell 7 were aligned transverse to the pile axis. Cell 9 had its shear web aligned with the horizontal load.

![Diagram of cell base showing strain-gauging of normal webs N1, N2 and N3](image)

**Figure 2. Load cell instrumentation**

The three strain gauge circuits are not independent and calibration of the load cell involves the determination of a 3 by 3 matrix relating normal load, eccentricity and shear load to the output voltages from the three circuits. Calibration was carried out by suspending known weights on a hanger attached to the cell via a calibration cap. The cells were expected to give an accuracy of better than ±5%.

The load cells along the shaft of the pile were fitted with top caps curved to match the curvature of the pile. All screw holes were sealed with Araldite and the clearance round each cell was filled with a soft-drying silicone to keep out sand particles and isolate the top of the cell from load in the pile.

3 TEST RESULTS AND INTERPRETATION

3.1 Testing Sequence
The instrumented model pile was first jacked into a prepared bed of dense sand to an embedment of 750 mm. The maximum jacking load required was about 26 kN of which 85% was taken at the pile toe. On removal of the jacking load, the pile was almost completely unloaded to a depth of 175 mm below the sand surface with tensile shear stresses below that depth. A normal stress amounting to about 26% of the maximum jacking load was retained at the pile toe. Jacking also gave rise to substantial radial stresses against the pile shaft, reaching a maximum value of about 500 kPa towards the pile toe. Removal of the jacking load led to the virtual disappearance of radial stress to a depth of 175 mm with radial stress increasing approximately linearly with depth below that, reaching a maximum value of about 200 kPa.

After the removal of the jacking load, the pile was subjected to incremental horizontal loading firstly in one direction followed by unloading and horizontal loading in the reverse direction. The horizontal load was applied at a height of 375 mm above the surface of the sand bed and to avoid any possibility of the pile yielding in bending the maximum level of horizontal load in either direction was limited to 0.47 kN. At each increment of horizontal load, readings were taken of the applied load, the pile deflections with depth and of the response of the load cells mounted in the pile.

### 3.2 Response to Horizontal Loading

The overall response of the pile to horizontal loading, represented by the plot of horizontal load versus pile deflection at the surface shown on Figure 3, was slightly non-elastic and resulted in significant irrecoverable pile deflection. The response on horizontal loading in the reverse direction was significantly more stiff than that on initial horizontal loading and final unloading restored the pile to its original location.

![Figure 3. Overall response to horizontal loading](image)

The response measured by the load cells indicated that essentially only the leading face of the pile (that half of the pile moving into the sand and undergoing compression) took part in the response of the pile to horizontal loading. Apparently no separation was induced between the sand and the back of the pile as horizontal loading produced a negligible change in the stresses against the back of the pile. Further, horizontal loading resulted in a negligible change in the stresses at the pile toe.

At each level of horizontal load, taking the normal load read from load cells 1 to 6 and 8 to be evenly distributed transversely indicated a total horizontal load of about 1.58 times that applied. The distribution of normal stress (aligned with the horizontal load) across the leading face of the pile is therefore not uniform but diminishes away from the centreline of the pile. Plots of the average normal stress with depth for each of the three levels of horizontal load are shown on Figure 4.

![Figure 4. Average normal stress versus depth](image)

The load cells also indicated that relative slip between the pile and the sand on flexing of the pile under horizontal loading gave rise to longitudinal shear stresses on the compression side of the pile. The transverse distribution of these stresses is likely to be similar to that of the normal stresses and on this basis the average shear stress may be plotted against depth as shown on Figure 5. It can be seen that significant flexing of the pile was confined to the upper half of the embedded length. The longitudinal shear stresses disappeared on removal of the horizontal load. Similarly, the stresses in the horizontal plane mobilised to resist the applied horizontal load essentially disappeared on removal of the load.

![Figure 5. Average longitudinal shear stress versus depth](image)
3.3 Transverse Distribution of Stress on Horizontal Loading

The readings taken from load cell 7, mounted at an angle of 66° to the line of horizontal loading, together with the average normal stress at the level of load cell 7 taken from Figure 4 may be used to estimate the transverse distribution of stress on horizontal loading. From these readings it is known that the radial stress induced by horizontal loading reaches a maximum at the pile centreline and at this location it is reasonable to assume that the transverse shear stress is negligible. Away from the centreline the radial stress diminishes and the transverse shear stress increases. It is considered that at the extreme edges of the pile both the radial stress and transverse shear stress must tend to zero as these stresses are negligible over the back of the pile. The approximate distributions of radial and transverse shear stresses at the level of load cell 7, which satisfy the above constraints and force equilibrium, are shown on Figure 6 for a horizontal load of 0.47 kN. These stresses may be resolved into stresses aligned with the horizontal load and perpendicular to it and these components are also shown on Figure 6. The approximate distribution of the resultant stress is shown on Figure 7 to be nonuniform and directed approximately radially towards the pile around much of its circumference.

![Figure 7. Approximate distribution of resultant stress](image)

Using equation (1) it was found that over the range of horizontal load applied the relationship between $E_s$ and depth $z$ is given approximately by:

$$E_s = 500 z^{3.0}$$

(2)

![Figure 6. Approximate transverse distributions of stress](image)

3.4 Soil Modulus from Experimental Results

The modulus of the sand on horizontal loading of the pile is given at any depth by:

$$E_s = \frac{p}{y} = \frac{(\text{Average normal stress})D}{y}$$

(1)

where $E_s$ is the soil modulus

- $p$ is the resistance to horizontal load per unit length of pile at any depth
- $y$ is the pile deflection at any depth, and
- $D$ is the diameter of the pile.

4 COMPARISON BETWEEN EXPERIMENT AND THEORY

The theoretical approaches for predicting the behaviour of a single pile subjected to horizontal loading are based on an elastic model of the soil. A solution is obtained by one of three methods.

(i) Solution of the differential equation for the bending of a beam supported by an elastic medium, based on Winkler's...
assumption (that the reaction of the medium is proportional at every point to the deflection of the beam at that point).

(ii) Solution by the boundary element method.

(iii) Solution by the finite element method.

In all methods the resistance to horizontal loading is assumed to be provided solely by a stress uniform across the width of the pile and directed parallel to the horizontal load. In general, no allowance is made for relative slip between the pile and the soil on horizontal loading which would lead to the generation of longitudinal shear stresses. The methods are numerically exact for a soil modulus which is constant with depth, but approximate solutions can be obtained for any distribution of modulus with depth. All three methods give similar solutions.

For the purposes of comparison a solution by Poulos (1971) using the boundary element method has been used. The horizontal load versus surface pile deflection predicted using this solution, assuming the experimentally determined soil modulus distribution with depth (given by equation (2)), is plotted on Figure 3. Comparison with the experimental curve also shown on Figure 3 shows that the theoretical solution overestimates the pile deflection for a given level of horizontal load. This results from the theoretical solution assuming an oversimplified distribution of stress resisting the applied horizontal load. The actual distribution of stress mobilised parallel to the horizontal load is approximately parabolic with a maximum value at the pile centreline. The stress mobilised perpendicular to the horizontal load, which is not considered in the theoretical solution, increases from an assumed value of zero at the pile centreline to a maximum value towards the edge of the pile. The perpendicular stress produces no net load on the pile but its existence implies that sand to the sides of the leading face of the pile contributes to the resistance of the pile/sand system to horizontal loading.

Flexing of the model pile on horizontal loading resulted in the generation of longitudinal shear stresses on the compression side of the pile over the upper half of its embedded length. The theoretical solution precludes relative slip between the pile and sand and so cannot allow for this effect. It is considered that ignoring the longitudinal shear stresses has negligible effect on the calculated horizontal pile deflections, although vertical pile displacements may be underestimated.

5 CONCLUSIONS

The following conclusions may be drawn from the work presented in this paper.

(i) The application of horizontal load to a pile jacked into a bed of dense sand is resisted by a nonuniform stress directed approximately radially towards the pile around much of its circumference and confined essentially to the leading face of the pile. The stress reaches a maximum value at the centreline of the pile, decreasing towards its edges, and essentially disappears on removal of the horizontal load.

(ii) Flexing of the pile on horizontal loading gives rise to longitudinal shear stresses on the compression side of the pile, most significantly over the upper half of the embedded length. These stresses disappear on removal of the horizontal load.

(iii) The experimental results indicated that the modulus of the sand was approximately proportional to depth to the power 3.0 with a constant of proportionality of 500, over the range of horizontal load applied.

(iv) Using the experimentally determined soil modulus distribution with depth, a representative theoretical solution overestimated the pile deflection for a given level of horizontal load, compared with the measured pile deflection. This is largely due to the theoretical solution not considering stress mobilised perpendicular to the horizontal load and therefore ignoring the contribution to the resistance of the pile/sand system to horizontal loading of sand to the sides of the leading face of the pile.

6 REFERENCES
