Post Failure Characteristics of UK Coal Measures Rocks with Special Reference to Mine Tunnel Stability

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SUMMARY This paper describes the investigations undertaken to determine the post-failure characteristics of several UK Carboniferous rock types under uniaxial and triaxial stress conditions. A number of empirical failure criteria have been employed in the analysis of the peak and residual strength values. The volumetric expansion of rocks has been measured and analysed. The importance of these evaluated parameters applied to the design and stability of tunnels is discussed.

1 INTRODUCTION

The application of rock mechanics to engineering problems involving underground excavations, is to predict and allow for the response of the rock to the forces imposed on it by the creation of an excavation. A knowledge of the physical and mechanical properties of rock material is of great importance in the design and assessment of the stability of any major mining excavation. Extensive research programmes have been conducted into the pre-failure strength characteristics of Coal Measures rocks and their associated discontinuities in the Department of Mining Engineering, University of Nottingham (Hassani, 1980a). Recent investigations into the evaluation of such parameters has included the examination of the complete rock post-failure characteristics and their associated volumetric expansion, (White, 1983). The post-failure characteristics of rocks in this investigation have been obtained by testing techniques employing a RDF-Howden electro-hydraulic servo-controlled testing system. The associated volumetric expansions are determined by specially designed apparatus to measure specimen dilation.

A number of researchers have proposed various theoretical formulae to predict the anticipated closure and support requirement for major access tunnels. The most recent of such proposals is that of Wilson (1980a) which highlights the use and importance of post-failure characteristics of Coal Measures rocks in evaluating circular tunnel closure in UK coal mines.

This preliminary investigation has been conducted to evaluate these parameters and to assess their use in theoretically based closure formulae.

2 ROCK PARAMETERS INVOLVED IN TUNNEL CLOSURE FORMULAE

The creation of an underground excavation produces a redistribution of the initial state of stress within the rock mass. UK Coal Measures rocks are frequently weak, and it is recognised that this redistribution of stress results in the development of a rock 'yield zone' surrounding such excavations. The existence and extent of such yield zones surrounding major access tunnels have been verified by measurements taken in a number of tunnels driven in Coal Measures rock, (Whittaker et al, 1983).

The tunnel support requirement is known to depend on rock strength characteristics, in situ stress condition, and the amount of closure allowed prior to support installation. Bray (1967), Ladanyi (1974) and Wilson (1980b) have developed theoretical design criteria to aid in the design of support requirement and anticipated tunnel closure. A detailed discussion of these design principles is beyond the scope of this paper; however, the design formulae proposed by Wilson (1980b), and currently employed by the NCB in tunnel design, is presented in order to highlight the parameters investigated by the authors.

2.1 Wilson Closure Formula

In order to develop a relatively simple equation for the evaluation of tunnel closure and the strength of tunnel lining, the following assumptions were made:

(a) The drivage has a circular cross-section.
(b) It is surrounded by a homogeneous, isotropic rock mass.
(c) Plane strain conditions exist.
(d) Virgin stress field is hydrostatic.

Considering the above assumptions, together with the theoretical stress conditions and rock expansion in the yield zone, Wilson (1980b) proposed the following equation:

\[ C = \frac{d}{E} \left[ \frac{(k+1)q+\sigma'}{(k+1)} \right] \left[ \frac{2q - \sigma}{q} \right] \frac{(2+\varepsilon)}{(k+1)} \]

where

- \( C \) = the diameter closure
- \( d \) = the driven diameter
- \( E \) = Poisson's Ratio
- \( v \) = Modulus of Elasticity
- \( k \) = Triaxial stress factor
- \( q \) = cover load
- \( \varepsilon \) = laboratory determined unconfined compressive strength of the rock
- \( p \) = lining resistance
- \( p' \) = an augmentation of the lining resistance brought about by the 'Cohesion' of the yielded rock
- \( F \) = a factor relating laboratory strength to in situ strength and takes into account the degree of jointing and fracturing
- \( \varepsilon \) = the expansion factor for rock dilation

4.4 Final closure, \( C_t \) (mm)

\[ C_t = \frac{(k+1)q+\sigma'}{(k+1)} \left[ \frac{2q - \sigma}{q} \right] \frac{(2+\varepsilon)}{(k+1)} \]

where

- \( C_t \) = final closure
- \( q \) = cover load
- \( \varepsilon \) = laboratory determined unconfined compressive strength of the rock
The parameters detailed above required investigation to determine their range of application.

3 POST FAILURE TESTING

The concept of controlled failure testing dictates that in order to obtain a complete stress-strain relationship for rock, total control must be achieved over the absorbed strain energy from the machine and the rate of displacement applied to the specimen. This can be achieved using two basic types of testing machine:

(a) By using a testing machine designed to obtain high values of longitudinal stiffness; in this type of machine, load is applied to the specimen by moving the machine crossheads together at a constant displacement rate.

(b) By using a testing machine which incorporates a closed-loop electro-hydraulic servo-controlled system. A constant displacement rate is maintained by the closed-loop operation of the servo-system adjusting continuously the actual displacement to that of the required programmed displacement. This action also results in the system obtaining a high apparent stiffness.

In the last two decades laboratory testing equipment has been characterised by an increase in sophistication. The development of "stiff" testing systems, the use of electro-hydraulic servo-controlled feedback systems, and recently the use of micro-computers in control systems has enabled the control of deformation and failure in rock specimens to be effectively achieved.

Control over the rock failure process results in the attainment of complete post-failure stress-strain curves, thus allowing an evaluation of relevant rock strength characteristics. Previous research has shown that there are a number of testing parameters which can influence the shape and thus magnitude of a rock specimen's post-failure curve. In the absence of recognised standards for conducting post-failure tests, the authors conducted an extensive review of previous work together with an examination of some of the parameters which are likely to influence the results of rock post-failure properties.

Post-failure characteristics of various rocks have been investigated by several researchers. Wawersik (1969), Wawersik and Fairhurst (1970) and Wawersik and Brace (1971) completed a detailed analysis of the post-failure characteristics of a number of rock types. From this research, a classification for the behavioural characteristics of rock in its post-failure state was proposed. It was based on a specimen's axial stress-strain relationship, as shown in Fig. 1. Rock types were termed 'stable' and 'unstable' or 'Class I' and 'Class II', respectively. A Class I rock is one in which work must be done on the rock to promote continued failure within the test specimen, resulting in a negative slope of the axial stress-strain curve. A Class II rock is one in which the failure process in the post-failure region is self-sustaining due to stored energy within the specimen. Thus, to control failure energy must be extracted from the specimen, resulting in a positive slope of the axial stress-strain curve.

Monotonically increasing displacement rate dictates that the complete failure curve for Class I rock type will be obtained, Curve OABC, Figure 1. The implications of these test conditions on a Class II rock type are apparent. The post-failure unloading path will be that of a Class I rock type, and not that which is required for controlled failure, OATC, as shown in Fig. 1. The specimen thus fails violently. Wawersik and Brace (1970) achieved controlled failure of Class II rock specimens, tested in a stiff testing machine. By rapidly backing off the platens at points of instability, the applied load to the specimen was reduced thus controlling the amount of excess strain energy within the specimen.

In a servo-controlled test system, this technique is achieved by optimising the feedback control signal. Optimum control is achieved when the feedback control signal is located for maximum sensitivity in detecting movements associated with specimen failure. In general, specimens develop cracks perpendicular to the least principal stress. In a uniaxial compression test, the displacement perpendicular to the loading axis should be measured to optimise the feedback control signal, Husdon et al (1972). Furthermore Hudson et al (1972) successfully controlled the failure process of Class II rock types in a servo-controlled testing system utilising this control technique.

3.1 Testing System

The test system employed for this investigation was a RDP-Howden electro-hydraulic servo-controlled testing system, comprising of:

(a) straining frame;
(b) SL 2000 analogue control console;
(c) hydraulic power pack;
(d) Apple II Micro-computer system.

The testing frame comprises of four steel columns fitted with variable daylight crossheads which is automatically clamped (hydraulically) to the columns. The crosshead is positioned by twin electrically driven screws. A double acting servo-controlled equal area actuator is located on the top crosshead. The testing frame is designed to apply maximum compressive and tensile loads of 1000 kN over a total working stroke of 100 mm with an overall machine stiffness of > 2,500 kN/mm. The SL 2000 analogue control console provides all the necessary feedback control system to perform closed-loop servo-controlled testing. An Apple II micro-computer was
3.2.1 The effect of strain rate

In order to evaluate the effect of strain rate on the complete uniaxial compressive strength characteristics a preliminary series of tests were performed upon four different types of Coal Measures sandstone. Cylindrical test specimens of height 100 mm and diameter 50 mm were prepared to minimum preparation tolerances given by ISRM (1972). Axial strain rates employed were $1 \times 10^{-5}$, $1 \times 10^{-5}$, $1 \times 10^{-6}$, $1 \times 10^{-6}$, $4 \times 10^{-7}$, $7 \times 10^{-7}$, $1 \times 10^{-7}$ mm/sec which were equivalent to applied specimen displacement rates of 1 mm in 100, 1000, 5000, 10,000, 25,000, 50,000 and 100,000 seconds.

The following conclusions were reached from the experimental results:

(a) For a decrease in rate of strain below $1 \times 10^{-5}$ there was little variation in the measured values of uniaxial compressive strength and elastic deformational moduli. Similar findings have been reported by Shanya and Dhir (1972). In general better specimen post-failure control is achieved for slower strain rates.

(b) A decrease in strain rate results in a greater proliferation of micro and macro-cracks resulting in a gradual loss in specimen load-bearing capacity.

(c) The experimental evidence suggests that as strain rate decreases the unloading portion of the load-displacement curve steepens, and in some cases can result in the experimental output of a Class II rock failure. Therefore, sandstone with high values of porosity were found to fail with typical Class I failure curves.

3.2.2 The effect of specimen geometry

With regard to the above findings, the strain rate of $2 \times 10^{-6}$ was used in order to investigate the effect of specimen diameter size and height to diameter ratio upon the complete post-failure characteristics. A medium grained sandstone was used for the purpose of this investigation. Rock specimens were prepared at five different diameter sizes of 25, 37, 50, 61 and 100 mm with height to diameter ratios of 1, 2 and 3.

The following conclusions were reached from the experimental results:

(a) Elastic moduli did not vary with a change in specimen diameter or height to diameter ratio.

(b) For specimens of one particular height to diameter ratio, i.e. 1, 2 or 3, there is no significant change in the obtained values of uniaxial compressive strength.

(c) There is a marked decrease in the strength of rock specimens with a height to diameter ratio of 1 to 2 for all specimen diameters. However, the decrease of height to diameter ratio from 2 to 3 is far less significant. This confirms the findings of Mogi (1966).

(d) Specimen geometry significantly affects the mode of specimen failure and the subsequent shape of the obtained post-failure curve.

The majority of the findings presented in the above two sub-sections are in close agreement with other researchers in this field, namely, Hudson et al (1972).

3.3 Uniaxial Compressive Strength Tests

Uniaxial compressive strength tests were performed on a range of Coal Measures rocks listed in Table 1.
TABLE I

PHYSICAL AND MECHANICAL PROPERTIES OF TYPICAL COAL MEASURE ROCKS

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Density kg/m³</th>
<th>Porosity %</th>
<th>U.C.S. MPa</th>
<th>Tensile MPa</th>
<th>E₀ GPa</th>
<th>ν₅₀</th>
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</thead>
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<tr>
<td>F.G.* Sandstone</td>
<td>2,433</td>
<td>6.06</td>
<td>52.63</td>
<td>4.47</td>
<td>9.36</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M.G.* Sandstone</td>
<td>2,292</td>
<td>13.70</td>
<td>57.89</td>
<td>3.46</td>
<td>8.88</td>
<td>0.226</td>
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<td>Derbyshire</td>
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<tr>
<td>Maltby Siltstone</td>
<td>2,728</td>
<td>7.30</td>
<td>48.35</td>
<td>8.95</td>
<td>15.16</td>
<td>0.214</td>
</tr>
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<td>Selby Mudstone</td>
<td>2,565</td>
<td>13.90</td>
<td>37.69</td>
<td>2.78</td>
<td>7.50</td>
<td>0.278</td>
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<tr>
<td>Cotgrave Seat earth</td>
<td>2,448</td>
<td>-</td>
<td>20.17</td>
<td>2.10</td>
<td>4.63</td>
<td>0.321</td>
</tr>
<tr>
<td>Blackshale Coal</td>
<td>1,371</td>
<td>4.90</td>
<td>17.13</td>
<td>2.03</td>
<td>5.54</td>
<td>-</td>
</tr>
</tbody>
</table>

* F.G. = Fine grain, M.G. = Medium grain.

Prior to the uniaxial strength tests the physical properties of the rock types were determined and are presented together with the mechanical properties in Table 1. In the absence of recognised testing standards for the attainment of complete post-failure curves the authors adopted the following testing standards based upon the findings of their preliminary investigations. The uniaxial compressive strength tests were conducted upon specimens of 50 mm diameter with height to diameter ratio of 2, and at an applied rate of strain equal to 2 x 10⁻⁶ mm/mm/sec. These standards were adopted to minimise the influence of specimen geometry, and the effect of rate of strain upon the specimen complete stress-strain curve as well as taking into account the practical size limitations of specimen and test duration. Several tests were conducted on each rock type in order to evaluate the validity and repeatability of the results. Typical complete stress-strain curves for each individual rock type are given in Figure 3.

The complete stress-strain curves of the various Coal Measures rocks shown in Figure 3 indicate that the failure of the rock specimens may be categorized as Class I type, after Wawersik (1968). Furthermore, it is clearly shown that the load bearing capacity of the rock specimens in the post-failure region is dependent upon the applied axial deformation. Consequently, at an axial deformation of greater than 3 mm or 3% axial strain there is a total loss of load bearing capacity. This is a result of gross 'slabbing' and 'shearing' of the specimen resulting in total disintegration of the rock specimens. The loss of load-bearing capacity with deformation is an important factor to be taken into consideration in the application of complete rock strength characteristics to design of rock excavations.

3.4 Triaxial Compressive Strength Tests

The triaxial compressive strength tests were conducted on several Coal Measures rocks in order to evaluate the complete post-failure characteristics under confined stress conditions. This has enabled the determination of the peak and residual strength values under different confining pressures. Furthermore, with specially designed apparatus, measurements of specimen dilation were obtained in order to evaluate a relationship for the volumetric expansion of rock under triaxial failure conditions.

The triaxial testing apparatus consisted of a conventional Hoek cell accommodating a 50 mm diameter test specimen with a length of 100 mm. The confining pressure is applied through a manually operated hydraulic system and monitored by a 70 MN/m² (= 10,000 psi) pressure transducer to an accuracy of 0.007 MN/m² (= 1 psi). Measurements of specimen dilation are obtained by maintaining a constant confining pressure in the Hoek cell throughout the experiments via a manually operated fine needle valve bleed system which is connected in series with the hydraulic confining system. Volumetric expansion of the specimen is obtained by measuring the volume of displaced oil required to maintain a constant cell pressure. Cell pressure was maintained to a value of ± 2 psi with the displaced oil measured to an accuracy of 0.02 cc.

To obtain complete post-failure characteristics of rock specimens under triaxial stress conditions larger axial deformations are required compared to the uniaxial case. Therefore, from practical

Figure 3 Complete stress-strain curves for typical Coal Measures rocks (uniaxial tests)
3.4.1 Triaxial strength test results

Triaxial strength tests were conducted at seven different confining pressures of 0.345, 0.69, 1.72, 3.45, 6.9, 13.8 and 27.6 MPa (as shown by curve numbers 1 to 7 respectively in Figs. 4 and 5). Figure 4a, b, c, d, e and f show typical families of complete stress-strain curves for each individual rock type under investigation. Axial deformation at which a constant residual strength is sustained varies from 1% to 3% for the range of confining pressures employed in this investigation. A maximum of 5% axial deformation is applied to the rock specimens at all confining pressures in order to ensure that the residual strength values are obtained. The peak and residual strength values for each individual rock type under different values of confining pressure are measured. Generally, \( \sigma_1 \) and \( \sigma_3 \) relationship for both peak and residual strength values are approximated to a linear equation, which may be expressed by:

\[
\sigma_1 = A\sigma_3 + C \tag{2}
\]

where

\( \sigma_1 \) = the peak or residual axial stress
\( \sigma_3 \) = the applied lateral stress
\( A' \) = a constant commonly referred to as the 'triaxial stress factor'.
\( K' \) = the value of uniaxial compressive strength.

In practice it is, however, evident that the relationship of \( \sigma_1 \) and \( \sigma_3 \) for both peak and residual strength values are not linear and scientifically it is more realistic to represent these relationships by employing a non-linear regression analysis. A power curve with y axis intercept for peak values (model no. 1 in Table 2) and power curve together with power curve with y axis intercept for residual values (model no. 2 and 3 in table 3) are used for these analyses. The power curve (model no. 1 in table 3) is employed because theoretically and experimentally it is proved that when \( \sigma_3 = 0 \) at ultimate axial deformation value of \( \sigma_1 \) for broken material will be zero. A relatively good coefficient of correlation 0.98 are obtained for all the regression analyses. The comparison of the coefficient of correlation indicates that power curve with y axis intercept results in the best fit to the data for both peak and residual strength values. Therefore the stress factor \( K' \) is variable depending upon the lateral pressure employed.

The shear strength properties of rocks under triaxial conditions may be evaluated by analysis of the relationship of calculated shear strengths and the normal stress. This is obtained by either linear or non-linear mathematical curves fitting techniques. The simplest method is the Coulomb-Navier linear failure criteria given by the following equation:

\[
\tau = \tan \phi \sigma_n + C \tag{3}
\]

where

\( \tau \) = shear strength
\( \phi \) = angle of internal friction
\( \sigma_n \) = normal stress
\( C' \) = intercept of y-axis known as 'apparent cohesion'.

Obert and Duvall (1968) by employing a similar configuration of Mohr Circles have approximated and evaluated the shear strength and angle of internal friction by the following simplified mathematical solutions:

\[
\frac{\Delta \sigma}{\Delta \sigma_1} = \tan \beta = \frac{3}{K} = \frac{\sin \phi + 1}{\sin \phi - 1} \tag{4}
\]

\[
\tan \phi = \frac{\tan \beta - 1}{2 \tan \beta} \tag{5}
\]

\[
\tau = \frac{(1 - \sin \phi)}{2 \cos \phi} \tag{6}
\]

Such analyses are an approximate solution to the triaxial data. However, due to their simplicity they have been used for the development of design formulai, Wilson (1980 a,b). Due to the non-linear nature of \( \phi \) with respect to \( \sigma_3 \), it applies that the relationship of \( \tau \) and \( \phi \) will also be a non-linear function with respect to the normal stress. Hassani (1980a) in research on the failure criteria of Coal Measures rocks examined a number of mathematical models to represent these failure criteria. The recommended non-linear models were employed for this investigation together with the linear relationships for comparison, as listed in tables 2 and 3.

The linear analysis of the \( \sigma_1 \) and \( \sigma_3 \) values indicate that the constant \( A' \) (equal to the triaxial stress factor) for Coal Measures rock ranges between 2-6 for peak and residual \( \sigma_3 \) values. However, as previously mentioned, due to the non-linear nature of these relationships the value of stress factor will be variable depending upon the lateral pressure employed.

3.4.2 Volumetric expansion

The volumetric expansion of the rock specimens was continually monitored and measured during the triaxial test. The results of volumetric strain are calculated as the ratio of change in volume to original volume of rock specimens. Figures 5a, h, c, d, e and f show the change in volumetric strain with respect to applied axial strain and confining pressure. The results indicate that the measured quantity of volumetric strain is a function of the applied confining pressure, and axial strain. Initially, it is found that there is an 'elastic' compaction of the material. Upon fracture initiation there is a rapid increase in specimen volumetric expansion. This sudden increase is markedly reduced with increasing lateral pressure. In some cases, at very high confining pressures the 'elastic' compaction of the material is greater than the specimen dilation due to fracturing and therefore the resultant output is one of negative volume change.

Graphical representation of the data, as shown in Fig. 6, indicated that an exponential decay function could be used to evaluate the relationship between volumetric strain and confining pressure. An empirical relationship of the form:

\[
V = A e^{-\frac{b}{\sigma_3}} + C
\]

was used to evaluate the experimental data. Fig. 6 shows that for an increase in axial strain there is a resultant change in volumetric strain. Therefore, curve fits were undertaken at 2, 3, 4 and 5% axial deformation to analyze the effect of applied axial strain to the empirical relationship of \( V \). The results of volumetric expansion for typical Coal Measures rocks are presented in Table 4.
### TABLE II
RESULTS OF TRIAXIAL TESTS FOR PEAK STRENGTHS

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Rock Type</th>
<th>Coal</th>
<th>F.G. Sandstone</th>
<th>M.G. Sandstone</th>
<th>Seatearth</th>
<th>Siltstone</th>
<th>Mudstone</th>
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<td>A</td>
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<td>24.37</td>
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<td>n</td>
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### TABLE III
RESULTS FOR TRIAXIAL TESTS FOR RESIDUAL STRENGTHS

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<tr>
<th>Model No.</th>
<th>Rock Type</th>
<th>Coal</th>
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<th>M.G. Sandstone</th>
<th>Seatearth</th>
<th>Siltstone</th>
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<td>15.23</td>
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4. PRACTICAL APPLICATION OF TEST RESULTS

The results of the pre- and post-failure strength characteristics of typical Coal Measures rocks associated with UK underground coal mining, presented here, may be used directly to assist in the design of underground excavations. In order to evaluate the effect of such parameters on tunnel stability, comparisons are made with the theoretical tunnel design formulae proposed by Wilson (1980a, b). The effect of the failure criteria proposed by the authors may be discussed in relation to tunnel stability analysis.

Equation 1 is the final form of the equation interrelating the excavation closure to the lining strength and the rock properties, the values of Poisson’s ratio (v), Modulus of elasticity (E) and uniaxial compressive strength; representative values are given in Table 1. The driven diameter of the tunnel (d), cover load (q) and lining resistance (p) can be readily obtained. The parameters which have a significant influence on the final estimated diametric closure of a tunnel and require close examination and discussion are those obtained from post failure triaxial strength tests.
The stress factor \((k)\) is assumed by Wilson (1980a, b) to be a constant and is obtained from a linear relationship of \(\sigma_1\) with \(\sigma_0\). The values of the stress factor \((k)\) for typical Coal Measures rocks are presented by a constant \(A\) in model 2 in Table 2 and model 3 in Table 3 for both peak and residual strength values respectively. The linear relationship of \(\sigma_1\) with \(\sigma_0\) for residual strength values is used by Wilson (1980a, b) to determine the augmentation to the lining resistance brought about by the cohesion of the yielded rock \((P')\).

The parameter is given as

\[
P' = \frac{\sigma_0'}{K - 1}
\]

where

- \(\sigma_0'\) = the uniaxial compressive strength of broken rock material

The value of \(P'\) is approximated to 0.1 MPa by Wilson employing triaxial results obtained from granulated material.

From the linear regression analysis of the residual triaxial strength test represented by model 3in
The non-linear relationship of $\sigma_1$ and $\sigma_2$ values are found to be best represented by either a power curve or power curve with cohesion (y axis intercept) as shown in Tables 2 and 3. The use of power curves dictates that at zero confining pressure the uniaxial strength of broken rock material to be zero. This is found to be in agreement with experimental results obtained from the post failure strength test.

Figure 2 indicates that with sufficient axial deformation the residual uniaxial strength of broken rock material for all Coal Measures rocks reduces to zero value. With regard to dependency of such parameters on the axial deformation the authors believe that the power curve with cohesion (y axis intercept) may also be used where ultimate residual strength is not reached. Employing these non-linear relationships, however, causes the deduction of the value of $P'$ to be very complex. In order to incorporate such non-linear function to design formulae such as Wilson's, an iterative method of computation must be used. Such prediction methods of tunnel closure employing non-linear failure criteria are presently being developed at the Department of Mining Engineering, University of Nottingham.

### Table V

<table>
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<th>Rock Type</th>
<th>$k$</th>
<th>$\sigma_0'$</th>
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</table>

The behaviour of the Coal Measures rocks under triaxial conditions are represented by a number of empirical solutions. The normalized power curve relationship (model 4, Table 2) is found to represent the best fit to the peak of strength data. This is confirmed by Hassani et al. (1980b) and Brook (1979). The power curve fit to residual strength values (model 4, Table 3) also represents the best fit to such data. This relationship is not normalized because the value of uniaxial compressive strength of broken rock material ultimately is zero.

The closure prediction of a tunnel driven at depth, requires a function which takes into account the volumetric expansion of the rock when failure occurs. In the theoretical analysis by Wilson an expression factor $e$ is incorporated which is taken to equal a constant value of 0.2. The bulk factor of granulated rock is approximately 1.5, therefore $e$ must be less than 0.5, hence

$$0 < e < 0.5$$

However, the authors have shown that the volumetric strain of a rock specimen is a function of the applied confining pressure and the axial deformation, given by

$$V = A e^{-\sigma_3/C} + C$$

where $A$, $B$, $C$ are constants

$V$ is volumetric strain.
It is indicated from this work that this function will provide a good assessment of the volumetric expansion of the rock yield zone surrounding underground excavations. Table 4 gives the values of the constants A, B and C for axial deformation of 2, 3, 4 and 5%. It is proposed at present that the values of the constant required should be those of 5% axial deformation. Further research is being conducted to analyze the effect of larger axial deformation on the values of A, B and C.

5 CONCLUSIONS

In this investigation complete and controlled stress-strain curves representing post-failure characteristics of typical Coal Measures rocks under uniaxial and triaxial stress conditions have been successfully obtained. The effect of the strain rate, specimen size and height to diameter ratio upon such rock characteristics have been evaluated. It is recommended that post-failure tests should be conducted on specimens of 50 mm diameter with length of 100 mm, at strain rates of 2 x 10^{-6} \text{ mm/mm/sec} for uniaxial tests, and 4 x 10^{-6} \text{ mm/mm/sec} for uniaxial tests. It was found that 5% axial deformation would be sufficient to obtain the ultimate residual strength for Coal Measures rocks.

The investigations reported here underline the importance of taking post-failure behaviour of rocks into account in tunnel design assessment work. The results give an indication of how confining stress can greatly affect the behaviour of rocks around mining excavations. It is important, however, to ensure that a standardised testing procedure is rigidly adhered to in post-failure testing of rocks.

6 ACKNOWLEDGEMENTS

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7 REFERENCES


