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The Pyramids, Large Scale Rock Mechanics Modelling

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SUMMARY The history of pyramid building in ancient Egypt is interpreted as a trial and error approach to the understanding of material behaviour. The resulting structures can be analysed to provide parameters relevent both to rockfill and to bedded sedimentary formations, in open cut.

1 INTRODUCTION

In rock mechanics, the application of test results to field conditions often permits a degree of speculation. In general, larger tests are considered more realistic, but reality can be expensive. The ancient Egyptians, however, appeared undaunted by economic constraints and modelled rock behaviour on a scale not available today even to the most expensively funded research/investigation programme. Thirty tonne blocks were transported as a normal procedure from Aswan to Giza, a distance of some 800 km, while blocks of 200 tonnes were lifted into place during construction.

Leaving aside aesthetic considerations, the building of pyramids can be seen as full scale model tests. Based on the repeated observations of the limits of material behaviour, or what we might now call past experience, these megalithic architects have provided us with a legacy which is of direct application to geomechanics.

2 HISTORY OF PYRAMID CONSTRUCTION

Table 1 lists the relevent data on the major pyramids, extracted from Edwards (1947). For the purposes of the following discussion, several categories are proposed.

2.1 Rockfill Pyramids

The forerunners of the pyramids were low rectangular and ephemeral tombs called mastabas, built until the early centuries of the third millennium B.C. There appears to have been no transition from these to the larger pyramidal structures, and the Step Pyramid, designed by Zoser about 2680 B.C., stands as a singular advancement. With a height of almost 70 m., the Step was built around a core of local stone, no doubt placed by hand, and finished with an outer casing of smallish, almost square, limestone blocks. The outer faces incorporated five berms, or steps, to give average slopes of 45 degrees on the E. and W. faces, and 48 degrees on the N. and S. The former 1:1 slopes might well have been chosen for convenience of construction, the latter provide a measure of the inclination at which rockfill was able to stand, presumably when densely packed.

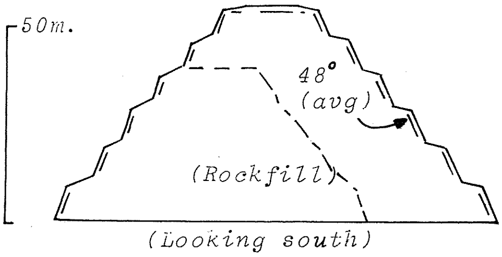

Figure 1 Outline of the Step Pyramid

Now, it is reasonable to suppose that this area of Egypt has been subject to earthquakes in the last four and a half thousand years. Assuming the rockfill to be cohesionless, a simple formula can be used to analyse the structure,

$$\tan\theta = \frac{\tan\phi - a}{1 + a\tan\phi}$$

where θ is the critical slope, a is the ground acceleration as a proportion of gravity, and ϕ is the angle of shearing resistance of the rockfill. Taking 48 degrees as safe, the pertinent angles of shearing resistance, for various earthquake forces and zero pore pressures, are tabulated in Table 2.

The next pyramid, that of Sekhemket, was also a step type, also of rockfill, but with side slopes of 49½ degrees attempted. This pyramid is no longer

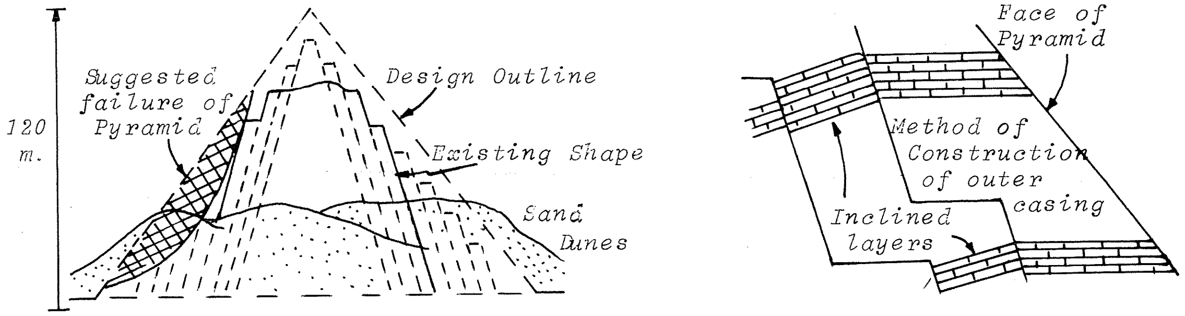

Figure 2 Pyramid of Medium, with detail of construction technique

TABLE 1

Pyramid	Height (m)	Slope (°)	Construction Materials	Remarks
Step (Zoser) 2680 B.C.	63	45 & 48	Core of local stone, outer facing of small limestone blocks	Intact. Core possibly represents dense rockfill
Sekhemket	70	49½	as above	Rubble
Medium c. 2600 B.C.	119	53	Stone core; six thick masonry coatings, inclined inwards	Badly damaged; possible slip at 200 to 250' height
Bent	102	54½ & 43½	Similar to Medium	Intact; slopes reduced at approx. 160' height
Dashur	91	43½	Limestone blocks laid horizontally	Intact; first true pyramid
Cheops c. 2550 B.C.	146	<52	Similar to Dashur	Intact, outer facings gone
Cephren	143	>52	Similar to Dashur	Intact, " " "
Mycerimus	66	>51	Similar to Dashur ?	Intact
Sahure c. 2480 B.C.	49	51½	Sand, stones and Nile muds	Rubble
Pepi II c. 2200 B.C.	52	53	Similar to Sahure	Rubble; massive girdle at base to increase stability

standing, but whether less care was taken with the rockfill, or whether 49½ degrees was just too steep under possible earthquake conditions, is not known. Nonetheless, this does suggest that the 48 degrees of Zoser was close to the critical slope.

2.2 Layered Structures

Within a hundred years of the Step, pyramids were being encasedwith more than just an outside veneer of limestone blocks. Medium (c. 2600 B.C.) appears to be the first of the new type. Here, slabs of limestone were laid, apparently without mortar, so as to slope inwards at about 15 degrees and form batters of 75 degrees. An overall outside slope of 53 degrees was achieved by stepping, Fig. 2. Like the pyramid of Sekhemket, this is no longer fully intact and the shaded part of the figure is all that remains. Such a relic, all the same, is useful since it could indicate failure over a height of about 75 m., as sketched. Further, the residual standing portion provides information on material strength and this is discussed later.

Of similar construction is the Bent Pyramid, Fig.3. Begun at even steeper slopes, 54½ degrees, this went to around half way when drastic changes were made in design. The pyramid was finished "in haste" at reduced slopes of 43½ degrees.

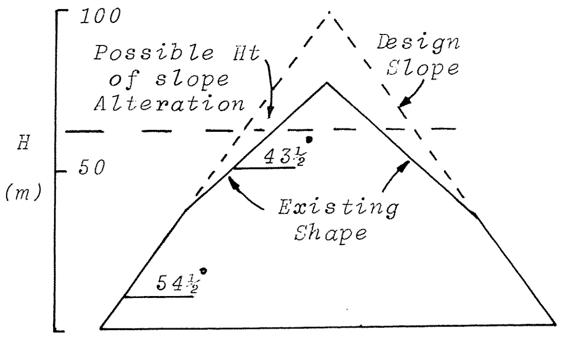

Figure 3 The Bent Pyramid

The original batters of this pyramid could well have been chosen for convenience; a slope of 54½ degrees means that the corner ridges rise up at a simple 1 :1 slope. The reason for the sudden change in design at half height is less easily inferred, but it would not be surprising if the change had been necessitated by threatened failure of the slopes, during construction. Whether this was earthquake initiated or not is difficult to say, but the height at which the change was made could well have been close to that at which completion of the structure would have been permitted by cannibalisation of the outside layers of the original steeper batters. Perhaps 60 m. is a reasonable guess.

Following on from the possibly unpleasant experiences of the Bent Pyramid, the architect of the pyramid of Dashur favoured a conservative approach, using slopes of only 43 degrees overall. This pyramid is well preserved and, despite its conservative slopes, it incorporates a design innovation which later builders also used: the limestone slabs were now laid horizontally, instead of sloping inwards.

The main pyramids at Giza came after that of Dashur and all incorporated steep and stable slopes to their full heights of almost 150 m.: Cheops at 51 degrees 52 minutes; Cephren at 52 degrees 20 minutes and Mycerimus at just under 51 degrees. All are well preserved today, although their outside veneers have long since slid off.

Because of their sizes, these Giza pyramids can be taken as indicative of safe slopes for a semi-infinite height, in horizontally layered limestones. Allowances for earthquake forces can again be made, approximately, giving angles of shearing resistance for this horizontally layered situation, Table 2.

2.3 Stones and Mud

Compared with the monumental Giza phase, later pyramids are inferior structures, built largely of sand, stones and Nile mud. Attempts were made to keep to the old design slopes of around 50 degrees, but the material composition was unable to oblige - especially when the Nile muds dried out. These structures are now generally little more than heaps of rubble. The pyramid of Pepi II suggests that evidence of instability had asserted itself during construction; the base is surrounded by a massive girdle for stabilisation.

Whether through shortages of material, or natural disasters, is uncertain, but the practice of pyramid building went out of fashion soon after this time. Smaller brick pyramids were built, around a millennium and a half later, in the Sudan. Photographs show these structures to be quite steep and consequently subject to slope failures but, to the writer's knowledge, there is little data on them.

TABLE II

Pyramid Type	Slope (°)	Angle of Shearing Resistance		
		a = 0	a = 0.1	a = 0.2
Rockfill, Step	48	48	54	59
Horiz. layers (Giza)	52	52	58½	64

2.4 Summary

A simplified approach to stability analyses can be made from a height versus slope plot, as shown on Fig. 4. This plot incorporates not only the extant pyramids and the known failures, but also the stable relics of the failed structures, when available. The data suggest a family of curves for the different types of construction. For comparison, a similar plot for the Teewah Sands, a horizontally bedded, faintly indurated sand, exposed in sea cliffs in S.E. Queensland, is also given (James 1980).

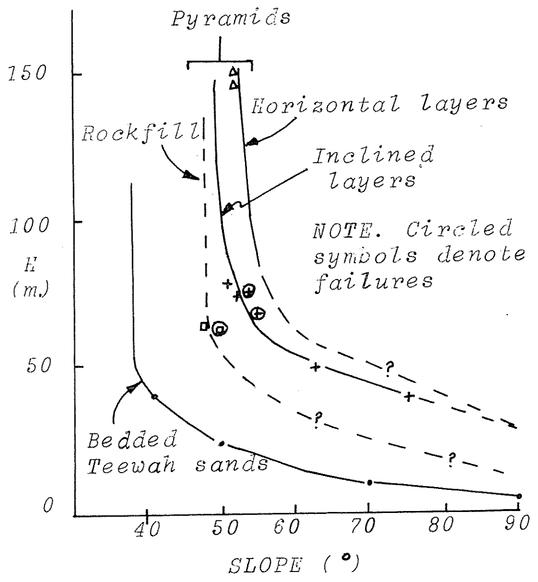

Figure 4 Slope versus Height in Pyramid Structures

Two significant points are illustrated by the interpretation of Fig. 4.

Firstly, the curve based on the inclined layer pyramid suggests that some apparent cohesion is acting at low stress levels, despite the historical notes as to the absence of mortar. This is perhaps not unexpected, where limestones and sandy limestones are used and exposed to the weather. Alternatively, this apparent cohesion could reflect, instead, the marked curvature of the failure envelope typically found in tests on rockfills at low stress levels, (Charles 1979, 1980). It is interesting to note that this particular effect dies out rapidly, to become of little or no significance by heights of 60 to 80 m. At such heights, stress levels on potential shear planes might be of the order of 200 to 300 kPa.

Secondly, the curves suggest that, for stability, the angle of shearing resistance for the inclined layer situation could be several degrees less than that for the horizontal layering. This idea is taken further in Section 3.2.

3 APPLICATION

As mentioned, Egypt has almost certainly been subject to earthquake forces over the last four and a half thousand years. The angles of shearing resistance which must then have applied to keep the structures stable are encompassed by the values given in Table 2. Based on this assumption, the pyramid experience provides two main applications for present day geomechanics.

3.1 Rockfills

The Step Pyramid suggests that a 48 degree slope, for a semi-infinite height in densely packed rockfill, has a probability of being seismically stable. A value of ϕ up to 55 degrees might therefore be reasonably attributed to this material. This is considerably higher than traditionally employed in design, although values of this order have been recorded in low stress level tests on rockfills, (Marachi et al 1972, Charles & Watts 1980). The writer has also measured basalt screes at slopes up to 50 degrees, in special circumstances, i.e. where dynamic effects are absent.

One of the conclusions of the Marachi tests was the suggestion that the ϕ value reduced with increasing "grain" size. This was not encountered in the tests by Charles and Watts, nor does the pyramid experience appear to support this trend. This finding has important economic consequences for dam design, and steeper batters than traditionally employed ought, in many cases, to be stable, provided protection is given to the outermost zones. Care with the treatment and placing of rockfill might therefore pay off by reducing fill quantities, which would certainly be of benefit in third world countries where rockfill could be placed by hand.

3.2 Open Cuts

Measurement of the angle of shearing resistance of rock with regular planes of weakness has been studied in the laboratory by Brown (1971) and others. The problems of size effects associated with laboratory testing are overcome by the full scale models of layered rocks from the pyramid of Medium to that of Cephren. These can be analysed as follows:

The maximum ϕ value for the horizontal-layered situation can be extracted from Table 2. With due allowance for earthquake forces, a value approaching 60 degrees would not be unreasonable. When the beds dip gently into the face, as at Medium, this value is reduced slightly, and a ϕ of around 55 degrees might be a first estimate. The Bent Pyramid, at 54½ degrees to some 60 m. height, gives a lower limit to this. The minimum strength of the layered situation occurs when the beds dip out of the face at approximately the value of the angle of shearing resistance along the bedding. This value is not available from the pyramids, but it is typically amenable to measurement in the laboratory or field. Here, a value of ϕ in the vicinity of 45 degrees is suggested for clean joints. By contrast, the strength at right angles to the bedding is more difficult to estimate, but might take some intermediate value between the extremes just given, depending on joint spacing, etc. A value around 50 degrees does not appear out of the question, again for unweathered joints.

Using the above figures, a curve for the angle of shearing resistance against bedding orientation can be drawn, for stability purposes. This has been done in Fig. 5 and the pyramid experience is given by curve A.

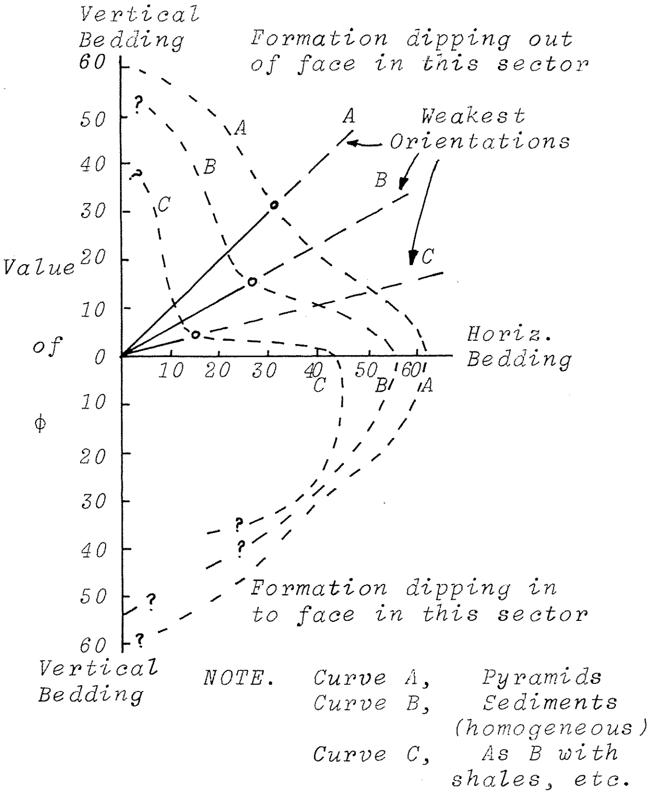

Figure 5 Variation of ϕ with bedding dip for stability of high rock slopes

In practice, of course, the problem of strength and stability is compounded by the effects of flexural slip between beds. This effect might justifiably be taken as ubiquitous, (Casinader 1980, Deere 1973 and James 1981).

Curve B then represents the situation in nature, allowing for flexural slip in a homogeneous bedded limestone or sandstone formation. In the absence of argillaceous beds or partings, a ϕ value somewhere in the vicinity of 30 degrees could obtain along the bedding planes. Again, this bedding plane strength is the most simple to measure or otherwise determine from field observations. Theoretically, mass rock strength should be unaffected by planes of weakness at right angles to the major stress. However, in practice, some allowance for the effect of flexural slip might be advisable, even in the horizontally bedded situation; a value of ϕ slightly than for the pyramids is therefore suggested, perhaps 55 to 57 degrees. Using these values and theprinciples stated earlier, curve B is presented as a guide to rock mass strength available for stability in high rock slopes of homogeneous limestone or sandstone. Obviously, major discontinuity patterns, if they exist, could be treated as bedding planes.

Curve C represents the situation where the formation contains argillaceous beds or seams. Here, flexural slip will be concentrated along the boundaries of the weaker members and will reduce the value of ϕ to perhaps as low as 15 degrees. Some effect of these weak seams might also be manifest where the bedding is horizontal, or dipping slightly into the face, and a hypothetical reduction has been made to allow for this. Curve C, it is suggested, would probably be the most commonly applied in practice.

Observations made on open cut mines in Queensland (Mallett & Wooltorton, 1981) lend some support to this suggestion. The authors indicate that high wall failures are common when the beds dip out of the face, almost irrespective of that dip. The reason for this can be seen from curve C: the relevent strength for stability drops dramatically for even very minor dips out of the face, and it remains very low until the dips become almost subvertical.

4 CONCLUSIONS

The pyramids have been proposed as rock mechanics models which have stood the test of time. Shear strength parameters deduced from their performance tend to confirm the higher range of values found in laboratory tests at low stress levels.

The above discussions have not included the effects of cohesion in the rock mass. This parameter often constitutes the greatest unknown in both short and long term stability problems. However, in many cases, it appears apparent cohesion can be explained as largely the result of curvature of the Mohr envelope, Fig. 4; and that this effect drops off rapidly by around 60 to 80 m. height. Since 50 m. is often the height at which high wall stability assumes importance in many open cut coal mines, it could be argued that the effect of cohesion should be ignored, provided full advantage is taken of the angles of shearing resistance at low stress levels. Such an approach would help to simplify not only many design problems, but also back analyses of failures in open cuts.

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