Assessment of Applicability of Four Empirical Strength Criteria for Intact Coal

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SUMMARY The applicability of four empirical strength criteria, namely, Bieniawski, Hoek and Brown, Johnston and Ramamurthy has been assessed for intact coal by using the published triaxial test data. For nonlinear regression analysis, the PC package FFIT has been selected. The best fit has been found by minimising the sum of the squares of the relative errors between the function and the data points. Although unique values of the constants in all criteria have been determined with good coefficients of determination ranging from 0.83 to 0.86 for overall data, a wide variation has been noticed in the values of the constants when individual data sets have been analysed. A strong negative correlation has been observed between B in Bieniawski's criterion and uniaxial compressive strength, σ_c (coefficient of determination, $r^2 = 0.86$), m in Hoek and Brown's criterion and σ_c ($r^2 = 0.71$) and B in Ramamurthy's criterion and σ_c ($r^2 = 0.88$). The following empirical strength equation best describes the triaxial strength test data analysed for intact coal:-

$$\frac{\sigma_1}{\sigma_c} = 1 + B\,(\frac{\sigma_3}{\sigma_c})^{0.6}$$

where σ_1 = axial compressive stress at failure, MPa;
σ_3 = confining pressure, MPa;
σ_c = uniaxial compressive strength, MPa; and
$B = 7.2075 - 1.7263\,(\log \sigma_c)^2$.

An estimate of the axial compressive stress at failure at any confining pressure can be made if only the uniaxial compressive strength of the intact coal is known.

1. INTRODUCTION

The estimation of the triaxial strength of coal is essential in the design of pillars in underground coal mines. The size of pillars determined should allow maximum coal recovery while maintaining overall stability.

The theoretical triaxial strength criteria based on the actual mechanism of fracture do not fit the experimental results properly and to overcome this problem, many empirical criteria have been formulated for rocks. The strength criteria can be written in terms of either

(1) principal stresses, σ_1 and σ_3 at failure such as

$$\sigma_1 = \sigma_c + a\,\sigma_3^{\,b} \tag{1}$$

or

(2) normalised principal stresses at failure obtained by dividing the principal stresses, σ_1 and σ_3 at failure by the relevant uniaxial compressive strength, σ_c such as

$$\frac{\sigma_1}{\sigma_c} = 1 + B\,(\frac{\sigma_3}{\sigma_c})^{\alpha} \tag{2}$$

In equations (1) and (2), a, b, B and α are constants.

Equation (1) was proposed by Murrell (1965) whereas equation (2) was proposed by Bieniawski (1974a). Equation (2) permits the direct comparison of a number of tests on the same plot. A typical relationship between σ_1 and σ_3 or $\frac{\sigma_1}{\sigma_c}$ and $\frac{\sigma_3}{\sigma_c}$ at failure for coals is a nonlinear one.

Four empirical strength criteria proposed by Bieniawski (1974a), Hoek and Brown (1980a, b), Johnston (1985) and Ramamurthy (1986) have been selected to assess their applicability for intact coal by using the published triaxial test data. These authors have not analysed data for coal and the appropriate values for constants are not available for the same.

2. SUMMARY OF THE FOUR CRITERIA

2.1 Bieniawski's Criterion

The criterion proposed by Bieniawski (1974a) is:-

$$\frac{\sigma_1}{\sigma_c} = 1 + B\,(\frac{\sigma_3}{\sigma_c})^{\alpha} \tag{3}$$

where B = 3.0 for siltstone and mudstone;
= 4.0 for sandstone;
= 4.5 for quartzite; and
= 5.0 for norite and
α = 0.75 for all rock types.

Yudhbir, Lemanza and Prinzl (1983) modified the Bieniawski's criterion and proposed the following equation:-

$$\frac{\sigma_1}{\sigma_c} = A + B\left(\frac{\sigma_3}{\sigma_c}\right)^{\alpha} \quad (4)$$

where A = 1 for intact rocks;
= $0.0176\,Q^{0.65}$
where Q = NGI rating for rock mass.
B = 2.0 for tuff, shale and limestone;
= 3.0 for siltstone and mudstone;
= 4.0 for sandstone and quartzite; and
= 5.0 for norite and granite and
α = 0.65 for all rock types.

The details of NGI rock mass classification are given by Barton, Lien and Lunde (1974).

2.2 Hoek and Brown's Criterion

The criterion proposed by Hoek and Brown (1980a, b) is:-

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(s + m\frac{\sigma_3}{\sigma_c}\right)^{0.5} \quad (5)$$

where m and s are material constants. σ_c is the uniaxial compressive strength of the intact rock i.e. of a laboratory size specimen (say a 50 mm diameter by 100 mm long core) which is free from discontinuities such as joints or bedding planes. For intact rocks, s = 1 and m depends on rock type as follows:-

7 for dolomite, limestone and marble;
10 for mudstone, siltstone, shale and slate (normal to cleavage);
15 for sandstone and quartzite;
17 for andesite, dolerite, diabase and rhyolite; and
25 for amphibolite, gabbro, gneiss, granite, norite and quartz-diorite.

They also suggested approximate relationships between the constants m and s and the rock mass ratings developed by Bieniawski (1974b) and Barton et al. (1974) and the latest recommendations were given separately for disturbed and undisturbed rock masses (Hoek and Brown, 1988).

Substitution of σ_1 = 0 in equation (5), and solution of the resulting quadratic equation for σ_3, gives the uniaxial tensile strength of a rock, σ_t as:-

$$\sigma_3 = \sigma_t = \frac{1}{2}\sigma_c\left[m - (m^2 + 4)^{0.5}\right] \quad (6)$$

2.3 Johnston's Criterion

The criterion proposed by Johnston (1985) is:-

$$\frac{\sigma_1}{\sigma_c} = \left[\left(\frac{M}{B}\right)\left(\frac{\sigma_3}{\sigma_c}\right) + 1\right]^{B} \quad (7)$$

where M and B are constants. These constants depend upon σ_c as follows:-

$$M = 2.065 + k\,(\log \sigma_c)^2$$

$$B = 1 - 0.0172\,(\log \sigma_c)^2$$

where k = 0.170 for dolomite, limestone and marble;
= 0.231 for mudstone, shale, slate and clay;
= 0.270 for sandstone and quartzite;
= 0.659 for amphibolite, gabbro, gneiss, granite, norite and grano-diorite; and
= 0.276 for all rock types combined (overall)
and
σ_c = uniaxial compressive strength, kPa.

When $\sigma_1 = 0$, σ_3 becomes tensile strength, σ_t and

$$\frac{M}{B} = -\frac{\sigma_c}{\sigma_t}$$

This is the only criterion which suggests that the values of the constants are not only dependent on rock type but also on uniaxial compressive strength of the rock.

The equation (7) has also been proposed by Sheorey, Biswas and Choubey (1989).

2.4 Ramamurthy's Criterion

The criterion proposed by Ramamurthy (1986) is:-

$$\frac{\sigma_1}{\sigma_3} = 1 + B\left(\frac{\sigma_c}{\sigma_3}\right)^{\alpha} \quad (8)$$

where B and α are constants.

The constant α was found to be between 0.75 and 0.85 and an average of value 0.8 was suggested for all rock types. He proposed the following values for B:-

1.8 for siltstone, clay, tuff and loess;
2.2 for shale, slate, mudstone, claystone and sandstone;
2.4 for limestone, anhydrite and rocksalt;
2.6 for quartzite, andesite, diorite, norite, liprite and basalt;
2.8 for marble and dolomite; and
3.0 for granite and charnockite.

This criterion is only applicable for all values of $\sigma_3 > 0$.

3. REGRESSION ANALYSIS OF TRIAXIAL TEST DATA

Regression models can be classified as:-

1. Linear - Simple
   - Multiple
2. Nonlinear.

In simple linear regression model, the term "simple" implies a single regressor variable, x, and the term "linear" implies linear in x.

$$y = \beta_0 + \beta_1 x + \varepsilon \quad (9)$$

where y = measured response variable;
β_0 = intercept;
β_1 = slope; and
ε = model error.

In multiple linear regression model, the term "multiple" implies multiple (more than one) regressor variables, x_1, x_2, ..., x_n, and the term "linear" implies linear in regressor variables, $x_1, x_2, \ldots\ldots, x_n$.

$$y = \beta_0 + \beta_1 x_1 + \ldots\ldots\ldots\ldots + \beta_n x_n + \varepsilon \quad (10)$$

where β_0, β_1, β_2, ..., β_n are the parameters that specify the nature of the relationship.

The regressor variables or the response variable are not necessarily in natural units. They may be transformations (log, square root, etc).

In a nonlinear regression model, at least one of the parameters enters the model in a nonlinear way.

$$y = a + b^{-cx} + \varepsilon \quad (11)$$

3.1 Estimation of Regression Coefficients

To find the best-fitting curve to match the plotted data, the sum of the squares of the errors between the data and the function evaluated with the parameter values is minimised. Although this procedure is widely used, relative errors can be quite high for low values of regressor variable. To overcome this difficulty, the sum of the squares of the relative errors can be minimised.

3.2 Example

The following triaxial test data refers to Uchitdih coal (Das and Sheorey, 1986). The value of m in Hoek and Brown's criterion is to be determined for this data assuming $s = 1$.

Minor principal stress, MPa	Major principal stress, MPa
0	35.3
7	76.8
15	99.0
25	124.3
35	147.5
45	161.5
60	169.3
70	187.4

Hoek and Brown's criterion is nonlinear. Unless appropriate software is available for nonlinear regression, it is not feasible to do the analysis by the use of a calculator. Hoek and Brown (1980a) recommended the following transformation to make it a simple linear model to determine the appropriate values for the parameters by the use of a calculator:-

$$(\sigma_1 - \sigma_3)^2 = \sigma_c^2 + m\,\sigma_c\,\sigma_3 \quad (12)$$

Equation (12) is of the form

$$y = a + b\,x \quad (13)$$

where $y = (\sigma_1 - \sigma_3)^2$;
$x = \sigma_3$;
$a = \sigma_c^2$; and
$b = m\,\sigma_c$.

x and y are known and a and b can be estimated by simple linear regression analysis by the use of a calculator.

It must be mentioned here that linearisation of a nonlinear model does not produce an equivalent model. A full discussion on regression analysis is given by Myers (1990).

By simple linear regression analysis, σ_c has been found to be 64.52 MPa (test value - 35.3 MPa) and m to be 2.51. Sheorey et al. (1989) also reported that σ_c estimated from simple linear regression after transformation has been significantly higher than the test value.

By nonlinear regression analysis through the minimisation of the sum of the squares of the errors, σ_c has been found to be 51.94 MPa and m to be 3.92. For this analysis, the PC package for statistical analysis, SPSS/PC+ has been used. "Eureka: The Solver" can also be used but limited to about 20 data points.

By nonlinear regression analysis through the minimisation of the sum of the squares of the relative errors, σ_c has been found to be 37.55 MPa and m to be 7.24. For this analysis, the PC package FFIT has been used (Hoyer, 1989).

The results of analysis of relative errors (in percentage) from the three methods of regression analysis are given in TABLE I. Figure 1 gives the three curves found from the three methods of regression analysis. From this example, it has been concluded that FFIT program serves well for nonlinear regression analysis of triaxial test data and it has been used for all analyses.

TABLE I

RESULTS OF ANALYSIS OF RELATIVE ERRORS IN % FROM THREE METHODS OF REGRESSION ANALYSIS

	Simple linear regression analysis after transformation	Nonlinear regression analysis using SPSS/PC+	Nonlinear regression analysis using FFIT
Mean	10.504	10.868	9.233
Std dev	14.244	8.724	5.647
Minimum	2.894	4.924	0.191
Maximum	45.287	32.037	18.963
Range	42.392	27.113	18.772

4. DATA FOR ANALYSIS

In order to examine the applicability of the four empirical criteria selected, the triaxial strength data for coals from two publications (Hobbs, 1964 and Das and Sheorey, 1986) have been used. The uniaxial compressive strength ranges from 2.2 to 51.4 MPa. Twenty six sets of data are available for analysis.

In these tests, the specimens used were of two sizes, 5.1 cm long by 2.5 cm diameter (Hobbs, 1964) and 4.2 cm long by 2.8 cm diameter (Das and Sheorey, 1986). For practical purposes, both sizes can be considered to be the same. For analysis, these coal specimens have been considered to be intact.

Coal, in general, contains three dominant planes of weakness, the bedding planes and the two families of cleat planes, the three families being orthogonal to one another. Hobbs (1964) reported the results which were similar for two orientations.

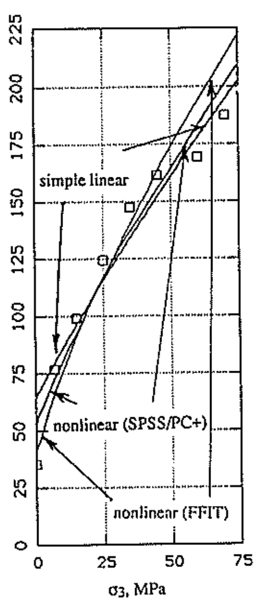

Figure 1 Plot of triaxial test data for Uchitdih coal and regression lines from 3 different regression models.

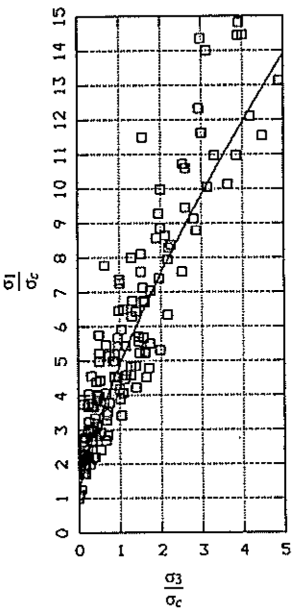

Figure 2 Plot of $\frac{\sigma_1}{\sigma_c}$ versus $\frac{\sigma_3}{\sigma_c}$ for all data along with regression line according to Bieniawski's criterion.

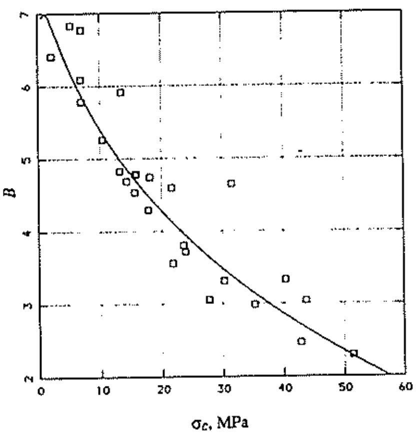

Figure 3 Plot of B versus σ_c for Bieniawski's criterion.

Although laboratory size coal specimens do contain planes of weakness, they can be considered to be intact. For example, if the criterion by Yudhbir et al. (1983) is used to describe coal strength behaviour, parameter A has to be near 1. From the analysis of coal test data, A has been found to be between 0.9959 and 1.0152.

Two of the criteria selected, namely, Hoek and Brown and Johnston, have been suggested to describe results in the "brittle" range only i.e. $\frac{\sigma_1}{\sigma_3} > 3.4$ (Mogi, 1966). However, Das and Sheorey (1986) concluded that the triaxial behaviour of coal did not become significantly ductile above the brittle limit. Hence, all the triaxial test data have been included in the analysis.

5. ANALYSIS

5.1 Bieniawski's Criterion

A plot of $\frac{\sigma_1}{\sigma_c}$ versus $\frac{\sigma_3}{\sigma_c}$ for all data along with the regression curve is shown in Figure 2. The values for B and α have been determined to be 3.90 and 0.7445 respectively. These values are in the range suggested by Bieniawski (1974a). The coefficient of determination has been found to be 0.857. The analysis of relative errors has indicated that 18.2% of the calculated values fall within 5% relative error, 27.9% within 10% and 40.3% within 15%. The details of this analysis are as follows:-

Mean - 22.891
Standard deviation - 19.767
Minimum - 0.181
Maximum - 103.112
Range - 102. 931

Analysis of individual data sets has given a range of values for B from 2.2910 to 6.8326 and for α from 0.4117 to 0.8687.

Analysis of these values in conjunction with the σ_c has indicated that there is a significant correlation between B and σ_c (Figure 3) and practically no correlation between α and σ_c. The relationship between B and σ_c is as follows:-

$$B = 7.0266 - 1.6228\ (log\ \sigma_c)^2 \qquad (14)$$

The coefficient of determination for this regression has been found to be 0.864.

Using the above equation for calculating B and the average value of 0.6 for α, axial stresses at failure, σ_1 and relative errors have been calculated and analysis of them has given that 39.6% of the calculated axial stresses fall within 5% relative error, 63.6% within 10% and 83.1% within 15%. The details of this analysis are as follows:-

Mean - 8.890
Standard deviation - 8.111
Minimum - 0.126
Maximum - 42.418
Range - 42.292

Taking α as 0.6, the values for parameter B have been recalculated for all the individual data sets. From this analysis, B has been found to be between 2.0663 and 7.7150 and the relationship between B and σ_c has been found to be as follows:-

$$B = 7.2075 - 1.7263\ (\log \sigma_c)^2 \qquad (15)$$

The coefficient of determination for this regression has been found to be 0.910.

Using equation (15) for calculating B and 0.6 for α, axial stresses at failure, σ_1 and relative errors have been calculated and analysis of them has given that 52.6% of the calculated axial stresses fall within 5% relative error, 82.5% within 10% and 91.6% within 15%. The details of this analysis are as follows:-

Mean - 6.250
Standard deviation - 6.123
Minimum - 0.003
Maximum - 31.715
Range - 31.712

5.2 Hoek and Brown's Criterion

The value for m has been determined to be 14.55. This value is in the range suggested by Hoek and Brown (1980a). The coefficient of determination has been found to be 0.847. Analysis of individual data sets has given a range of values from 5.3795 to 50.190 for m. Analysis of these values along with the σ_c has indicated that there is a significant correlation between them (Figure 4). The relationship between them has been found to be as follows:-

$$m = 35.512 - 10.764\ (\log \sigma_c)^2 \qquad (16)$$

The coefficient of determination for this regression has been found to be 0.7000.

Using equation (16) for calculating m, axial stresses at failure, σ_1 and relative errors have been calculated and analysis of them has given that 27.9% of the calculated axial stresses fall within 5% relative error, 51.3% within 10% and 69.5% within 15%. The details of this analysis are as follows:-

Mean - 13.427
Standard deviation - 12.381
Minimum - 0.142
Maximum - 66.562
Range - 66.420

5.3 Johnston's Criterion

The values for M and B have been determined to be 6.8889 and 0.6562 respectively. The coefficient of determination has been found to be 0.826. Analysis of individual data sets has given a range of values for M from 5.26 to 67.48 and for B from 0.33 to 0.78. However, no good correlation has been found either between M and σ_c or between B and σ_c as suggested by Johnston (1985).

5.4 Ramamurthy's Criterion

The values for B and α have been determined to be 3.955 and 0.6256 respectively. These values are in the range suggested by Ramamurthy (1986). The coefficient of determination has been found to be 0.868.

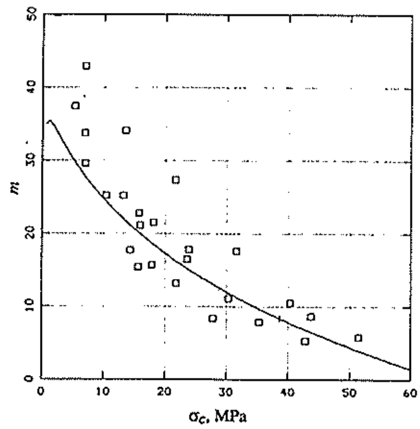

Figure 4 Plot of m versus σ_c for Hoek and Brown's criterion.

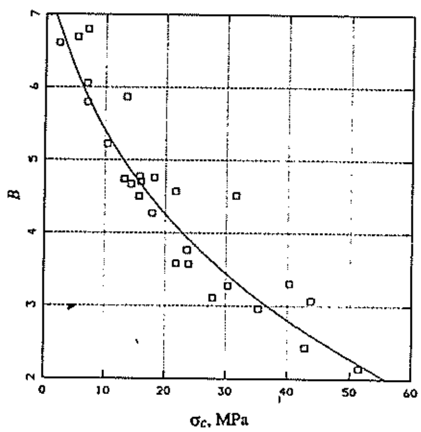

Figure 5 Plot of B versus σ_c for Ramamurthy's criterion.

Analysis of individual data sets has given a range of values for B from 2.15 to 6.80 and for α from 0.47 to 0.82. Analysis of these values in conjunction with the σ_c has indicated that there is a significant correlation between B and σ_c (Figure 5) and practically no correlation between α and σ_c. The relationship between B and σ_c has been found to be as follows:-

$$B = 7.0601 - 1.6579\ (\log \sigma_c)^2 \qquad (17)$$

The coefficient of determination for this regression has been found to be 0.878.

Using equation (17) for calculating B and the average value of 0.667 for α, axial stresses at failure, σ_1 and relative errors have been calculated and analysis of them has given that 39.6% of the calculated axial stresses fall within 5% relative error, 62.3% within 10% and 76.6% within 15%. The details of this analysis are as follows:-

Mean - 11.214
Standard deviation - 14.319
Minimum - 0.132
Maximum - 122.222
Range - 122.090

Taking α as 0.67, the values for parameter B have been recalculated for all the individual data sets. From this analysis, B has been found to be between 2.5805 and 9.1073 and the relationship between B and σ_c has been found to be as follows:-

$$B = 7.1551 - 1.6311\ (log\ \sigma_c)^2 \quad (18)$$

The coefficient of determination for this regression has been found to be 0.839.

Using equation (18) for calculating B and 0.67 for α, axial stresses at failure, σ_1 and relative errors have been calculated and analysis of them has given that 36.4% of the calculated axial stresses fall within 5% relative error, 63.0% within 10% and 81.2% within 15%. The details of this analysis are as follows:-

Mean - 10.458
Standard deviation - 12.130
Minimum - 0
Maximum - 103. 390
Range - 103.390

6. CONCLUSIONS

The selection of appropriate software for nonlinear regression analysis is crucial in the results obtained. The PC package FFIT has been found to be most appropriate for nonlinear regression analysis of triaxial test data. Out of the four criteria selected, Bieniawski's criterion with variable B (equation (15)) and a constant α of 0.6 appears to be the most appropriate one for coal. Ramamurthy's criterion with variable B (equation (18)) and a constant α of 0.67 comes next. There is very good correlation between parameter B of Bieniawski's criterion and parameter B of Ramamurthy's criterion (Figure 6). The relationship is as follows:-

$$B_B = 0.1244 + B_R \quad (19)$$

where B_B = parameter B in Bieniawski's criterion and
B_R = parameter B in Ramamurthy's criterion.

The index of determination for this regression has been found to be 0.997.

The value of B depends upon σ_c which in turn depends upon material and structural characteristics and experimental conditions. Since there is not much difference in α, B can be taken approximately as a measure indicating the effect of σ_3 on σ_1. The lower the uniaxial compressive strength, the higher is the pressure sensitivity of triaxial strength at low confining pressure.

An estimate of the triaxial strength of coal can be made by means of the Bieniawski's criterion but with a variable B dependent upon σ_c (equation (15)) and a constant α of 0.6, that has an accuracy sufficient for practical purposes. The only parameter required for this criterion is the uniaxial compressive strength which can be determined simply.

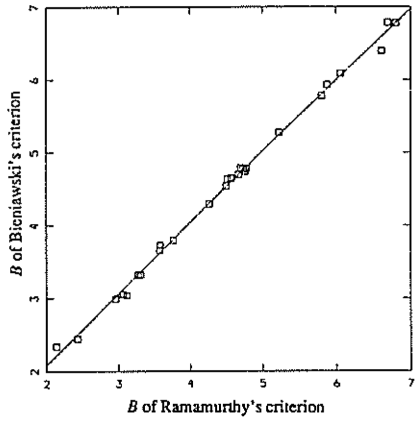

Figure 6 Plot of B of Bieniawski's criterion verus B of Ramamurthy's criterion

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