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Understanding and Applying Probabilistic Concepts to Rock Slope Design

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SUMMARY The basic concepts involved in using probability theory are outlined, and examples of alternative techniques to derive probabilities of failure for two-dimensional data are given. Extrapolation to three-dimensional data is discussed together with other practical aspects of using probabilistic techniques in rock slope design.

INTRODUCTION

The use of some form of probabilistic analysis in the design of rock slopes has become widespread in recent years (Refs 1-9). However, a 'black box' syndrome appears to be developing, with some misuse of probabilistic concepts occurring as a result.

This paper outlines some of the basic concepts involved in using probability theory, and, using practical examples, considers the following areas where errors are commonly made in rock slope design:

(i) assessment of kinematic populations;

(ii) determination of the correct technique to assess the probability of failure;

(iii) analysis of three-dimensional data.

Extrapolation of the techniques to include cohesion, groundwater pressures and earthquake forces is also discussed.

1 BASIC CONCEPTS

The primary reason for using a statistical treatment of data for the determination of the probability of failure of a rock slope, is the VARIABILITY we tend to observe in almost every parameter affecting its stability.

When we design a rock slope, we can use a number of alternative approaches, including:

(i) analysis of the slope with the mean values of the variables influencing the analysis, with application of a Safety Factor requirement to the result;

(ii) analysis of the worst possible combination of variable values, with application of a Safety Factor requirement to the result;

(iii) analysis using the measured variabilities, and design for the optimum probability of failure.

Neither of the first two approaches tells us what degree of conservatism is being used. Is a safety factor of 1.2 adequate for example? When nothing is known about the potential variation in safety factor in the analysis due to variation in the variables used to compute that result, the choice of a safety factor becomes meaningless. This then is the reason for analysing slopes using the measured variabilities.

Probability of Failure in its simplest form can be taken to represent *the likelihood that the Safety Factor of an individual analysis is less than unity*. Consider two situations as follows:

(i) one continuous bedding plane striking parallel to a very long slope;

(ii) two fractures which combine to form a wedge.

In both cases,

(i) the plane, or line of intersection, dips out of the proposed face, and

(ii) dips more steeply than the available friction.

In the first case, any and every portion of that slope has a safety factor of less than 1.0, and the probability of failure (Pf) of any portion of the slope or the whole slope, is therefore 1.0 or 100%. In the second case, any combining planes will fail but does this represent a probability of failure of 1.0 for the whole slope? In this instance, more information is required about the fractures forming the wedge. For example:

(i) if there is only one fracture in each of the two fracture sets, then only one isolated failure can occur. Its scale depends on the length of the fractures and their position in space with respect to the face so,

Pf (identified portion) = 1.0
Pf (whole slope) = 0.0;

(ii) if there are several fractures in each set then the LENGTHS and SPACING of the fractures are required to properly understand the probability of failure.

This introduces the concept that for other than planar failure on continuous planes, the *Probability of Failure of an individual combination of planes does not indicate the Probability of Failure of an entire slope.*

The technique of determining the Probability of Failure of an indivudal analysis will be demonstrated first, and its extension to a practical slope will be discussed later in the notes.

2 METHOD OF ANALYSIS

The following section shows the way the probability of failure is determined for an individual planar failure analysis. In the first instance, the fracture data will be considered to only have variation in dip. In reality the orientations of planes vary in direction as well as dip, and the extension of the ideas to this situation is discussed in Section 3. Note also that although this analysis is presented for planar failure there is no difference with 2-D or 3-D wedge analyses.

Assume that a slope is to be cut through a set of adversely dipping fractures. Fig. 1 shows histograms of values of dip (β) recorded from outcrops and oriented drill core, and values of friction angle (ϕ) recorded from shear tests, together with fitted normal distributions. Cohesion is assumed to be zero at this time, but is easily included in the analysis. (Refer to Section 2.2.) This Figure shows that orientation and friction data are typically normally distributed.

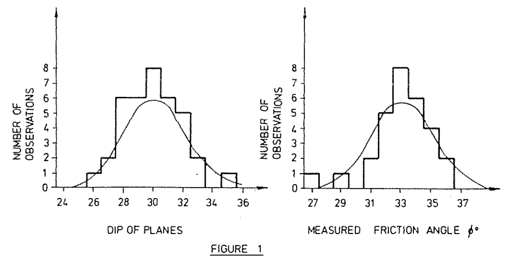

Figure 1 Histograms of dip and friction angle data with fitted normal distributions

From these data, the mean dip of the fractures (β) equals 30°, and the mean friction angle (ϕ) equals 33°. The Safety Factor determined from these mean values under dry conditions is given as

$$SF = \frac{\tan \phi}{\tan \beta} = \frac{\text{(available strength)}}{\text{(required strength to prevent sliding)}}$$

$$SF = \frac{\tan 33^{\circ}}{\tan 30^{\circ}} = 1.12$$

Note that we have not mentioned anything about the slope angle, which is what we are trying to determine! The slope must dip more steeply than the dip of the fractures in the same plane for a safety factor to have any meaning. i.e., The slope must be *kinematically* able to fail. The Safety Factor is then a measure of *kinetic* stability.

What is the likelihood or probability then that the Safety Factor is less than 1.0? Let us first assume that the slope we intend to cut is vertical, i.e. all the fracture planes are kinematically able to fail. Then the probability of failure Pf can be defined as the likelihood that the dip of the planes (β) out of the face exceeds the available friction (ϕ), assuming that all planes are continuous (e.g. bedding planes).

$$\text{i.e. } Pf = P(\beta>\phi) \quad (1)$$

If the slope is non-vertical, but formed at some angle α, there is also some likelihood that the slope will not undercut the planes. The Probability of Failure in this case is the probability that the planes will be undercut multiplied by the probability that, having undercut the planes, sliding will occur.

$$\text{i.e. } Pf = P(\alpha>\beta) \,.\, P(\beta>\phi) \quad (2)$$

This assumes that β and ϕ are not correlated, i.e. they are independent variables. Experience has shown that this is nearly always a valid assumption.

Referring to the normal distributions of the dips (β) and friction angles (ϕ), we can superimpose them on the same axis as shown in Fig. 2. The parameters for each normal distribution are as follows:

DIPS (β) Mean $\mu = 30^{\circ}$, st. dev. $s = 2.17^{\circ}$
FRICTION (ϕ) Mean $\mu = 33^{\circ}$, st. dev. $s = 2.15^{\circ}$

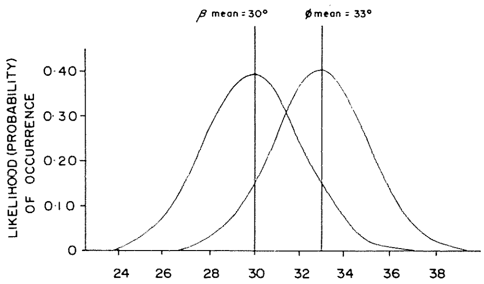

Figure 2 Dip and Friction Distributions

If these two distributions did not overlap, this would indicate that the one value is always greater than the other. Because there is an overlap, we can see that on some occasions (given by the convolution area which is shaded) the friction will be less than the dip of the planes, and failure will occur.

$$\text{i.e. } 0.0 < P(\beta>\phi) < 1.0$$

We can also represent any planned slope as a vertical line on this diagram. If this lies to the right of the dip distribution, then all planes are always kinematic. If the face dip intersects the distribution then again

$$0.0 < P(\alpha>\beta) < 1.0$$

Fig. 2 is therefore a simple visual way of understanding the relationship between kinematic and kinetic stability for any given situation.

2.1 Alternative Techniques for Determining Pf

There are several different statistical techniques which can be used to determine the probability of failure, and three of these are discussed in more detail, viz.

(i) manipulation of statistical distributions;

(ii) Monte Carlo simulation;

(iii) Rosenbleuth's technique.

Each has its strong points and weak points, which are discussed. References are made to available texts where appropriate.

2.1.1 Manipulation of Distributions

In the simple example being analysed, the probability that the dip exceeds the available friction is given by

$P(\beta>\phi)$
or $P(\phi-\beta < 0.0)$

Using the statistics of normal distributions the β distribution can be subtracted from the φ distribution to find a third normal distribution (φ-β) with parameters

$$\mu\,(\phi - \beta) = \mu\phi - \mu\beta = 33^{\circ} - 30^{\circ} = 3^{\circ}$$

$$s\,(\phi - \beta) = \sqrt{s\phi^{2}+s\beta^{2}} = \sqrt{2.15^{2}+2.17^{2}} = 3.05^{\circ}$$

This distribution is shown in Figure 3, with the area where (φ - β) is negative shown shaded. This represents the proportion of the time when β > φ. φ - β = 0.0 represents limiting equilibrium.

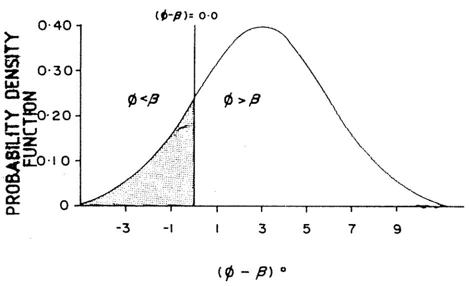

Figure 3 φ - β distribution

This area is calculated mathematically by the use of published tables of the Standard Normal Distribution (Ref. 10). These tables show the area under the curve as a percentage of the total area for a normal distribution with mean 0.0 and variance 1.0. Any distribution is 'normalised' by computing a value Z where

$$Z = \frac{x - \mu}{s}$$

and x = required ordinate
μ = mean of distribution
s = standard deviation of distribution

In this example we want the Z value corresponding to a value of φ - β = 0.0

$$Z_{0.0} = \frac{0.0 - 3}{3.05} = -0.982$$

From the tables of the standard normal distribution (1) this gives a value of 0.837. However, because Z is negative the smaller 'tail' of the distribution with a probability value less than 0.5 is required. The result is therefore subtracted from the total area of 1.0 and this gives

$$P\,(\phi - \beta) < 0.0 = 1.0 - 0.837 = 0.163 = 16.3\% \qquad (1)$$

A similar technique is used to determine the probability of the slope dip undercutting the fracture dips $P(\alpha>\beta)$.

e.g. For a slope angle of 32°

$$P(\alpha > \beta) = F(Z)$$

$$= F\left(\frac{x - \mu}{s}\right) = F\left(\frac{32-30}{2.12}\right) = F(0.943)$$

$$P(\alpha > \beta) = 0.8272 \qquad (2)$$

What this means is that at any point on a long slope cut at 32° there is an 83% chance that the slope dips more steeply than any single fracture. Assuming then that the friction available on the fracture at any point is independent of the dip, we can assess the probability of failure Pf at any point by combining (1) and (2). Hence $Pf=P(\alpha>\beta).\ P(\beta>\phi) = 0.8272 \times 0.163 = 0.1348 = 13.5\%$.

By calculating $P(\alpha>\beta)$ for various values of α, a graph of probability of failure Pf versus slope angle can be drawn up, as shown in Fig. 4.

Strengths: rapid manipulation, accurate

Weaknesses: only applies to normally distributed data

becomes more difficult to use for complex stability equations.

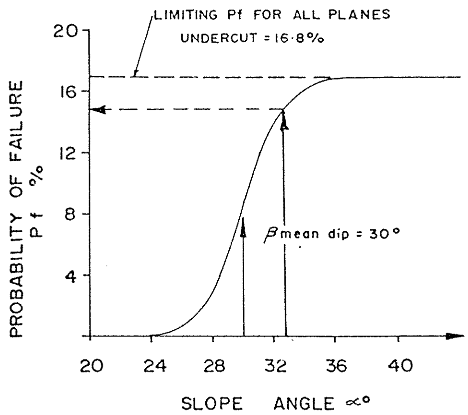

Figure 4 Probability of failure versus slope angle

2.1.2 Monte Carlo Simulation

In Monte Carlo simulation random numbers are used to sample distributions for variables which constitute the equations from which a safety factor is determined (Ref. 11). To facilitate this, it is usual to express any observed or calculated frequency distribution as a cumulative distribution. Fig. 5 shows the original dip data, together with the normal distribution, expressed as a cumulative distribution.

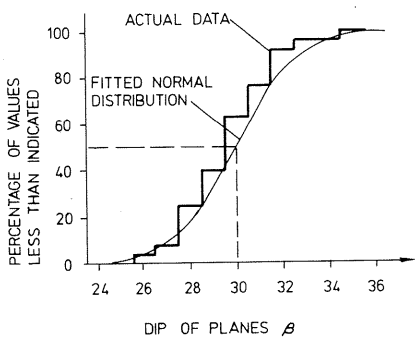

Figure 5 Cumulative histograms of dip and friction angle data with fitted normal distributions

By generating a random number between 0 and 100, a value for any variable can be sampled by representing the cumulative distribution by a series of steps or lines, or by an equation. There are no limitations on the shape of the distributions. After a value of each variable is selected, a safety factor is determined. By repetitive sampling, eventually a distribution of safety factors is developed. By keeping count of the individual analyses which generate a safety factor of less than 1.0, the probability of failure is simply given by

$$\frac{\text{the number of recorded failures}}{\text{the total number of analyses}}$$

Using this system, the slope angle is usually incremented at the end of each set of analyses, enabling a graph of Pf versus slope angle to be drawn up.

Strengths: ability to work with any mixture of distribution types (or actual data if preferred), any number of variables, and with any level of complexity of analysis.

Weaknesses: quantity of processing. Several thousand individual calculations of the safety factor may be required to evaluate a safety factor distribution to the desired accuracy, although 500 to 1000 appears to be satisfactory in many simpler cases;

computer processing almost essential for the volume of computations.

2.1.3 Rosenbleuth's Method

Rosenbleuth's Method (Ref. 8) is a technique whereby individual variables (which have distributions of values) are replaced by point estimates at fixed points of one standard deviation (=± 1SD) either side of the mean value. This is analogous to a simple case of statics where, for example, a uniformly distributed load can be replaced by point loads. The value of a variable γ at + 1SD is called $\gamma+$ and at -1SD is called $\gamma-$.

So in our example, the distributions of β and ϕ are replaced by the values of $\beta+$, $\beta-$, $\phi+$ and $\phi-$ where the + and - indicates + or - 1 standard deviation from the mean values.

For the Data collected

$$\beta+ = 30 + 2.17 = 32.17^\circ$$

$$\beta- = 30 - 2.17 = 27.83^\circ$$

$$\phi+ = 33 + 2.15 = 35.15^\circ$$

$$\phi- = 33 - 2.15 = 30.85^\circ$$

The next step is to determine the safety factor for every possible combination of the variables in the equation defining the safety factor. In this case, there are only four combinations for the two variables β and ϕ. In general, the number of combinations will be 2^n for n variables.

$$SF_1 = SF{++} = \tan\phi+/\tan\beta+ = \tan 35.15/\tan 32.17 = 1.119$$

$$SF_2 = SF{+-} = \tan\phi+/\tan\beta- = \tan 35.15/\tan 27.83 = 1.334$$

$$SF_3 = SF{-+} = \tan\phi-/\tan\beta+ = \tan 30.85/\tan 32.17 = 0.949$$

$$SF_4 = SF{--} = \tan\phi-/\tan\beta- \; \tan 30.85/\tan 27.83 = 1.131$$

The mean safety factor is given by

$$\mu SF = \frac{1}{n}\sum_{i=1}^{n} SF_i$$

$$= \frac{1}{4}(1.119+1.334+0.949+1.131)$$

$$= 1.333$$

and $$S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$= 0.0186$$

Therefore

$$S = 0.1364$$

Again, by using the standardised normal variable Z we can compute the likelihood of a safety factor of less than 1.0 occurring

$$Z_{1.0} = \frac{1.0 - 1.133}{0.1364} = 0.975$$

$$F(Z) = 0.836$$

This corresponds to a probability that $\beta > \phi$ of

$$\approx (1 - 0.836)$$

$$= P(\beta > \phi) = 16.4\%$$

i.e. For normally distributed data this yields the same results as the other methods. The probability of failure Pf would then be assessed by considering the slope angle as in Section 2.1.1.

Strengths: able to handle skewed distributions simply provided skewness β_1 is determined;

conceptually simple regardless of the number of variables, or the complexity of the equations involved;

able to handle correlated variables;

much less computer time required than for Monte Carlo simulation.

Weaknesses: cumbersome when only a few variables in the equations.

Obviously, for the given example, the simplest technique would be to use the subtraction of distributions. For more complicated situations, Monte Carlo simulation would be best <u>if</u> the computing is low cost. Otherwise, Rosenbleuth's method should be used.

2.2 Incorporation of Cohesion and External Forces

In the simple analysis presented here, no allowance has been made for cohesion, groundwater or earthquake forces. There are two ways to allow for them, viz:

(i) the equations for determining safety factor can be modified to include these forces;

(ii) the value of dip β can be 'corrected' to include these forces and keep the safety factor equation simple.

Considering the second alternative, McMahon (Ref. 1) has shown that the dip β can be converted to a 'critical dip' β_c by means of correction angles α as follows:

$$\beta_c = \beta(\text{dip}) + \alpha_c \text{ (cohesion correction)}$$
$$- \alpha_u \text{ (water uplift correction)}$$
$$- \alpha_e \text{ (earthquake correction)}$$

The angles are determined as follows.

$$\alpha_c = \sin^{-1} \left(\frac{c \cos\phi}{w.g}\right)$$

$$\alpha_u = \sin^{-1} \left(\frac{U \sin\phi}{w.g}\right)$$

and $\alpha_e = \tan^{-1} (k/g)$

where W = weight of block (tonnes)

U = water uplift force (kN)

k = earthquake acceleration (m/sec^2)

g = gravitational acceleration (m/sec^2)

c = cohesion force (kN)

Where applicable, these forces are calculated for a unit thickness of slope (i.e., two-dimensionally).

Each correction angle will itself be a variable, and Rosenbleuth's technique can again be used to determine the mean and variance of the distribution of the angles, and thence the Safety Factor.

2.3 Extension to Three-Dimensional Wedge Analysis

A three-dimensional wedge can be analysed using either the Monte Carlo or Rosenbleuth techniques. Using either technique, the probability of failure is still the probability of kinematic wedges forming from two given sets of joints, times the probability of the critical dip being exceeded. The correct determination of kinematic planes and wedges is discussed in the next section.

Using the 'critical dip' concept, the critical dip is given by

$$\beta_c = K_s \tan^{-1} (\beta - \alpha_u - \alpha_e + \alpha_c)$$

where $K_s = \dfrac{\sin \sigma A + \sin \sigma B}{\sin (\sigma A + \sigma B)}$

and σA, σB are the apparent dips of planes A and B in the plane normal to the line of intersection.

3 ASSESSING THE POPULATION FOR KINEMATIC ANALYSIS

So far, only the simple situation of a variation in dip *in the plane of the face* has been considered. When fracture orientation data are collected in the field, they are commonly plotted as poles on a stereographic or equal area projection. For kinematic instability, individual planes must have a component of dip (i.e. an apparent dip) out of the face, whilst wedges must have a line of intersection satisfying the same criterion.

In Fig. 6, the poles to three planes A, B and C are shown on an equal area projection, together with a proposed slope dipping at 60° in direction 315°. The hatched area represents the area within which individual poles will represent planes having a component of dip out of the proposed face. The curve enclosing this area is formed by the locus of poles to planes at 90° to the great circle defining the proposed face in the vertical plane. Therefore, points on this curve represent the normal to planes dipping parallel to the apparent dip of the face in the direction of dip of the plane. Any points outside this area represent planes which dip more steeply than the apparent dip, and therefore are kinematically unable to fail.

In the example, Plane A lies within the kinematic area, and is therefore kinematic. Planes B and C individually are not kinematic, but they form a wedge which has a line of intersection defined by the point D. Because this intersection dips less steeply than the proposed slope, (i.e. the intersection of planes B and C lies outside the great circle defining the slope face), the wedge is kinematic.

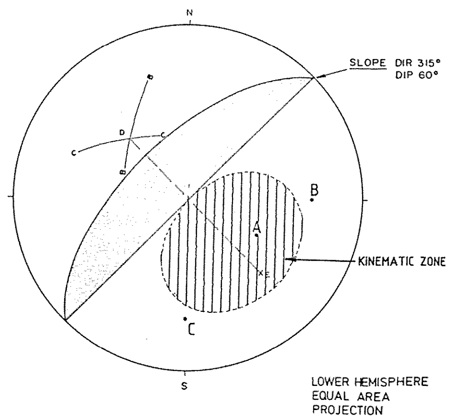

Figure 6 Testing for kinematic poles for planar and wedge analysis

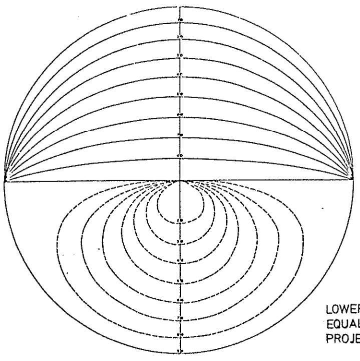

Figure 7 Kinematic stability chart

Wedges can be related to the kinematic area for planar failure shown on Fig. 6, by considering the normal to the line of intersection *in the plane containing the normals to planes B and C.* This is shown on Fig. 6 as point E, which is just the point at 90° to the intersection containing the vertical plane (i.e. through the centre of the projection). This point can be called the 'wedge point'. We can therefore make the following statement:

> *If the wedge point for two planes lies within the kinematic area for any given slope, that wedge is kinematic with regard to that slope.*

Fig. 7 shows a kinematic stability chart after McMahon (Ref. 3). This shows planes, and their respective kinematic areas, in increments of 10° dip. As can be seen the area increases in size for steeper dips, i.e. an increased percentage of planes are likely to 'daylight', as would be expected. For the extreme case of a vertical slope, the kinematic area represents the full half circle (i.e. quarter-sphere). This is then the area which defines the POPULATION from which we assess the probability of undercutting the slope.

We can therefore state that:

> *Planes or lines of intersection of planes which do not have a component of dip into the semi-circle contained by the direction of dip of the face, do not constitute part of the population for planar or wedge analysis respectively.*

This is important, as gross <u>underestimates</u> of the probability of undercutting planes or wedges can occur if all recorded data is used as the population in an analysis. Therefore, if the plane or wedge is indeterminate for a vertical slope, it should not form part of the population. To illustrate this point, consider the following example:

> An outcrop in which a rock slope is proposed was mapped, and the poles to sixty (60) fractures were plotted (see Fig. 8). Assuming a friction angle on all fractures of mean 30° and standard deviation 2°, what would be the probability of failure for a slope cut at 50° in direction 315° assuming all fractures are continuous?

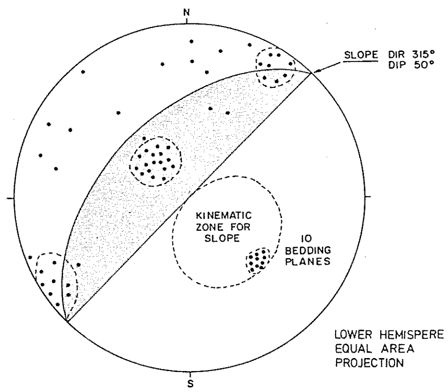

Figure 8 Determination of the correct population for probabilistic analysis

From Fig. 8, it can be seen that all fractures except 10 dip into the face, and are kinematically unable to fail. The remaining 10 have mean dip 44° and standard deviation 2° in the direction of dip of the slope. What is the correct population? Fig. 9, shows the distributions of friction angle and dip plotted together with the slope angle. It can be seen that, for the fractures dipping out of the slope,

$\beta > \phi$ at all times
$\alpha > \beta$ at all times

and hence the probability of failure of a 50° slope in this instance is given by

$$Pf = P(\beta > \phi)\ P(\alpha > \beta) = 1.0 \times 1.0 = 1.0$$

i.e. There is a 100% chance of a failure occurring. Provided that the fractures are continuous, this is exactly what we might expect.

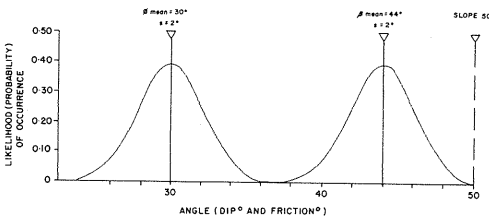

Figure 9 Stability analysis for data from fig. 8

Previously published techniques suggest that, the probability of failure for continuous joints is given by:

$$Pf = Pj$$

where $Pj = Na \frac{Nu}{N\varepsilon}$

and Na = Number of joint SETS
Nu = Number of joints in an unstable orientation for the slope being considered
Nε = total number of joints observed

For this example,

Na = 4, Nu = 10, Nε = 60

so $Pj = \frac{4.10}{60} = 0.67 = 67\%$

Using this theory, the addition of another fracture set dipping into the face will increase the probability of failure. The author believes that this should not be the case.

In summary, care must be taken in determining the correct population at the start of an analysis, particularly for wedge analyses when the lines of intersections or wedge points dictate kinematic stability, and not the poles to the individual planes.

4 USE OF THREE-DIMENSIONAL ORIENTATION DATA

The preceding notes have considered orientation data which is only variable in dip. This section discusses some aspects relating to the use of bivariate orientation data (i.e. variable in both direction and dip).

4.1 Determination of the Distribution Parameters

Fig. 10, shows contoured data of a fracture set taken from rock outcrops or diamond drill core. The typical features of such data are:

(i) rough axes of symmetry along a great circle and at right angles to it;

(ii) elongation of the data along one axis;

(iii) normally distributed orientations on both axes.

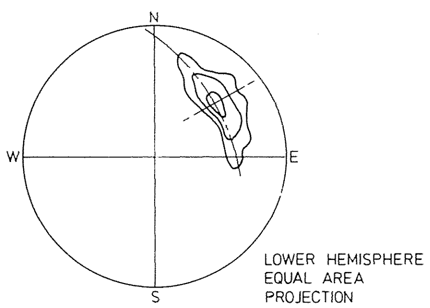

Figure 10 Typical pole concentration contours

In order to reproduce these data in a statistical analysis we need to be able to determine the following parameters:

(i) the mean orientation (direction and dip);

(ii) the axes of symmetry;

(iii) the angular standard deviations on both axes.

A technique which uses the matrix formed from the direction cosines of the unit vector (pole) to each measured plane has been reported by Markland (Ref. 12). The eigen vector associated with the greatest eigen value of this matrix is then the vector mean of the distribution and the other two eigen vectors are the normals to the planes of symmetry of the distribution. Whilst this technique can work well, the effect of unstable matrix evaluations in computations for elongated distributions may be pronounced. A visual check of plotted data on the stereonet should always be made to check the output of this type of analysis. Alternatively, a technique used by Morriss and Stoter (Ref. 5) may be adopted.

Having determined the mean orientations and the axes of symmetry, it remains to determine the standard deviations of the data on these axes. Techniques have been developed by Fisher (Ref. 13) and McMahon (Ref. 1), and will not be discussed here.

4.2 Use of Three-Dimensional Data in Analyses

In order to make use of the measured orientation data, it is necessary to regenerate it in the analytical model. If Monte Carlo simulation is being used, it is simple to select a plane (or planes) for a single analysis using normally distributed random numbers along both axes of symmetry to determine a pole orientation. An alternative technique, if using Rosenbleuth for example, is to use the extremes of the distribution (say two standard deviations from the mean on each axis), plus the centroid. With all other variations in the analysis accounted for, the variation in safety factor versus orientation can be assessed.

It is recommended that other than for checking ranges of safety factor, Monte Carlo simulation is adopted once sampling of three-dimensional data is undertaken, as confusion can easily result from attempting to incorporate the bivariate distribution into other techniques.

5 OTHER DATA REQUIRING RESEARCH

This paper has endeavoured to give some practical ideas about the use of probabilistic slope design techniques. However, space precludes discussion of other areas which the author believes require more (or at least published) research. These include

- The incorporation of joint spacing and length data into the modelling of failure plane shear strength characteristics, and the implication of these factors on 'probability of occurrence';
- The evaluation of waviness and roughness distributions from drill core or face mapping data. (The variations measured on the stereonet from drill core are not variations of the mean dip of the planes, but include roughness/waviness measured on the scale of the drill core);
- case studies on the detailed application of these ideas to the mining environment.

6 ACKNOWLEDGEMENTS

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