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The Use of the Finite Element Method for the Stability Analysis of Earth and Rockfill Dams

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SUMMARY A simplified procedure for predicting the embankment stability using stresses calculated from finite element analyses is presented and discussed. This method is based on the principal assumption that the component materials along the failure arc considered provide resistance proportional to the local safety factor of the relevant elements of a finite element grid, the length of failure arc in each material zone and the shear strength of the materials in each zone. Applications of the method to some existing earth and rockfill dams are presented and results are compared with conventional limit equilibrium methods. A review of existing methods of slope stability analyses is also made. It is found that the proposed method provides a rational basis for determining the safety factor of embankment slopes.

NOTATION

c	cohesion
d, F	Poisson's ratio parameters (Hyperbolic stress-strain model)
F_{FE}	Factor of safety based on finite element method
F_{FE1}	F.E safety factor based on stress level
F_{FE2}	F.E safety factor based on shear stress
F_{FE3}	New definition of F.E safety factor
F_{LE}	Factor of safety based on limit equilibrium methods
G	Poisson's ratio parameter (Hyperbolic model)
K	Modulus number (Hyperbolic model)
n	Modulus exponent (Hyperbolic Model)
R_f	Failure ratio (Hyperbolic Model)
s	Shear strength
ΔL	Increment of length along the failure arc
γ	Unit weight
σ	Normal stress on the slip surface
σ_1, σ_3	Major and minor principal stresses
$(\sigma_1-\sigma_3)_f$	Principal stress difference at failure
τ	Shear stress required for equilibrium
ϕ	Angle of internal friction

1 INTRODUCTION

Geotechnical Engineers with the ready access to computers frequently use the finite element method to solve their design problems.

The application of the finite element method to analysis of stresses and movements in earth and rockfill dams has been well established over the past 10-15 years: however, the use of this method in predicting the stability of dams has not received wide attention.

A simple procedure for predicting embankment stability using the stresses calculated from finite element analyses is presented in this study. The advantages this method has over the conventional limiting equilibrium methods are the ability to predict the safety factor of a general slip surface and the ability to accommodate complex material behaviour and loading conditions.

2 LIMIT EQUILIBRIUM STABILITY ANALYSIS METHODS

Limit equilibrium stability analysis methods have been used widely and successfully by the profession although these methods are subject to the following theoretical objections (Wright et al., 1973).

(i) Arbitrary assumptions, mostly concerning the locations or directions of side forces on slices, are employed so that the normal stress on the shear surface may be determined using only the conditions of static equilibrium, without consideration of the stress-strain characteristics of the soil.

(ii) Most of the equilibrium methods (Bishop's simplified method, 1955; Janbu's Generalized Procedure of Slices, 1957 and Morgenstern and Price method, 1965) involve the assumption that the factor of safety is the same for every slice; an assumption valid only at failure, when the factor of safety is equal to one for every slice.

(iii) Some of the equilibrium methods, including the Ordinary (Fellenius) Method of Slices, Bishop's simplified method, and the wedge methods based on force equilibrium do not satisfy all conditions of equilibrium.

Comparisons of limit equilibrium methods of stability analysis have been published previously (e.g. Fredlund and Krahn, 1977; Duncan and Wright, 1980). Table I briefly compares the five most widely used methods, in the order in which they appeared in literature.

It is found that all equilibrium methods of slope stability have the following characteristics:

(i) the use of the same definition of the factor of safety

$$F_{LE} = s/\tau \qquad (1)$$

(ii) the use of the implicit assumption that the stress-strain characteristics of the soils forming the slopes are non-brittle, and that the same value of shear strength may be mobilized over a wide range of strains along the slip surface.

TABLE I

COMPARISON OF EQUILIBRIUM METHODS

Method	Equilibrium conditions satisfied: Overall moment	Ind. slice moment	Vertical force	Horiz. force	Shape of slip surface	Assumptions regarding side forces
Ordinary (Fellenius) Method of Slices 1936	Yes	No	No	No	Circular	Resultant of side forces is parallel to the base of each slice
Bishop's Simplified Method - 1955	Yes	No	Yes	No	Circular	Resultant of side forces is horizontal (no vertical side forces)
Janbu's Generalized Method of Slices 1957	Yes	Yes	Yes	Yes	Any	Location of side force resultants on sides of slices (location can be varied)
Morgenstern and Price's Method 1965	Yes	Yes	Yes	Yes	Any	Pattern of variation of side force inclination (θ) from slice to slice : $\theta = \lambda f(x)$. The value of f(x) is assumed at each interslice boundary, and the value of λ is an unknown.
Spencer's Method 1967	Yes	Yes	Yes	Yes	Any	Side forces are parallel (θ = constant). Corresponds to f(x) = constant in Morgenstern and Price's method.

(iii) the use of some or all of the equations of equilibrium to calculate the average value of τ and σ to determine the shear strength using the Mohr-Coulomb failure criterion

$$s = c + \sigma \tan\phi \qquad (2)$$

(iv) the use of explicit assumptions to supplement the equations of equilibrium. Since the number of equilibrium equations is smaller than the number of unknowns in the problem, all methods employ assumptions to enable solution.

3 THE FINITE ELEMENT STABILITY ANALYSIS METHOD

3.1 Previous Investigations

The use of finite element stresses to calculate safety factor of embankments was first introduced by Kulhawy et al. (1969), using the same definition of safety factor employed in limit equilibrium analysis procedures. Subsequently Wright et al. (1973) used these definitions and studied the variation of factor of safety along the shear surfaces for various slope angles and Poisson's ratio values. Some comparisons were made between the limit equilibrium and the finite element stability analysis methods.

Two definitions were used, namely, the safety factor based on stress level (F_{FE1}) and the safety factor based on shear stress(F_{FE2}).

3.1.1 Factor of safety based on stress level (F_{FE1})

This method uses the values of stress level or fraction of strength mobilised, $(\sigma_1-\sigma_3)/(\sigma_1-\sigma_3)_f$. The reciprocal of this ratio is interpreted as being equal to the value of safety factor against local failure within each element of the finite element mesh. It is assumed that the minor principal stress is the same at failure, as for the mobilised stress state.

The stress levels of elements along the failure surface are multiplied by the corresponding lengths of failure arc ΔL. These values are then summed over the total number of elements and divided by the full length of failure arc to obtain a weighted average value of stress level for the shear surface. The safety factor F_{FE1} is defined as the reciprocal of this average value.

3.1.2 Factor of safety based on shear stress (F_{FE2})

In this method, the factor of safety is defined with respect to the value of shear stress for a potential failure surface. This factor of safety is expressed as the ratio $\frac{\Sigma(c+\sigma\tan\phi)\Delta L}{\Sigma\tau\cdot\Delta L}$ in which the summation (Σ) indicates that both the shear strength and the shear stress are summed over the segments of arc (ΔL) within each element along the shear surface. The value of σ, the normal stress on the shear surface is assumed to be the same in the equilibium and failure states. Values of normal stress and shear stress for different shear surfaces (failure arcs) are calculated from the finite element stresses.

3.2 New Definition of F_{FE}

The definitions F_{FE1} and F_{FE2} do not take into account the strength of the materials and the local safety factor simultaneously. A stronger material zone which has not fully mobilised its strength could resist the slip failure even when the other material zones along the slip surface have reached the failure state.

A new definition F_{FE3}, combining F_{FE1} and F_{FE2} which accomodates the safety factor against local failure within each element and its shear strength is proposed in this study. It is based on the prinicipal assumption that the component materials along the failure arc provide resistance proportional to the local safety factor of the relevant elements of the finite element grid, the length of failure arc in each material zone and the shear strength of the materials in each zone.

The safety factor against local failure is defined similarly to that in F_{FE1}. The minor principal stress is assumed to be the same at failure, as for the mobilised stress state.

The shear strength is defined by the Mohr Coulomb criterion. It is also assumed that the normal stress on the shear surface is the same in the equilibrium and failure states.

The safety factor is expressed as the ratio

$$F_{FE3} = \frac{\sum \left\{ \frac{(\sigma_1 - \sigma_3)f}{(\sigma_1 - \sigma_3)} \cdot (c + \sigma \tan\phi)\, \Delta L \right\}}{\Sigma \{(c + \sigma \tan\phi)\, \Delta L\}} \qquad (3)$$

The summation being taken over the total number of segments of arc (ΔL) along the failure arc.

The steps involved in the calculation of F_{FE3} are as follows:

(i) choose the failure arc for which the factor of safety is to be determined;

(ii) using the stresses from the finite element analysis for the given loading condition, select the stress levels $(\sigma_1-\sigma_3)/(\sigma_1-\sigma_3)f$, of the elements (with centroids nearest to the mid point of each increment of length ΔL) along the failure arc;

(iii) calculate the maximum shear strength corresponding to each increment of length (ΔL) of failure arc in each element using equation (2).

Note: The normal stress in equation (2) is determined using the stresses of the 4 elements (with centroids nearest to the mid point of the increment of length ΔL) in a first order finite difference equation.

(iv) substitute into equation (3)

It is noted that the assumptions involved in the limit equilibrium methods are eliminated in this definition. However, several other assumptions have been added, as stated above. Both limit equilibrium and finite element definitions of safety factor commonly employ the Mohr-Coulomb failure criterion.

4 APPLICATIONS

The safety factors based on Bishop simplified limit equilibrium method and the finite element method have been calculated for the upstream slope for the end of construction stage of Dartmouth and Talbingo Dams - the highest earth and rockfill dams in Australia. The minimum safety factors based on Bishop simplified method (F_{LE}) were determined using the computer programme BISHOP (SR & WSC, 1982). The finite element stresses and the corresponding safety factors (F_{FE}) were determined using the computer programme FEED (Adikari, 1981b).

Non-linear incremental finite element analyses were performed using hyperbolic stress strain model. Full details of the finite element investigations on Talbingo and Dartmouth dams are presented in Adikari (1981a) and Adikari (1982) respectively.

Figures 1 and 2 illustrate the embankment arrangements and the finite element meshes for Talbingo and Dartmouth Dams respectively. The material parameters used in the finite element and limit

TABLE II

EMBANKMENT MATERIAL PARAMETERS

Dam and Height	Symbols and Units	Core Zone 1	Filters Zone 2A	Transitions Zone 2B	Rockfill Zone 3
Talbingo 162 m	K	250	430	850	680
	n	0.45	0.45	0.82	0.44
	d	4.80	7.40	7.70	7.70
	F	0.05	0.14	0.07	0.06
	G	0.32	0.29	0.21	0.25
	R_f	0.86	0.60	0.76	0.71
	c (kNm^{-2})	85.0	0.0	0.0	0.0
	ϕ (Deg.)	23.0	45.0	45.0	45.0
	γ (kNm^{-3})	18.1	20.4	20.4	20.4
Dartmouth 180 m	K	350	600*		600
	n	0.41	0.68		0.31
	d	4.20	6.90		7.00
	F	0.40	0.48		0.22
	G	0.12	0.21		0.12
	R_f	0.84	0.72		0.71
	c (kNm^{-2})	60.0	0.0		0.0
	ϕ (Deg.)	29.0	45.0		43.5
	γ (kNm^{-3})	21.0	23.1		22.0

*Note: Zone 2 includes 2A and 2B filters and transitions.

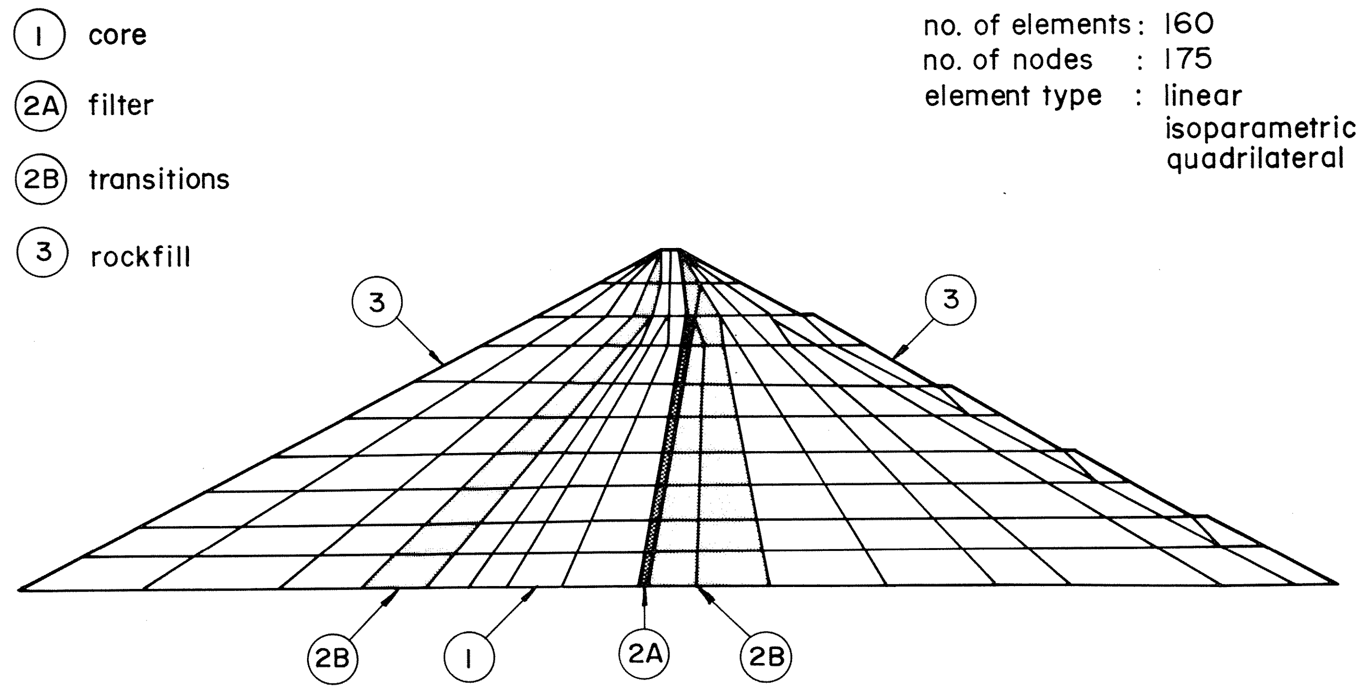

Figure 1 Embankment arrangement and the finite element mesh for Talbingo Dam

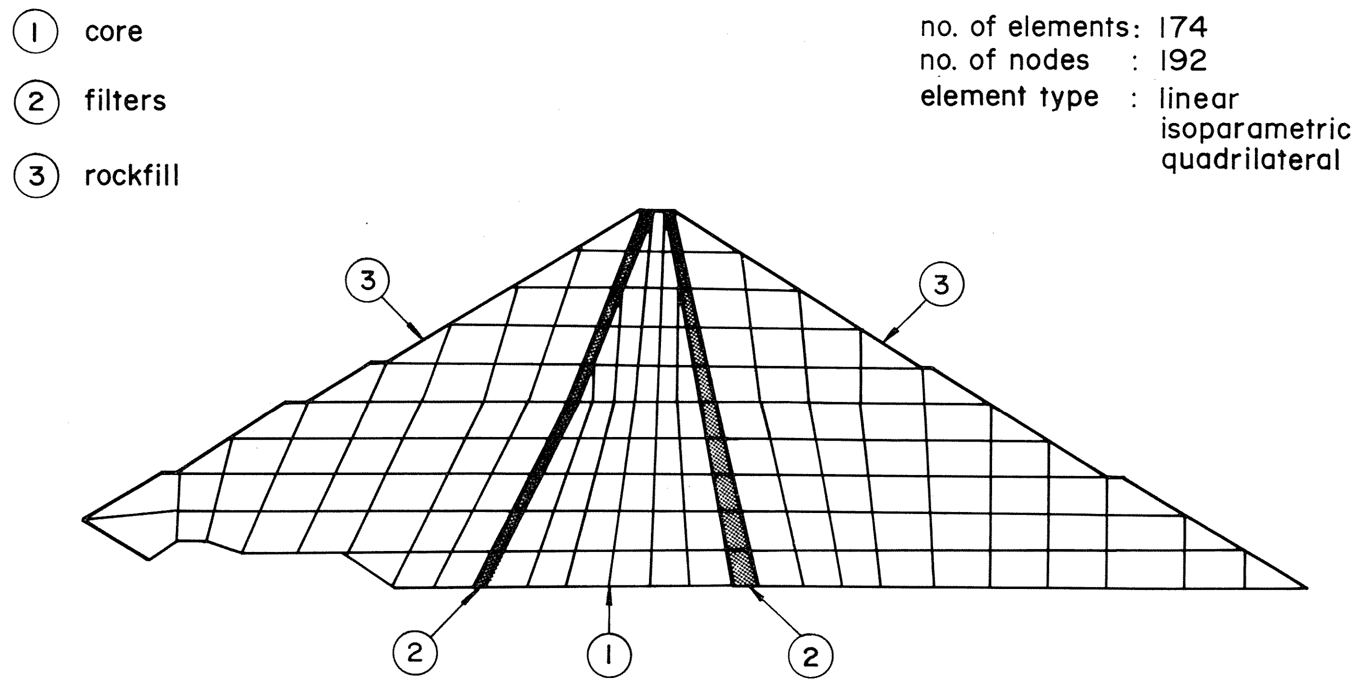

Figure 2 Embankment arrangement and the finite element mesh for Dartmouth Dam

TABLE III

COMPARISON OF FACTORS OF SAFETY

END OF CONSTRUCTION, UPSTREAM SLOPE

	Critical circle information*				Total stress factors of safety			
Dam	X (m)	Y (m)	Rad (m)	Based on	F_{FE1}	F_{FE2}	F_{FE3}	F_{LE}
Talbingo	42.5	400.0	400.0	F_{LE}	2.13	2.09	2.21	2.18
	48.7	401.4	400.0	F_{FE3}	2.04	2.20	2.17	2.18
Dartmouth	58.1	417.0	383.0	F_{LE}	2.00	2.05	2.13	2.02
	69.5	420.0	388.0	F_{FE3}	1.84	2.15	1.96	2.12

* Note: Origin of X,Y axis is the intersection point of upstream rockfill slope and embankment base.

equilibrium stability analyses are presented in Table II.

Table III summarises the safety factors obtained. It can be seen that the three values of F_{FE} are close to each other and to the F_{LE} value, confirming that the assumptions made in both the F_{FE} and F_{LE} definitions are justifiable. The agreement between F_{FE} and F_{LE} also indicates that the finite element stresses can be used successfully in determining safety factor values in dam design.

It should be noted that "end of construction" is not the most critical case in the overall stability of the two dams considered. Lower safety factors have been obtained for other load cases, such as rapid drawdown. These load cases need to be treated separately, using the effective stress analysis.

It is further observed that the F_{FE1} is the lowest of all safety factor values appearing in Table III. F_{FE2} and F_{FE3} are either higher or lower than F_{LE} depending on whether the critical circle chosen is based on F_{LE} or F_{FE3}. Validation of these observations for cases with safety factor values less than 2.0 is needed.

The finite element safety factor method has several advantages over the limit equilibrium methods. It is applicable to a general slip surface and the method can accommodate complex material behaviour, internal arrangements and loading conditions. Once the stress distribution within the embankment is determined using the finite element method, the safety factors for a large number of trial surfaces can be computed inexpensively.

In general, limit equilibrium methods are inaccurate at low and high safety factor regions ($F_{LE} < 1.0$; $F_{LE} > 2.0$). Due to their inability to represent the modulus and Poisson's ratio values of the component materials, they provide misleading safety factors when stress redistribution and large strains take place within different parts of the dam.

In addition, all finite element analyses give other important information such as stresses, movements and pore pressures at every point inside the dam, thereby giving the designing engineer a greater insight to the behaviour of the structure.

5 CONCLUSIONS

A stability analysis method based on the stresses within the soil mass of the embankments calculated using the finite element method was presented and discussed. The following conclusions are derived from the results presented:

(i) The finite element stability analysis method provides a rational basis for determining the safety factor of embankment slopes.

(ii) The assumptions involved in the finite element stability analysis method are different from those involved in the limiting equilibrium methods, giving two independent approaches to the assessment of the stability of earth and rockfill dams.

(iii) More investigations are needed (a) on the influence of the material model on F_{FE}, (b) on effective stress stability analysis and (c) on cases with lower safety factors. These areas are currently being investigated at the State Rivers and Water Supply Commission and Monash University.

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