Determination of Retaining Wall Stability using the Finite Element Method

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SUMMARY Conventional methods of stability analysis for retaining walls are compared with two approaches based on finite element analyses, namely the modified conventional analysis and the recently developed Nodal Displacement Method (NDM). The NDM has considerable potential as it is able to model construction sequences and gives due attention to soil and wall displacements as well as stresses. The physical meaning of safety factor is discussed and the NDM shown to be capable of giving a range of values, depending upon which problem variables are considered to be significant.

1. INTRODUCTION

The safety of gravity and semi-gravity retaining walls is conventionally assessed by limit equilibrium methods in which a failure mechanism is postulated and the restoring and disturbing forces or moments are compared. Many of these analytical methods are unable to take into consideration some of the significant variables likely to affect the stability of the retaining wall. These include the deformation properties of the backfill and foundation subsoil, the stiffness of the retaining wall, the soil-structure interaction and the associated construction sequence.

Application of numerical analyses such as the finite element method permits the calculation of stresses and deformations in walls and retained soil, with reasonably appropriate modelling of most major variables. However, the conversion of stresses and displacements into a meaningful measure of safety against wall instability is not straightforward and the use of stresses from an F.E. analysis in conventional calculations of safety factors against overturning and sliding is highly questionable, as these stresses will not necessarily be valid for a wall on the point of failure.

An alternative method, the Nodal Displacement Method, is currently under development and has shown considerable potential for calculation of the stability of slopes, foundations and retaining structures, in addition to giving valuable information on the deformations of these structures under working conditions. The method is described briefly in this paper and some applications to gravity and cantilever retaining walls discussed.

2. COMPUTER PROGRAM

The program used in this research was written by Goh (1984) and uses an elastic-ideally plastic formulation with either fully associated or non-associated flow rule. The mesh consists of either four or eight noded isoparametric quadrilaterals, with four or six noded slip elements at all soil/concrete interfaces. Incremental excavation and filling may be modelled so that realistic construction sequences may be simulated, e.g. excavation to foundation level, placement of concrete, backfill placing in compacted layers and subsequent rise in ground water level.

3. CONVENTIONAL ANALYSES

In conventional stability analyses it is assumed that fully active conditions have been mobilised behind a wall at the point of failure. Safety factors against sliding and overturning may then be calculated as indicated in Fig. 1 for simplified gravity and cantilever walls.

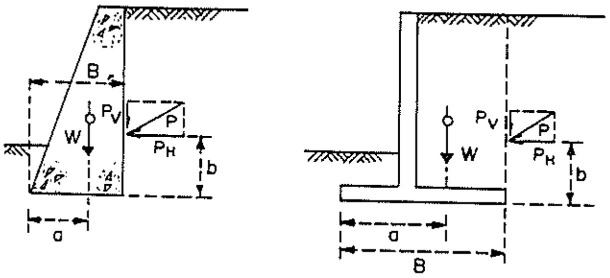


Figure 1 Forces acting on a retaining wall

$$F_{overturning} = \frac{W.a + P_v.b}{P_H.b} \qquad (1)$$

$$F_{sliding} = \frac{c_B.B + (W + P_v) \tan \delta_B}{P_H} \qquad (2)$$

where P_H and P_v, the components of active force, P_A, may be calculated by approaches such as given in CECP 2 (1951). The method pays no attention to soil stiffness and deformations are not calculated but assumed to be constrained to acceptable values by the choice of appropriate design values of safety factor.

4. MODIFIED CONVENTIONAL ANALYSES

If an F.E. analysis of a wall has been carried out, the stresses acting on the real or virtual back of a wall may be used to derive values of P_v and P_H for insertion in (1) and (2). These stresses are estimates of the actual values on the wall for its in-service condition and, for the active case, may be significantly higher than for fully mobilised active conditions. Their use in such a modified conventional analysis would therefore lead to lower values of safety factor than the simple conventional method. The F.E. stresses would of course be applicable to wall structural design and estimates of wall and soil displacements but, for stability analyses, at best the calculated safety factors could be looked upon only as indications of the likelihood of the initiation of large scale movements. Such safety factors could be used in design, but adopted values would need to be calibrated against experience.

5. NODAL DISPLACEMENT METHOD

Many of the problems and assumptions of the conventional and modified conventional methods may be avoided by using the Nodal Displacement Method (NDM). In the definition of safety factor used in the majority of limit equilibrium analyses

$$\tau_f = \frac{s}{F} = \frac{1}{F}(c + \sigma \tan\phi) \qquad (3)$$

i.e. the safety factor is a factor on soil strength and implies that, if the field strength should reduce to $\frac{1}{F}$ of its current value, failure would be imminent and large displacements would occur.

In the NDM analysis a series of F.E. calculations of mesh nodal displacements is carried out, while incrementally reducing - or increasing - strength parameters c and ϕ through application of a multiplication factor, N ($\gtrless 1$). The safety factor is then given by $\frac{1}{N}$ for the situation where displacements at critical nodes in the F.E. mesh show large increases for small changes in N, i.e. a failure mechanism is about to develop. This principle is illustrated in Fig. 2.

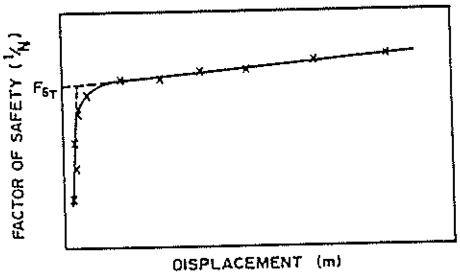
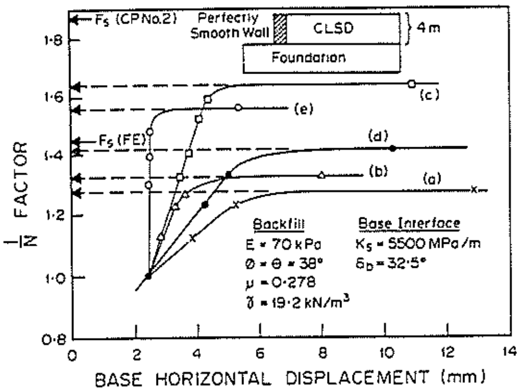


Figure 2 Principle of Nodal Displacement Method

The method possesses a number of obvious advantages - the failure mechanism need not be specified (and is indeed determined by the analysis), soil stress-strain properties are correctly allowed for, critical parameters may be pinpointed by varying them independently and in combination and no assumptions are required concerning changes in soil stresses from the equilibrium to the failure state.

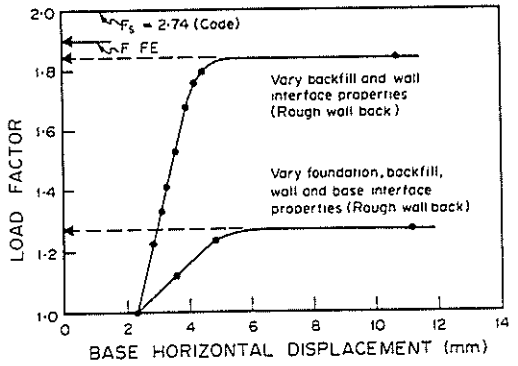
On the debit side considerable computer time may be needed, though with experience this can be minimised, the analysis is carried on to large strains which could limit the accuracy at large displacements and the final result depends on correct choice of critical node. These are not severe criticisms as the critical or turn-over point in the plot normally occurs at relatively small strains and can be clearly defined by simple geometrical constructions and it is usually obvious where excessive deformations will first become apparent, to guide in the selection of critical nodes. It is advisable to prepare nodal displacement plots for a number of presumed significant nodes and it will usually be found that the derived factors of safety fall within a narrow range.

Applications of the NDM to a range of problems have been given by Tan and Donald (1985), Donald et al (1985), Donald and Giam (1988) and Goh (1990).



vary properties of
(a) backfill, foundation, base (d) foundation, base
(b) backfill, base (e) base
(c) backfill

(a) smooth wall



(b) rough wall

Figure 3 NDM plots - gravity retaining wall

6. EXAMPLES

6.1 Gravity Wall

Results for a 4 m high, 1.4 m wide concrete gravity wall with a slightly clayey sand (CLSD) backfill and foundation are shown in Fig. 3.

Both smooth and rough wall/soil interfaces were considered and the design parameters are given in the Figure. The effective stress cohesion for the clayey sand, based on triaxial tests, was small enough to be ignored. The results for a smooth wall back are given in Fig. 3 (a) and for a rough wall back in Fig. 3 (b) and a summary is included in Table I. This Table also contains values of safety factor from conventional and modified conventional sliding and overturning analyses.

For the smooth wall the safety factors cover a wide range from 1.27 to 1.87, the latter being for a conventional sliding analysis. As expected, use of equilibrium F.E. stresses in a sliding analysis gives a lower value of F = 1.45. The NDM values range from 1.27 to 1.64, depending on which parameters are varied - the choice being from ϕ(backfill), ϕ(foundation) and δ(base). The unlikely separation of base interface from foundation soil behaviour has limited the combinations of interest to the five listed in Fig. 3.

TABLE I

COMPARISON OF RESULTS
(INCREMENTAL CONSTRUCTION ONLY)

SAFETY/LOAD FACTOR				
Conventional		Modified Conventional		Nodal Displacement
Sliding	Overturning	Sliding	Overturning	
CLSD BACKFILL AND FOUNDATION				
Smooth Gravity Wall (4 m ht.)				
1.87	1.54	1.45	1.54	(a) 1.27 (b) 1.33 (c) 1.64 (d) 1.41 (e) 1.56
Rough Gravity Wall (4 m ht.)				
2.74	2.55	1.90	2.29	(e) 1.59 (f) 1.27 (g) 1.84
Smooth Cantilever Wall (8 m ht.)				
2.07 (Rankine)	3.90	1.68	3.42	(a) 1.55 (c) 1.60 (e) 2.08
SDCL BACKFILL, CLSD FOUNDATION				
Smooth Cantilever Wall (8 m ht.)				
2.86 (Rankine)	7.83	1.39	3.04	(a) 1.07 (c) 1.25 (e) 1.20

Note: (a) = vary backfill, foundation and base interface prop. (b) = vary backfill and base interface prop.
(c) = vary backfill prop. only (d) = vary foundation and base interface prop.
(e) = vary base interface prop. only (f) = vary backfill, foundation, wall interface and base interface prop.
(g) = vary backfill and wall interface prop.

The conventional sliding factor equation may be rewritten as either

$$P_H = \frac{c_B}{F} . B + (W + P_v) \frac{\tan \delta_B}{F} \qquad (4)$$

or

$$F.P_H = c_B B + (W + P_v) \tan \delta_B \qquad (5)$$

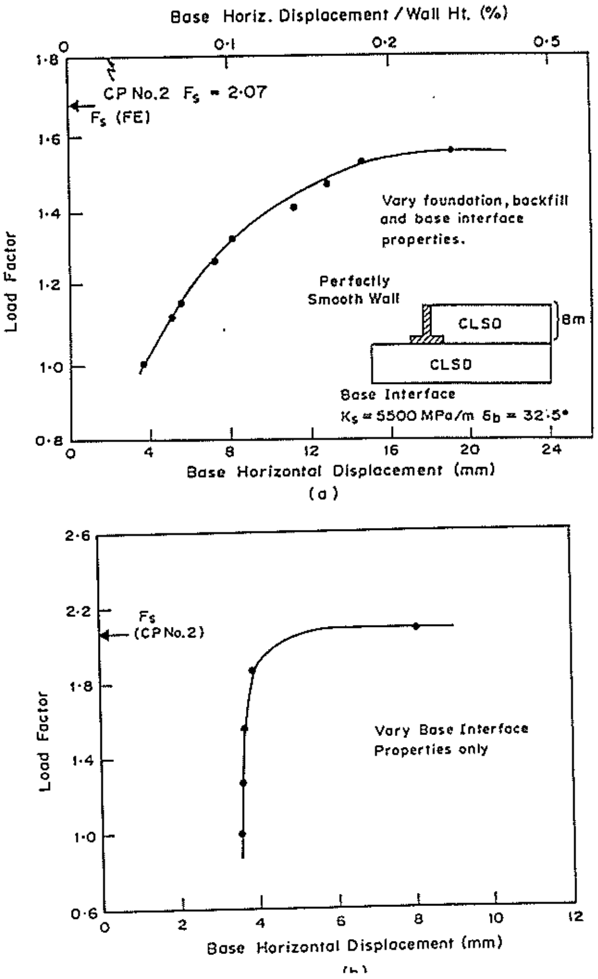
(4) may be interpreted as applying the safety factor only to the base interface properties, ϕ (backfill) and ϕ (foundation) being presumed reliably known, while (5) implies that only ϕ (backfill) is not known with confidence - i.e. a decrease in backfill tan ϕ to $\frac{1}{F}$. tan ϕ leads, approximately, to an increase of lateral thrust to $F.P_H$.

Neither of these interpretations necessarily represents real situations adequately, but it should be noted that cases (e) and (c) in Fig. 3 are closest to the above interpretations, and the value from (c) of F = 1.64 is not greatly different from the conventional F = 1.87. The value from (e) of F = 1.56 is very close to the conventional overturning value of F = 1.54 but it is worth emphasising that the NDM analysis does not assume a specific failure mechanism but calculates the wall displacements, which include both translational and rotational components. The lowest value of F = 1.27 arises from allowing ϕ (backfill), ϕ (foundation) and δ(base) all to vary by equal proportions, implying that the design values for all three could be equally, unconservatively in error simultaneously. This seems an unlikely occurrence and judgement would have to be exercised when deciding which of the many safety factors is most relevant.

For a rough wall back with δ = 29° the safety factors are generally increased (Table I), though the lowest NDM value remains at 1.27, for simultaneous variation of all four significant parameters. Varying only the backfill ϕ and wall back δ gives F = 1.84, which is still well below the conventional (Code) values for sliding and overturning, though close to the modified conventional value of 1.90. The difference between 1.27 (f) and 1.84 (g) highlights the importance of base interface properties for stability, a conclusion which is reinforced by the results for the smooth wall also. However, other analyses for an increased foundation stiffness (modular ratio 5 : 1) showed that the base interface influence decreases with increasing modular ratio.

6.2 Cantilever wall

Results for an eight meter high cantilever wall are shown in Fig. 4 and Table I for a clayey sand (CLSD) foundation and either a clayey sand or a sandy clay (SDCL) backfill. The SDCL has properties c = 16 kPa, ϕ= 28° while the CLSD has the same properties as in the previous example. Varying all properties together gives the lowest NDM value of F = 1.55, while varying only the base interface properties gives F = 2.08, very close to the conventional sliding factor of F = 2.07. With the SDCL backfill and the critical $\frac{1}{N}$ value of 1.25, for variation of backfill properties only, lateral pressures on the virtual wall back were still significantly higher than fully mobilised active values and $\frac{1}{N}$ = 1.46 , with accompanying lateral movements of 33 mm, was required for full active state



(a)

(b)

Figure 4 NDM plots - cantilever wall

mobilisation. This indicates that the conventional assumption that lateral pressures will reduce to active values just prior to failure at small displacements may not always be valid. It is also noteworthy that all three NDM values for the SDCL backfill fall in a narrow range, while the conventional analyses give much higher safety factors.

7 DISCUSSION

From the preceding results it may be seen that the concept of safety factor in retaining wall design is a rather imprecise one, unless the conditions and restrictions under which the safety factor has been derived are clearly defined. In some cases a physical meaning may be attached to a particular definition or method of analysis, as was done with (4) and (5), yet it is not immediately obvious what is the correct definition for a particular situation. The Nodal Displacement Method seems to have several advantages over conventional methods, in particular its avoidance of the need to postulate a failure mechanism and its ability to separate the influences of major variables, either individually or in various groupings. The designer may then use knowledge of which parameters are likely to be the least reliable when making a choice of safety factor definition. Application of the NDM obviously needs additional research, particularly in allowing different degrees of variation for various parameters - something akin to a limit state partial load factors approach. Guidelines will also have to be drawn up for acceptable safety factors for the various definitions, as some of the differences in Table I are alarmingly large. Problems involving water behind the wall have not yet been investigated and cohesive backfills obviously require additional investigation.

Based on experience to date it is recommended that the lowest NDM safety factor be calculated, varying all parameters simultaneously, and a value of between 1.25 and 1.50 be accepted for cohesionless soils. For walls where stability is not a problem but deformations are critical, the NDM provides all data required for displacement-controlled design.

8 CONCLUSIONS

An alternative method of retaining wall analysis has been presented, using the Nodal Displacement Method applied to finite element analyses. As the method includes calculations of displacements as well as pressures it provides a more realistic modelling than conventional or modified-conventional limit equilibrium solutions. The method is promising, but further work is required before it may be used in routine practice with confidence.

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