Strain Compatability and Design Criteria for Reinforced Earth

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SUMMARY Semi-empirical design methods are applied successfully to design conventional reinforced earth, geotextile reinforced earth and nailed soil. However, as new types of reinforcing elements are tried in a variety of types of soils and rocks, there is considerable scope for further study of the most appropriate design assumptions and techniques for analysis and design of soil and rock reinforcement.

A method of analysis is described, which is a modification of a working load, displacement based analysis known as the displacement method. The progressive development of displacement as construction proceeds is modelled, as is the stress-strain behaviour of the soil.

Results of analyses using the modified displacement method and analyses using an elasto-plastic finite difference program are studied to provide insight into the effect of the method of determining the normal stress distribition on the failure surface and the significance of choice of Factors of Safety.

1. INTRODUCTION

Semi-empirical design methods are applied successfully to design conventional reinforced earth (Juran et al, 1978), geotextile reinforced earth (Jewell, 1990) and nailed soil (Guillox et al, 1983) and there has been extensive research and field studies of these types of problems. However, as new types of reinforcing elements are tried in a variety of types of soils and rocks, there is considerable scope for further study of the most appropriate design assumptions and techniques for analysis and design of soil and rock reinforcement.

The present work uses a modified "displacement method" of analysis to study the significance of various design assumptions and to provide some insight into choice of appropriate Factors of Safety. It is thought that displacement methods, while relatively simple and expedient, offer significant advantages because they can show the inter-relationship between the relative stiffness of the various components of the system and the safety with respect to failure of these individual components.

The displacement method, as used by Gourc (1986) and others, has been modified to account for the progressive development of displacement as construction proceeds and to model the stress-strain behaviour of the soil in a simple way. A series of potential log spiral "failure" surfaces are considered and for each the rotational displacement required for equilibrium is calculated. The critical surface is chosen as that which undergoes the maximum decrease in potential energy to come to equilibrium.

2. DESIGN METHODS FOR REINFORCED SOIL SYSTEMS

In general, design methods for reinforced soil systems are currently based on analysis of the at-failure condition, and employ limit equilibrium methods of analysis or the limit analysis concept (Juran and Schlosser 1978). In these methods it is most common that the reinforcement force required for equilibrium is calculated, based on the assumption that the soil resistance is fully mobilized.

Most commonly, the Factor of Safety is defined as the maximum available reinforcement tension (limited by either reinforcement strength or soil-reinforcement interface strength), divided by the calculated reinforcement tension required for equilibrium.

The methods currently used for design do not allow the calculation of tensions developed under working loads, and therefore do not allow the assessment of the local Factor of Safety at the level of each reinforcement, with respect to reinforcement rupture or pullout.

The fundamental limitation of the current design methods in this respect is that the requirement of strain compatibility between the soil and the reinforcement is not considered. Strain compatibility must be considered if the load sharing between the soil and the reinforcements and between the reinforcements themselves is to be determined, and hence local Factors of Safety calculated. (The significance of local Factors of Safety will be discussed in some detail in a subsequent section.)

3. THE DISPLACEMENT METHOD

A displacement compatibility design method was proposed by Gourc et al. (1986) and Delmas et al. (1986), and developed further by Ratel (1987). The proposed method, known as the "displacement method", was based on the following assumptions:

- during construction, the active zone (see Figure 1) undergoes a rigid body translation or rotation. No allowance was made for the progressive development of the movement as construction proceeds. The boundary between the active zone and the resistant zone (termed the "failure surface" on Figure 1), is the line along which shear strains are concentrated, and also the locus of maximum tension.
- the relationship between the local deformation of the reinforcement at the potential failure surface, and the

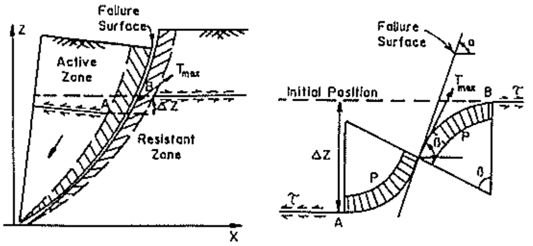


FIG. 1. **Anchored Membrane Concept for Geotextile Reinforcement (Gourc et al. 1986)**

normal stress acting on the reinforcement through the zone of local deformation is based on the concept of the reaction modulus of an elastic soil.

- the normal stress on the failure plane is determined using a conventional slope stability method of slices (based on the Fellenius assumptions).

The displacement of the active zone that is required to develop sufficient reinforcement tension to ensure global equilibrium of the active zone is calculated for a range of potential failure surfaces. In the determination of global equilibrium, the contribution of shear stresses in the soil to the forces resisting failure is limited to the ultimate resistance reduced by a desired Factor of Safety. The required level of reinforcement is determined such that the developed displacements do not exceed acceptable limits. The application of such a criteria for a reinforced embankment in a practical sense is not clear, since the displacements develop progressively as construction proceeds.

The method as proposed by Gourc et al. (1986) does not therefore fully consider the strain compatibility between the two components of the system since it is based on an assumed Factor of Safety on soil strength.

3.1 Modifications to the Displacement Method

An analysis technique (the "modified displacement method") has been developed by the principal author and is discussed in the following sections. This method is based on the displacement method as outlined above, but which differs from the method as proposed by Gourc et al. (1986), Delmas et al. (1986) and Ratel (1987) in two important aspects:

- the progressive development of displacement as construction proceeds is modelled.
- an attempt is made to model the shear stress - shear strain behaviour of the soil.

A computer program, MDISMET (Modified Displacement Method) has been developed to carry out the large number of calculations required.

3.1.1 Modelling of Construction Process

In the originally proposed formulation of the displacement method, equal translational or rotational displacements (depending on the shape adopted for the failure surface) are experienced at all locations along the failure surface. Consequently, the only difference between the compatible reinforcement strains for the different reinforcing layers is due to changes in the inclination of the failure plane.

It is readily apparent that the displacement of the active zone develops progressively as the construction of a reinforced embankment proceeds. The compatible reinforcement strains are therefore not only a function of the failure surface geometry, but also of the location of the reinforcement. The reinforcements towards the base of the wall develop strain progressively as the material above is placed, and hence the reinforcement strains and tensions are greatest in the lower reinforcements.

In the proposed extension of the displacement method, the displacement required to develop equilibrium is determined *for each stage of construction*, considering the contribution of the layers of reinforcing *that have been installed at that stage of construction*. The final reinforcement strains are equal to the sum of the strains that have been imposed during each of the construction stages.

3.1.2 Modelling of Stress-Strain Behaviour of the Soil

In the proposed extension of the displacement method, the soil is modelled as an elastic, perfectly plastic material with a yield strength defined by the Mohr-Coulomb criteria (see Figure 2). The method adopted is similar to that adopted by Cooper (1988) in the analysis of progressive failure of slopes, except that the soil is assumed not to strain soften, and that a variation of δ_{peak} with normal stress is not considered.

The active zone is assumed to displace as a rigid body. The shear stresses developed along the failure plane are calculated as a function of displacement through the displacement modulus S (see Figure 2), but are limited to the shear strength of the material. The determination of the displacement modulus is currently based on the assumption of a value for δ_{peak} (values in the range of 10-15mm have been used). Definition of the displacement modulus in terms of the shear modulus of the soil is difficult and has not been attempted, since it is necessary to determine the width of the shear zone, which would be a function of the relative reinforcement stiffness.

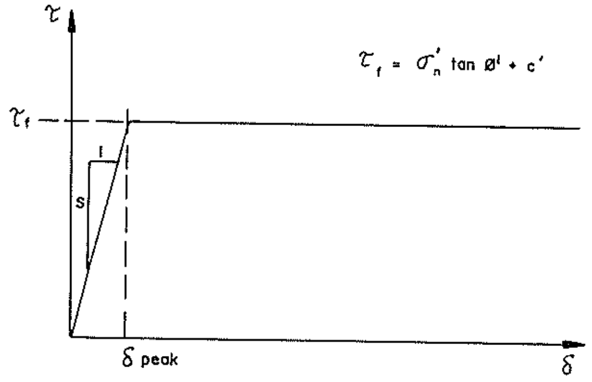


FIG. 2. **Stress - Strain Relationship for Soil**

3.1.3 Calculation Methodology

In the proposed extension of the displacement method, the active zone is incrementally displaced until the resisting contributions of the soil and the reinforcement are together equal to the disturbing forces. This calculation is carried out for a range of potential failure surfaces. The choice of the critical failure surface is based on the determination of the active zone which undergoes the maximum decrease in potential energy to come to equilibrium.

The reinforcement is considered to provide support to the active zone in two ways:direct support of the active zone through the action of the normal stresses between the reinforcement and the soil in the vicinity of the failure plane (Figure 1), and indirectly through increasing the normal stress on the failure surface (increased lateral confinement).

Separate Factors of Safety can be defined for the soil and for the reinforcement, either locally or globally. Further discussions on the choice and significance of Factors of Safety will be presented in a subsequent section.

4. CHOICE OF THE NORMAL STRESS ASSUMPTION TO BE USED IN THE DISPLACEMENT METHOD

The shape of the failure plane will have a significant effect on the choice of the most appropriate normal stress assumption. In the proposed development of the displacement method, analysis of vertical reinforced walls is based on the assumption that the failure surface will take the form of a log-spiral, which is vertical at its intersection with the ground surface behind the wall. A typical failure surface geometry is illustrated in Figure 4, which is consistent with experimental results published (Bolton et al., 1982).

The difference in normal stress on failure surfaces of such geometry, as determined using a method of slices based on the Fellenius assumptions and the Bishop assumptions has been investigated. Figure 3 illustrates the variation in normal stress with failure plane inclination and mobilized friction angle, as calculated by the two methods. It can be seen that for failure plane inclinations up to about 40 degrees, there is very little difference between the two methods, but that for steeper failure planes the difference is significant. It is therefore significant that the inclination of the failure plane shown in Figure 4 varies from about 55 degrees at the base of the wall up to 90 degrees at its intersection with the ground surface.

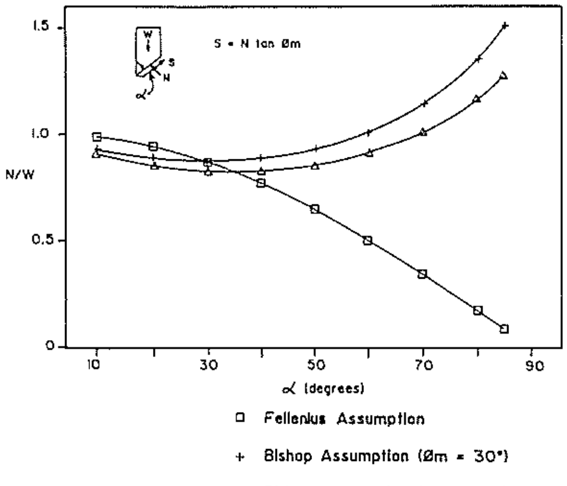
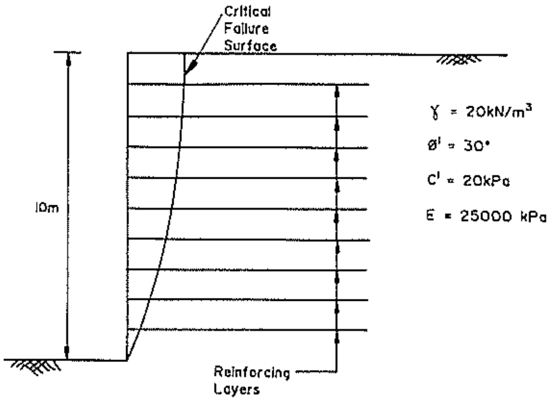


FIG. 3. Variation of Normal Stress on Slice Base with Base Angle for Fellenius and Bishop Assumptions

4.1 Effect of Normal Stress Assumption on the Results of the Displacement Method

Analyses were carried out for a vertical reinforced earth wall using the modified displacement method, to determine the significance of the normal stress assumption on the results of the calculation. Two cases were considered, with reinforcement of different tensile stiffnesses. The conditions analysed are depicted in Figure 4.

The formulation used for calculating normal stresses based on the Bishop assumptions was modified from the conventional formulation to make allowance for the dependance of the shear stress along the failure plane on the displacement of the active zone. The formulation used for the calculation of normal stresses based on the Fellenius method was not modified from the commonly used formulation.



CASE	REINFORCEMENT MODULUS (kN/m)
1 (Relatively Inflexible Reinforcement)	62800
2 (Flexible Reinforcement)	4000

FIG. 4. Conditions Analysed with Modified Displacement Method and with FLAC

The normal stress distributions calculated for the two cases are illustrated in Figure 5, and the effect of the normal stress distribution on the calculated reinforcement tensions is illustrated in Figure 6. The reported results are for the critical failure surface (based on a maximum decrease in potential energy), which is illustrated in Figure 4. It can be seen that the calculated reinforcement tensions are significantly greater if the calculation is based on normal stresses derived from the Fellenius assumptions.

4.2 Finite Difference Analyses to Determine the Most Appropriate Normal Stress Distribution

Since the results of the modified displacement method were significantly dependant on the assumed normal stresses on the failure surface, finite difference analyses were carried out in an attempt to determine which of the two normal stress assumptions that had been considered was more appropriate for use in the modified displacement method, or indeed if neither were appropriate.

The two cases analysed with the modified displacement method were analysed with the finite difference program FLAC. The finite difference grid used for the analysis is illustrated in Figure 7. The construction process involving the sequential placement of layers of fill and reinforcing was modelled. The soil was modelled as an elastic, perfectly plastic material with a yield strength defined by the Mohr-Coulomb criteria, and the reinforcements were modelled with the cable elements provided by FLAC (which have zero bending stress).

The FLAC analysis calculated normal stress distributions (on the critical failure surface that is illustrated in Figure 4) are included on Figure 5. It can be seen that while there is reasonable agreement between the distribution calculated by FLAC and that calculated by using the Bishop assumptions, the correlation between the FLAC calculated normal stresses and those calculated using the Fellenius assumptions is poor. It would therefore seem reasonable that calculation of normal stresses using a method of slices based on the Bishop assumptions would be more appropriate than using a method based on the Fellenius assumptions.

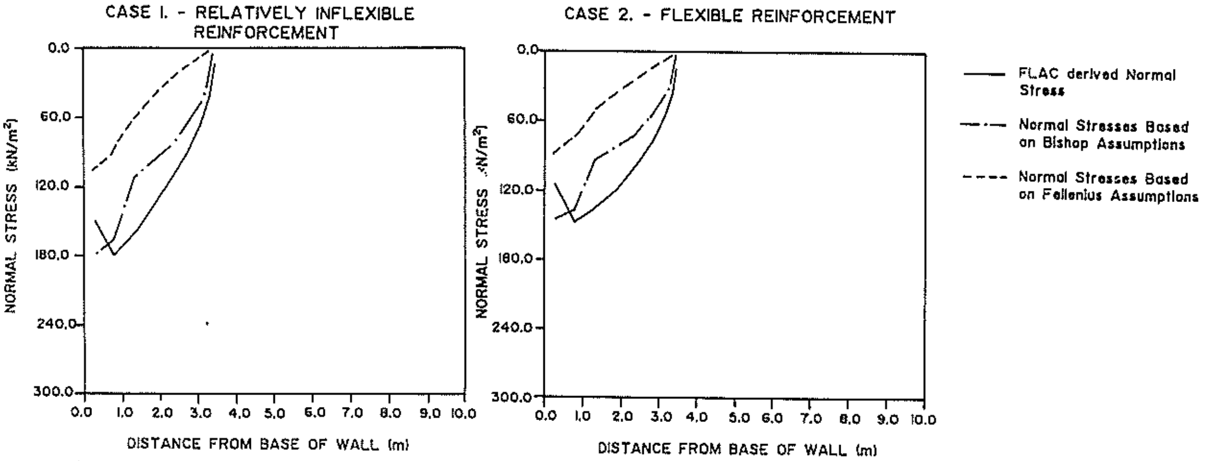


FIG. 5. Variation of Norml Stresses on Critical Failure Plane for Various Normal Stress Assumptions

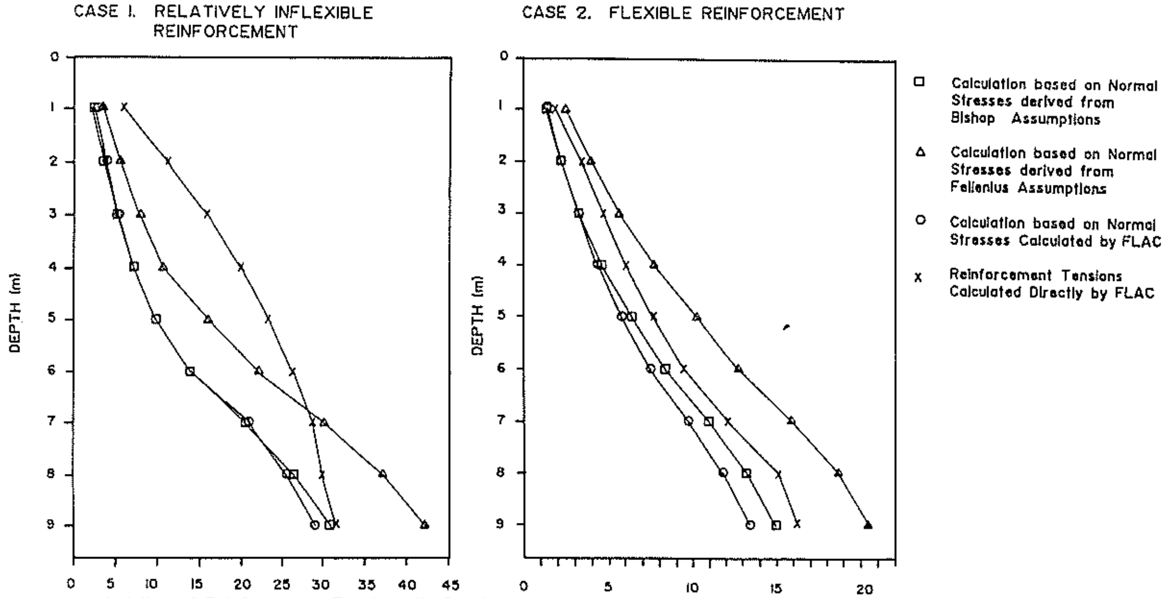


FIG. 6. Variation of Reinforcement Tension with Depth for Various Normal Stress Assumptions

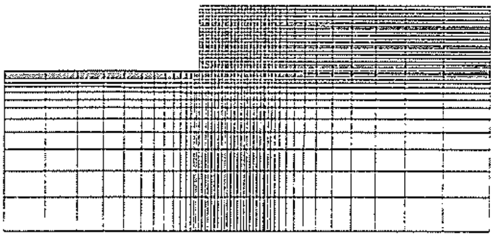


FIG. 7. Finite Difference Grid

This is supported by the comparison of reinforcement tensions that are calculated by the modified displacement method, using the normal stress distribution calculated by FLAC, with the reinforcement tensions calculated using the other normal stress distributions (Figure 6). Again there is a better correlation between the results from the FLAC derived normal stresses and the Bishop derived normal stresses than between the results from the FLAC derived normal stresses and the Fellenius derived normal stresses.

The reinforcement tensions that were calculated directly by FLAC are also illustrated in Figure 6. It can be seen in the case of the stiffer reinforcement that although the maximum reinforcement tension calculated by the modified displacement method using the Bishop assumptions is in good agreement with the maximum value calculated by FLAC, the distribution of reinforcement tensions is significantly different, with the total required reinforcement tension being much less when calculated by the modified displacement method.

In the case of the more flexible reinforcement, there is reasonable correlation between the tensions calculated by the modified displacement method using the Bishop assumptions, and the tensions calculated by FLAC.

The results suggest that the displacement method is more applicable to reinforcements which are flexible in tension than those which are relatively inflexible in tension.

5. SIGNIFICANCE AND CHOICE OF FACTORS OF SAFETY

5.1 General Comments

In general, reinforced soil systems can fail either by rupture of the reinforcements or by pullout of the reinforcements. The consequences of the two types of failure are different, and hence different Factors of Safety are appropriate for the different modes of failure.

Design methods for reinforced soil systems which are based on limit equilibrium concepts commonly calculate a global Factor of Safety on reinforcement rupture or on reinforcement pullout, with no Factor of Safety being applied to soil strength.

The assumption that all reinforcing elements have reached their ultimate strength at failure presumes a ductility and continuity of construction that may be difficult to achieve in practice (Bolton and Pang, 1982). Bolton and Pang (1982) and Juran and Schlosser (1978) identified the progressive nature of failure (in the case of failure by reinforcement rupture), in which the most highly stressed element ruptures first, followed in rapid succession by the remaining elements. This indicates that redistribution of the forces in the most highly stressed element to other elements should not be relied upon. However, limit equilibrium methods automatically redistribute stress away from the most highly stressed element so that all reinforcements have the same calculated Factor of Safety. Evidently, for the case of reinforcement rupture, the safety of a particular design is more directly related to the Factor of Safety against failure of the most highly stressed element than to a global Factor of Safety.

If the ultimate reinforcement pullout load was known with a high degree of confidence, it would seem reasonable to accept lower Factors of Safety on reinforcement pullout if a high Factor of Safety is calculated for the soil shear stresses. Intuitively it would seem that redistribution of stresses to the soil would be possible if a reinforcement failed by pulling out since it maintains the load that it carried at the point of failure, unlike a reinforcement which fails by rupturing and must shed all the load that it was carrying. A similar argument applies to the redistribution of stresses from the soil to the reinforcements once it has reached its ultimate strength. It is noted that the adoption of lower Factors of Safety would require the magnitudes of the loads carried by the soil and the reinforcement to be similar.

It is apparent that a method of calculation is required in which the tension in each individual reinforcement is determined and in which the proportions of the disturbing force carried by the soil and the reinforcements are determined, in order to rationally assess the risks associated with a proposed design.

5.2 Significance of Relative Stiffness

It is apparent that the sharing of load between the reinforcements and the soil is important when considering the appropriate Factors of Safety to be applied.

The effects of relative stiffness on load sharing and on Factors of Safety are illustrated in Figure 8. In this figure it is assumed that the influence of the reinforcement can be taken as an increase in shear resistance along the potential failure plane.

The benefit provided by the reinforcement as shown in Figure 8 is due to direct support to the active zone through the interaction of the reinforcement and the active zone in the vicinity of the failure surface, and due to indirect support provided by increased lateral confinement. The relative magnitudes of these components is not readily apparent but has significance in terms of the ability of the system to redistribute load between the two components.

It can be seen that for the same global Factor of Safety, the local Factors of Safety for the two components are significantly different. It is therefore apparent that if redistribution of stresses between the soil and the reinforcement is not possible, a design method is required in which strain compatibility is considered so that the load sharing between the components of the system can be determined. It would also seem that the acceptable Factors of Safety for the two components should include some provision for uncertainty in the modulus values.

5.3 FLAC Analyses of Load Redistribution

Non-linear finite difference analyses using the computer program FLAC were carried out to determine whether load can effectively be transferred from one component of the system to the other if it has insufficient strength. Analyses were not carried out for the case of reinforcement rupture, since the cable elements provided in FLAC will yield, but do not have a limiting strain at which rupture occurs.

The global Factors of Safety on soil strength and reinforcement pullout (calculated from the FLAC results), for the two cases considered previously, are tabulated below.

Table I - Global Factors of Safety for Cases 1 and 2

Case	Global FOS_{soil}	Global FOS_{rft}
Case 1 Relatively Inflexible Reinforcement	1.75	1.88
Case 2 Flexible Reinforcement	1.4	2.1

Further analyses were carried out using FLAC, in which the strengths of the soil and the reinforcement were separately reduced by the amounts shown above (strength reductions were considered for only one of the two components in any one of the analyses conducted). For example, in the first analysis based on Case 1, the strength of the soil was reduced by a factor of 1.75, and the reinforcement strength was not altered. The calculated global Factors of Safety, after strength reduction, are tabulated below.

Table II - Global Factors of Safety after Strength Reduction

Case	Global FOS_{soil}	Global FOS_{rft}
Case 1 with soil strength reduced by 1.75	1.0	1.25
Case 1 with reinforcement strength reduced by 1.88	1.64	1.46
Case 2 with soil strength reduced by 1.4	1.0	1.42
Case 2 with reinforcement strength reduced by 2.1	1.35	2.33

Failure of the wall did not occur for any of the strength reduction scenarios considered. However, for the case of relatively inflexible reinforcement and reduced soil strength, the results indicated that the system was marginally stable, and that the addition of any extra load would have caused failure. Although the calculated global Factor of Safety for the reinforcement was 1.25, the lowest four reinforcements were at the point of pullout failure.

It can be concluded that there is significantly more capacity, within the systems considered, for load to be transferred from the reinforcements to the soil (under conditions of reinforcement pullout), than for load to be transferred from the soil to the reinforcement. It is apparent that the major contribution of strength improvement provided by the reinforcement is in the form

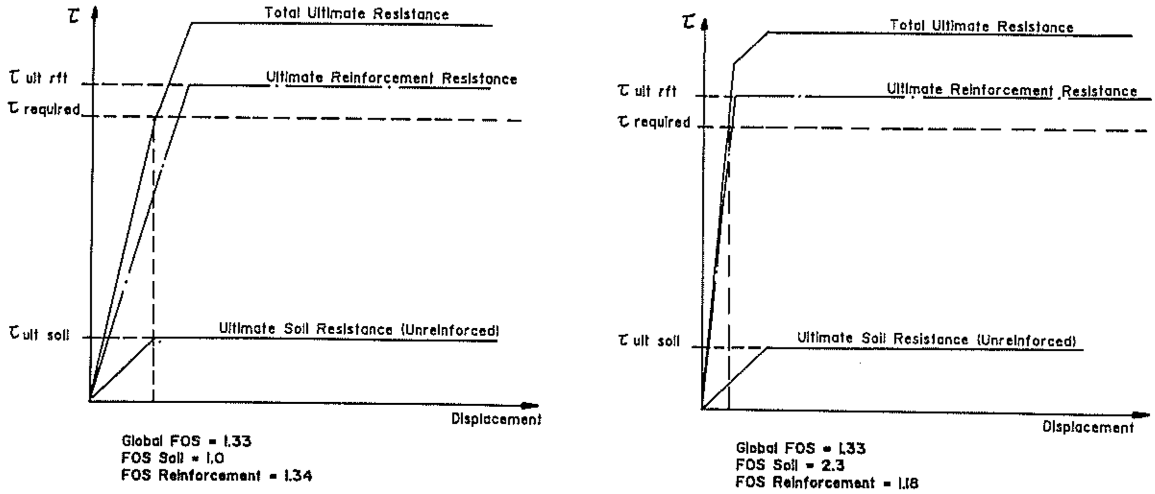


FIG. 8. Effect of Relative Stiffness on Load Sharing and Factors of Safety

of increased lateral confinement. Under conditions of reduced soil strength, this contribution is decreased also, hence the inability of the system to redistribute load from the soil to the reinforcement.

The results indicate that some reductions in the acceptable Factor of Safety may be considered on reinforcement pullout, if high Factors of Safety are calculated for the soil and if the ultimate pullout resistance of the reinforcements is known with confidence. However, similar reductions would not be available for the acceptable Factor of Safety on soil shearing.

6. CONCLUSIONS

The modified displacement method which has been developed offers the advantages of being able to model the influence of relative stiffness on the sharing of load between the soil and the reinforcement, and the progressive development of displacement as construction proceeds.

For the relatively steep potential failure surfaces encountered in reinforced earth systems, it appears that it is more appropriate to calculate normal stresses for use in the modified displacement method based on the Bishop assumptions rather than the Fellenius assumptions.

From the limited comparisons between results for the modified displacement method and more rigorous elasto-plastic finite difference solutions, it is suggested that the displacement method may be more appropriate where reinforcement is relatively flexible in tension.

Design methods for reinforced soil systems which are based on limit equilibrium concepts commonly assume that the ultimate strength of all elements is equally developed, and a global Factor of Safety is calculated. Intuitively, it would seem that redistribution of load would be possible in the cases of reinforcement pullout and full development of soil shear resistance, but that redistribution would not be possible in the case of reinforcement rupture. The ability of the system to redistribute load has significance for the choice of appropriate Factors of Safety for the various components.

Experimental evidence indicates that redistribution of load is not possible in the case of reinforcement rupture. Limited results from finite difference analyses indicate that redistribution is possible in the case of reinforcement pullout, but not in the case of full development of soil shear resistance.

The work has illustrated the potential benefits of the proposed modified displacement method in developing an insight into the significance of load sharing between the two components of reinforced earth systems, and the Factors of Safety for the individual components. Further work is to be directed towards correlation with the results of field and laboratory studies.

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