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Use of Mathematical Models to Predict Effects of Mining Under Stored Waters

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SUMMARY Since 1904 the Sydney Water Board has been in a position where coal mining has taken place beneath the stored waters of some of its major storage dams. The conflict in interests between the desire to extract coal and the desire to maintain the integrity of the storages has led to the present position where all mining applications near the stored waters must be approved by the N.S.W. Dams Safety Committee. The Board is entitled to make submissions to this Committee concerning each application and, in order to develop supporting evidence, Board's officers have been involved in the development of mathematical models to predict the subsurface effects of mining. It is anticipated that this work will lead to a time when both the structural and hydrogeological effects of mining can be predicted using analyses such as displacement discontinuity, finite element or finite difference, thus allowing an estimatation of the amount of stored water percolating into the storage basin rock strata as a consequence of coal mining.

1. HISTORY

In 1880 the N.S.W. Government Gazette proclaimed the Sydney Metropolitan Catchment Area, approx. 900 square km of virgin land south west of Sydney. This land was to be initially used as a catchment area with runoff being diverted through weirs, tunnels and canals to Prospect Reservoir and hence to the Sydney distribution network. Towards the end of the century the need for a storage reservoir became apparent and in 1902 work commenced on a 56 m high mass gravity dam on the Cataract River. Prior to this construction the Board's concern in the catchment area was with pollution. After the construction however, there were new concerns related to the newly developed coal mines under the catchment areas: the possible effects of subsidence on the dam and appurtenant structures and the possible losses of stored water caused by rock fracturing. These concerns are still strongly felt by the Board.

Throughout this century underground mining has moved progressively closer to the Board's dams and is now carried out underneath the stored waters of Cataract, Avon, Cordeaux and the Upper Cordeaux Dams and in the vicinity of the stored waters of Warragamba Dam. During these years the conflict between the Board and the Department of Mines (now the Department of Mineral Resources) remained for the most part unresolved.

In July, 1974 the N.S.W. Minister for Public Works appointed Mr. Justice Reynolds to hold an Inquiry into the dispute. The report of the Inquiry (Reynolds 1976) recommended that restricted mining be allowed inside a 26.5 degree angle of draw marginal zone (calculated from a vertical line at the full supply line of the storages) with the degree of mining varying with depth of cover:

(i) less than 60 m - no mining.

(ii) between 60 m and 120 m - bord and pillar mining (see Figure 1) with a maximum bord width of 5.5 m and pillars greater than the maximum of 15h (h = height of extraction) or d/10 (d = depth of cover).

(iii) greater than 120 m - panel and pillar mining (see Figure 1) with a maximum panel width of d/3 and pillars greater than the maximum of 15h or d/5.

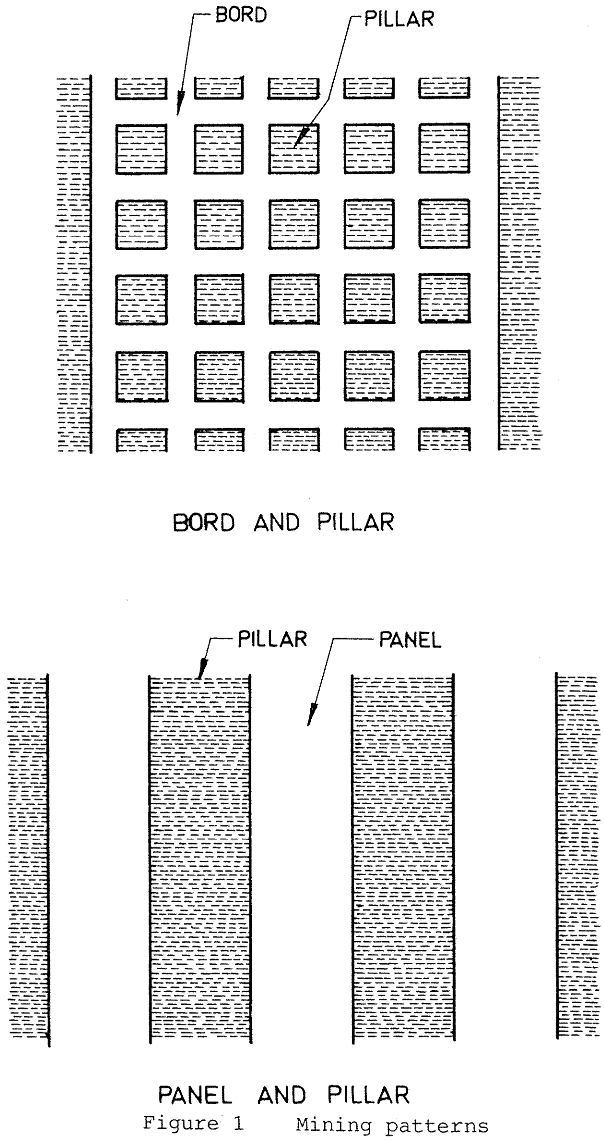

Figure 1 Mining patterns

Following release of the report the Board made submissions to the N.S.W. Government concerning the recommendations. As a result of these submissions the Government decided not to adopt the recommendations of Mr. Justice Reynolds but rather to pass legislation giving the newly formed N.S.W. Dams Safety Committee control of mining approvals in the vicinity of the Board's dams and stored waters on an individual basis. This Committee now examines every application to mine inside a restricted zone (approx. 50 degree angle of draw) and makes recommendations to the Minister.

2. MATHEMATICAL MODELLING (1975 - 1976)

As part of the Inquiry the Board engaged Golder Associates to carry out mathematical modelling of the mining under stored water problem. Golder Associates used both the finite element analysis and the displacement discontinuity analysis (both two dimensional) to examine the effects of underground extraction.

The finite element analysis used plane strain solid elements together with joint elements based on the formulation of Goodman (1968). The joint elements are linear, zero thickness elements which separate the solid elements and allow controlled slip or separation of the solid elements. They have an elastic behaviour based on normal and shear stiffness and, in a displacement discontinuity or non-linear finite element solution, have a non-elastic behaviour. This non-elastic behaviour is based on cohesion and friction angle in the shear direction and no tension/full compression (after closure) in the normal direction.

Golder Associates (1975) first analysed an 180 m wide excavation in a coal seam at a depth of 90 m. Separate runs were undertaken with one, three and seven bedding planes above the coal seam. The last run gave three times the surface subsidence of the first as well as increases in the areas of tension. Further analyses were then carried out for a panel and pillar system similar to that proposed under the stored waters to assess the affects of loss of pillars (e.g. due to fire and fretting). These results showed significant increases in strata disturbance.

Golder Associates then proceeded to analyse the strata with vertical as well as horizontal joints. The solution of such a problem with finite elements involves considerable data preparation and high computer costs. For these reasons Golder Associates (1976) investigated the possibility of using the displacement discontinuity method. This method was developed in the mid 1960's for analyses of mining problems at depths where it was desired to model a fairly simple excavation in an infinite homogeneous medium. A modification to the program (Crouch 1976) introduced a free horizontal natural surface and this allows the program to be used for the analysis mining near the surface.

The displacement discontinuity runs first analysed the same problems as the finite element runs to compare the two different techniques - the results were essentially identical. Then runs were carried out with panel and pillar layouts at depths of 90 and 270 m and with horizontal and vertical jointing. Both runs gave extensive cracked areas at the surface with the latter run showing vertical cracks of 260 mm width and 80 m depth and a cracked zone extending to a distance equivalent to an angle of draw of 50°.

The Golder analyses were not checked against any actual mining case and hence their results were theoretical only. However, they did give subsidence and strain results which were consistent with what would be expected and defined zones of tension, compression and cracking.

3. MATHEMATICAL MODELLING (1976 - 1983)

3.1 Available Packages

At present the Board has four programs that could be used for mining analyses, namely:-

(i) DDJ2D - the two dimensional displacement discontinuity analysis used by Golder Associates during the Mining Inquiry and subsequently purchased by the Board.

(ii) FINELP - a two dimensional finite element program based on a six node triangular element but with anistopic material properties, non linear (no tension) capabilities and a six node gap element.

(iii) SAGE - a two dimensional finite difference program with non linear capabilities (Dames and Moore, 1980).

(iv) SAPIV - a two/three dimensional finite element program. This program has not been used for mining problems.

3.2 Experience to Date

DDJ2D and FINELP have been used to analyse several mining situations and these analyses have yielded considerable information concerning the advantages and disadvantages of the programs and the techniques required to enable their most efficient use.

DDJ2D has the advantage of simplified data input, especially where a large number of joints are to be used. The iterative solution enables full non-linear joint properties (as opposed to the linear isotropic strata properties) to be used and the restart facility enables material modifications to be evaluated with little extra cost. The major disadvantages are that it models only a horizontal natural surface and will handle only linear material properties in the strata.

In any particular jointed strata problem FINELP requires far more elements than does DDJ2D. This larger number of elements, plus the poor bandwidth usually obtained and the large number of iterations required, necessitate substantial computer resources for even a modest run. However, FINELP's ability to have anisotropic and/or non-linear strata properties permits use of the more economical "micro-joint" model. In this type of model areas of irregular and discontinuous jointing are included within the strata material properties rather than by the introduction of nominal joints (i.e. "macro-joint"). This approach significantly reduces the size of the problem and the cost of each run compared to the macro-joint model.

The loading for the model is applied automatically in DDJ2D which calculates the normal and shear forces on each element from an initial stress field based on the depth below the free surface. With FINELP this loading is the preferred, although not only, method but involves the prior calculation of the stress field around the mine excavation. This calculation can be either based on the overburden weight and a horizontal component thereof (as in DDJ2D) or obtained from a previous finite element

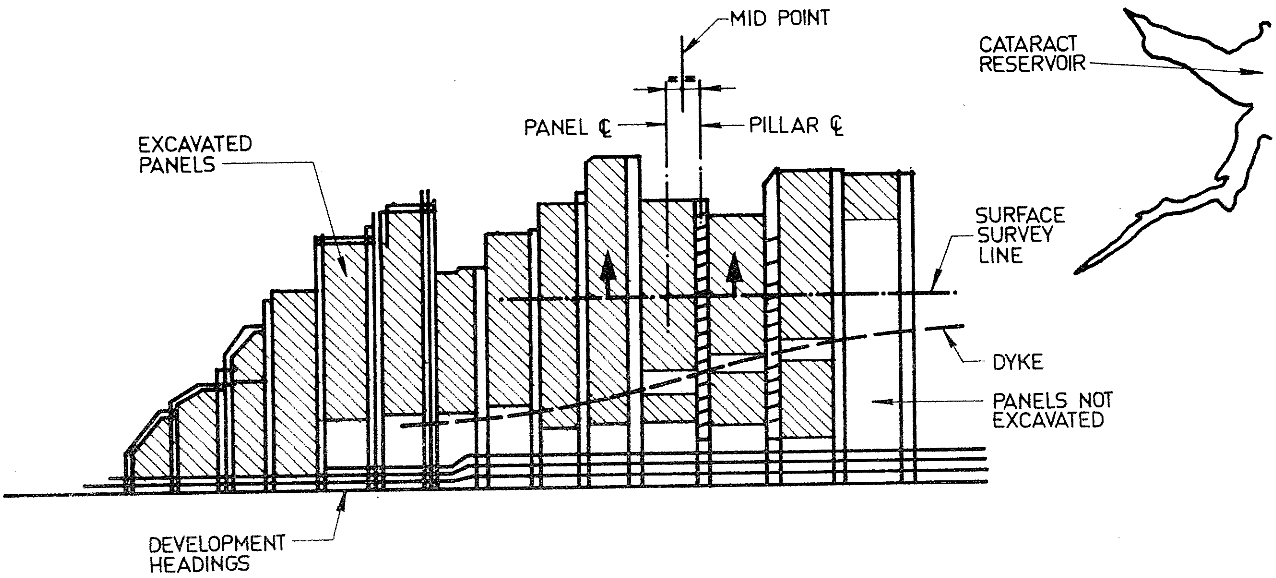

Figure 2 Plan of case study

run or a combination of both. Thus FINELP is more complicated but is a more versatile analysis technique.

Because DDJ2D is iterative it can handle the mine roof/floor closure automatically by changing the parameters when closure is achieved. With the FINELP elastic solution this is not possible and the final roof/floor situation must be anticipated. To model the roof/floor closure the corresponding roof and floor nodes are displaced towards one another and then locked together. If the closure is elastic then the displacement is equal to the seam thickness. If the roof goafs the displacement is some fraction of the seam thickness to allow for bulking of the rock. If the final results show vertical tension at these nodes then the assumption of roof/floor contact is incorrect and the nodes should be freed and the problem rerun.

The program SAGE is at present being implemented on the Board's computer and no detailed results are available. At this stage it is expected that it will prove preferable to DDJ2D and FINELP.

4. CASE STUDY

4.1 Description

The capabilities of the programs DDJ2D and FINELP were checked by analysing a mined area adjacent to Cataract Dam storage where total extraction of an upper seam had taken place in 1938 and panel and pillar extraction of a lower seam had recently been completed. Surface subsidences had been measured for this latter extraction. The details are shown in Figures 2 and 3.

The upper seam workings consisted of bord and pillar first workings of a 2100 mm thick seam followed by removal of the pillars to give total extraction. This created a goafed (ie. roof fractured and collapsed) area above this seam and a surface deformation that would have been somewhat less than the 2100 mm because of the bulking of the goafed rock.

With the extraction of the lower coal seam the effect of the collapse of each panel roof would have extended past the upper seam. Thus it would have disturbed the original goafed rock and resulted in some secondary consolidation of this material with correspondingly larger surface subsidence than would otherwise have been the case. This increased subsidence is estimated at a constant 200 mm.

The measured survey subsidence during extraction of the lower seam varied between 1150 mm over the panel and an average of 600 mm over the adjacent pillars. These values reduce to 950 mm and 400 mm respectively when the 200 mm secondary settlement is considered. These latter values can be considered the field values for comparison with the calculated values.

The thickness of the lower seam is 1300 mm but to allow for bulking of the goaf rock a roof/floor closure limit of 1000 mm has been assumed in all analyses.

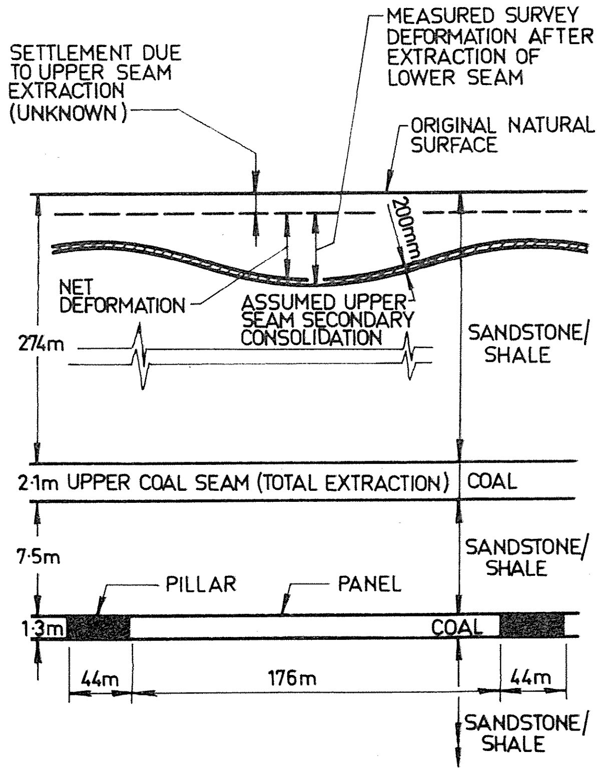

Figure 3 Section of case study

4.2 Material Properties

As part of the Mining Inquiry, tests were carried out (Bhattacharyya, 1976) to determine the rock properties of the southern coalfields. These tests gave a range of results (Young's modulus, Poisson's ratio, unconfined compressive strength, etc.) for the different strata. However, for these analyses, an average set of properties was selected for the entire strata as required by DDJ2D. The properties selected were based on the above and on other experience and comprise the following:

Strata:	
Young's modulus (Y)	7000 MPa
Poisson's ratio (u)	0.24
Shear modulus (G)	2820 MPa (ie. E/2(1 + u))
Density	2560 kg/m³
Horizontal Tectonic Force	50% of overburden
Mine pillars:	
Young's modulus	3000 MPa
Poisson's ratio	0.28
Horiz joints:	
Normal stiffness	2800 MPa/m
Shear stiffness	1100 MPa/m
Friction angle	40°
Vert. joints:	
Normal stiffness	1400 MPa/m
Shear stiffness	550 MPa/m
Friction angle	20°

4.3 DDJ2D

Three displacement discontinuity runs were undertaken to model the following situations:

1. rock strata unjointed,
2. rock strata separated by horizontal joints only,
3. rock strata separated by horizontal and vertical joints to give the grid shown in Figure 4.

The unjointed run showed the capability of the displacement discontinuity analysis, with the ability to model a large elastic problem with a small number of elements. The run was set up to model a semi infinite body with five panels extracted. Fifty two elements were used (40 for the mined area and 12 for the intact pillars). The results are given in Table 1 and represent an idealised elastic rock situation.

With the addition of the horizontal joints (run 2) the overburden changed from a 285 m deep elastic beam to a set of elastic beams up to 50 m thick. This increased strata flexibility allowed an increased closure of the extracted area although there was still no roof/floor contact.

The vertical joints in run 3 were added as non continuous sets at a nominal 44 m spacing near the surface and reducing to 11 m spacing above the mine seam. This model now had 507 segments, generated with only 35 lines of code. However, whereas convergence was quickly (and cheaply) achieved in the first two runs this run had not achieved convergence after 400 cycles. The results in Figure 4 are the values at cycle 400 and show a complete roof collapse over 60% of the panels with some goafing of the blocks above the panels. Despite the roof collapse the model still gave an essentially uniform surface deformation. It should be noted that similar problems analysed have reached convergence and have shown complete block collapse above the panels with large slips between adjacent blocks.

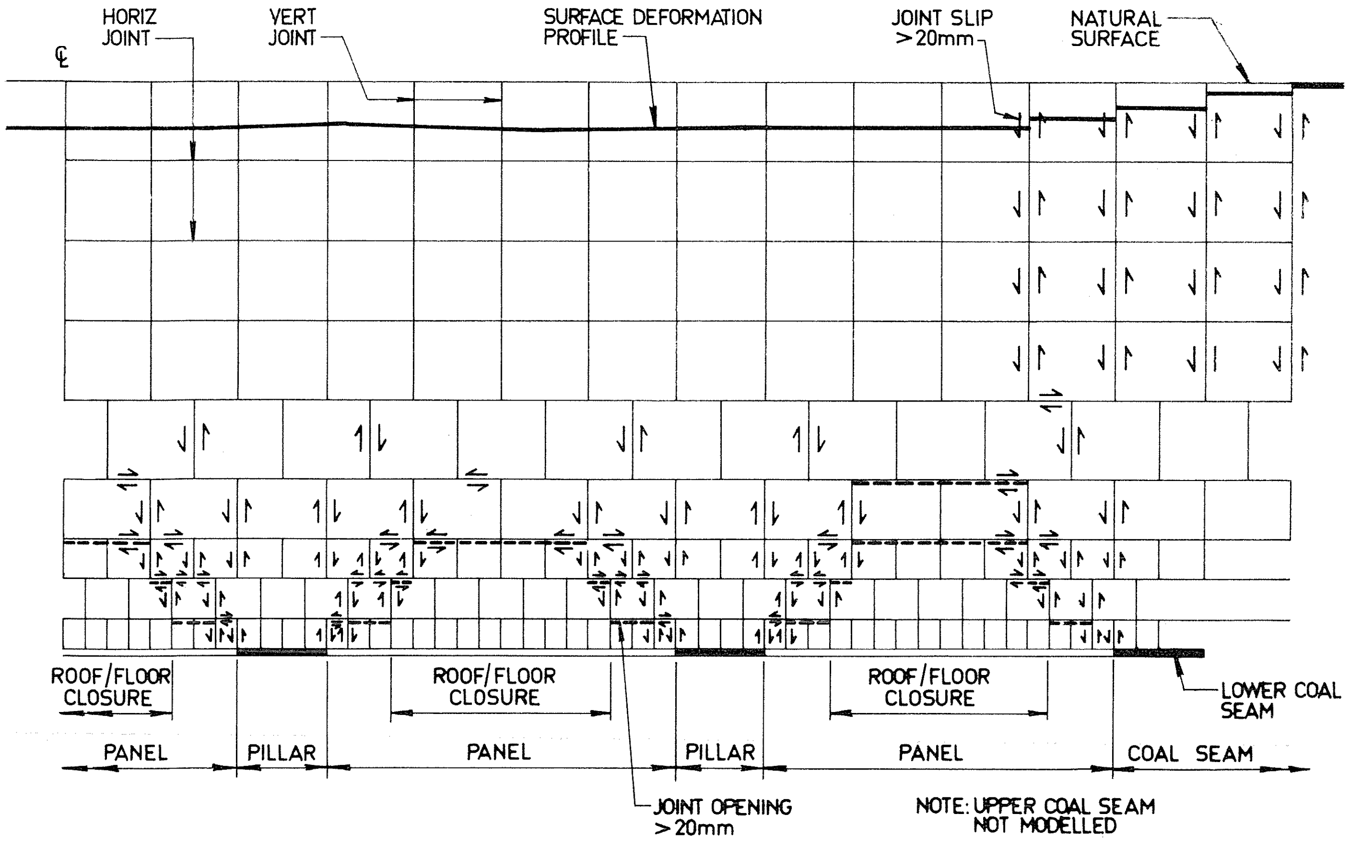

Figure 4 Case study - displacement discontinuity analysis (run 3)

TABLE 1

CASE STUDY RESULTS

ANALYSIS (E = 7000, G = 2820 unless stated)	DEFORMATIONS (mm)					RELATIVE COST
	SURFACE AT PANEL CENTRE	SURFACE AT MID POINT	SURFACE AT PILLAR CENTRE	MINE ROOF AT PANEL CENTRE	MINE FLOOR AT PANEL CENTRE	
Survey Measurements	1150	850	600	-	-	
Allowing 200 m for secondary consolidation of upper seam	950	650	400	-	-	
DDJ2D						
1. Strata unjointed	298	297	292	418	-173	1
2. Horiz. joints	335	333	327	816	-177	40
3. Horiz. and vert. joints (400 cycles)	362	354	344	847	-153 *	300
FINELP - ISOTROPIC						
1. Elastic	303	303	301	392	- 79	10
2. Non linear no tension (40 cycles)	313	313	312	516	- 79	10
FINELP - ANISOTROPIC						
1. E(vert) = 700, G = 800	731	731	731	865	-135 *	10
2. E(horiz) = 700, G = 800	765	721	672	908	- 92 *	10
3. G = 800	760	723	681	901	- 99 *	10
4. G = 100	931	760	384	974	- 26 *	10

E = Young's modulus in mPa
G = shear modules in mPa

* = roof/floor closure achieved

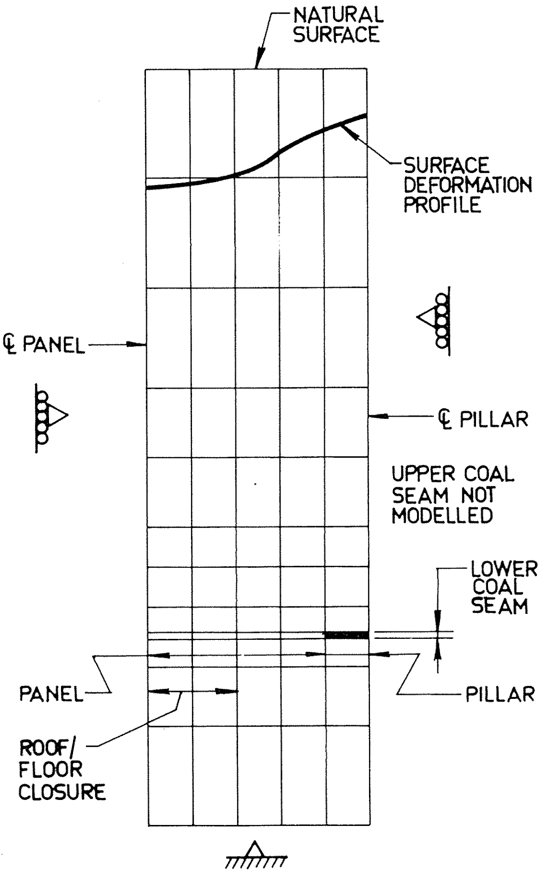

Figure 5 Case study - finite element analysis (run 4)

4.4 FINELP

Because of the dual symmetry of the problem the finite element grid was set up to model from the centre line of one panel to the centre line of the adjacent pillar with 60 elements and 215 nodes forming a rectangular mesh as shown in Figure 5. The mesh chosen was very coarse in the area of the mine pillar with a large aspect ratio (22 m by 1.3 m) but this was considered acceptable as accurate pillar stresses were not required. No joint elements were used.

The initial elastic isotropic run gave a surface deformation and roof floor deformation pattern nearly identical to DDJ2D run 1. A non-linear (no tension) analysis was also carried out but this only gave a small increase in deformations and still preserved the overall uniform surface deflections. The non-linear run results in Table 1 are for 40 cycles with roof/floor convergence still to be achieved.

Following the isotropic runs, four runs were carried out using anisotropic (transversely isotropic) material properties. The material properties used plus the results of these runs are included in Table 1 with vertical subsidence contours of the last run shown in Figure 5. These runs show that the surface profile for this analysis is primarily dependent on the vertical value of Young's modulus (to alter the average surface deformation) and on the shear modulus (to alter the relative deformation over panel c.f. pillar).

4.5 Summary

For the three DDJ2D runs and the two isotropic FINELP runs the calculated subsidence figures did not correlate well with the field subsidence figures of 950 mm and 400 mm. In particular the calculated figures for these runs showed generally uniform subsidences of less than 360 mm. Better correlation, however, was obtained from the four anisotropic FINELP runs. For the last of these runs the correlation was very good with values of 931 mm and 384 mm.

These results show that best correlation for future runs is most likely to be achieved using the "micro-joint" model (ie. no modelled joints, anisotropic strata properties). At present DDJ2D does not model anistropic strata, although the program could be so modified. In this modified state it is anticipated that DDJ2D could predict subsidence values which would also correlate well with the field values.

Whilst the correlation for the anisotropic micro-joint model is mathematically good, the question must be asked whether the assumed geotechnical parameters (E, G, etc.) are in fact representative of the geological conditions. The values selected for E and u were taken from laboratory test results and hence have some justification. The value selected for G (=E/70) to give the best correlation is considered realistic, and compares with values of E/110 (Rockaway and Elifrits, 1978) and E/20 (Wardle and Enever, 1983), both in similar horizontally bedded sedimentary strata. To further resolve the question will entail the analysis of different mining situations including areas where there has been no roof collapse.

All of the above runs treated the problem as a one stage solution and made no attempt to model the progressive collapse of the roof. Modelling this behaviour would probably show the need for a non-linear analysis with a no tension material that had a limited shear strength based on a Mohr Coulomb or similar yield criterion. This could be handled in the finite element and finite difference methods but not in the displacement discontinuity method.

5. FIELD MONITORING

The results in the above case study were correlated with field subsidence measurements (as discussed in 4.5 above) and the anticipated mine roof behaviour. This correlation is made difficult by the presence of a second seam but more generally before reliance can be placed on the results, subsurface correlation is required.

The Board is proposing to drill several investigation holes in an endeavour to provide subsurface data before and after a planned longwall extraction in the proximity of Cataract Dam spillway. The investigation program will include subsurface strain monitoring, geological logging, surface survey, surface seismic traverse, water table measurement, water testing, etc. The location and depth of the holes will be determined following two dimensional mathematical analyses. The monitored results will be compared with the results from the mathematical model and used to further its development.

6. PROGRAM OPTIONS

The work during the Mining Inquiry and subsequent work with DDJ2D was primarily a "macro-jointing" approach where the true jointing situation is modelled by using joint elements at discrete spacing in vertical and horizontal planes. This approach yields a "blocky rock" type of analysis where the properties of the joints have greater influence than the rock properties.

In addition to the purchase of SAGE the Board obtained an option on program RAGE (Dames and Moore 1981) which analyses the movement of discrete blocks (with the option of rounded corners) that can separate, slide and/or rotate. This program is still "state of the art" with limited documentation but its advantage is that it is a program designed to analyse the strata as a set of independent blocks. If sufficient geological information was available to show that independent blocks were present in a mining situation then RAGE would be investigated.

The finite element modelling has concentrated on the "micro-joint" approach where the jointing is allowed for in the bulk material properties. This approach gave the most successful results in the case study and seems to be the preferable choice for future work.

All of the above discussion has centred on a two dimensional approach. While the majority of cases can be successfully analysed by two dimensional methods some will require a three dimensional model to give the full stress/deformation picture. It is considered that, because of the complexity of three dimensional data preparation and computer costs, economical three dimensional modelling will only be possible with the "micro-joint" approach.

7. HYDROGEOLOGICAL ASSESSMENT

All analyses to date have been structural analyses based on the total stress approach (ie. no allowance for pore pressure). However, from the Board's point of view, the problem is additionally a hydrogeological one involving loss of stored water from the Board's reservoirs. For this reason later studies will be directed towards flow of water through the rock mass based on an assessment of the permeabilities before and after mining. Groundwater Resource Consultants (1983) have postulated that, if the initial permeability is known (e.g. from borehole testing), the post mining permeability can be estimated using the Poiseuille equation:

$$Q = \frac{\gamma}{12u} \; lb^3 I$$

where

Q = rate of flow
γ = specific weight of water
u = dynamic viscosity
l = length of segment normal to flow
b = segment aperture
I = hydraulic gradient

With such an approach it may be possible to convert post mining strains to increases in permeability and hence to calculate the water losses from the storages as a consequence of mining. Both the finite element and finite difference techniques have the ability to analyse hydrogeological models.

8. CONCLUSIONS

The early mathematical modelling used the finite element and displacement discontinuity analyses to estimate surface subsidences and strata stresses for proposed mining patterns. The results were consistent with what judgement might suggest but, because of lack of specific geological information and lack of observed behaviour, could only be an indication of real behaviour.

The case study presented is the first step in the process of verification of a mathematical model by correlation with observed behaviour. Good correlation with surface field data was obtained from the finite element program using realistic values of Young's modulus (based on material testing) and shear modulus (from the parametric study but within limits used by others). This correlation was based on a homogeneous (no joints) anisotropic material and could have been achieved by any of the other analysis techniques considered (finite difference, displacement discontinuity).

The next step in the verification of the model will be correlation with surface and subsurface data from other mining situations to determine whether other material parameters (eg. non linearity) need to be considered. Success in this step will enable surface and subsurface deformations to be confidently predicted for future mining activities. The surface predictions will be used to determine whether the current mining exclusion limits around the Board's dams are adequate to ensure their safety. The subsurface predictions will be used to assess the strata permeability and thus allow calculation of the potential loss of stored water attributable to the proposed mining.

9. ACKNOWLEDGEMENTS

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