

General Report: Analytical and Probabilistic Methods

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SUMMARY This General Report reviews the papers allocated to the session on Analytical and Probabilistic Methods. It also gives a brief review of the other important developments in these areas that have taken place recently. The methods of analysis covered include finite element, boundary element and distinct element techniques, stochastic methods, limit states approach, and risk assessment in rock engineering.

1. INTRODUCTION

Computers have now become so prevalent that their use in geomechanics is no longer confined to research laboratories. Many practicing geotechnical engineers are now using them routinely in their daily work and have done so for most of the past decade. The availability of relatively cheap personal computers and work stations has made possible the solution of a host of different tasks in geotechnical engineering, including laboratory data logging, spreadsheet analysis, and even the solution of complicated boundary and initial value problems, that a generation ago would have remained unsolved or at best would have required drastic simplification, idealisation and approximation.

Difficult non-linear and time dependent problems, i.e. problems that are deterministic in nature, can now be solved using numerical methods. Furthermore, sophisticated stochastic analysis can also be performed routinely and cheaply using the computational power of modern computers.

Has this ready access to computational power resulted in improvements in our understanding of our geotechnical problems, greater confidence in our solutions to these practical problems, and more reliable predictions of behaviour? Has the practice of geotechnical engineering been generally enhanced because of the greater levels of sophistication that may now be included in our analytical and probabilistic tools?

One of the objectives of this paper is to address these broad questions by briefly reviewing recent developments in the fields of analytical and probabilistic methods in geotechnics. A second and important aim of the paper is to summarise and review the findings of the twelve papers submitted to this conference that have been grouped together in this category. Where relevant, their part in advancing our knowledge in this area will be stressed.

The areas of analytical and probabilistic techniques are extremely diverse. To report on the many developments would be a lengthy task and one that is inappropriate for this conference. For this reason this paper will deal with only the most recent developments (particularly those taking place over the past two years), and the treatment

will be necessarily selective, reflecting the writer's own interests and familiarity with the literature.

2. PAPERS IN THIS SESSION

The summary of the papers presented in this session will indicate the theoretical concepts that have been discussed and the practical problems to which these theories have been applied.

Twelve papers have been grouped under the general heading of Analytical and Probabilistic Methods. At least four may be described as dealing mainly with deterministic methods of analysis, one describes an expert system, two deal with the application of grey system theory and fuzzy mathematics to geotechnical problems, one describes an entirely new concept for the analytical assessment of risk in rock engineering, and the remainder elaborate on probabilistic techniques. The applications include piles, slopes, footings, dams and sub-surface exploration. This break down into sub-categories is summarised in Table I.

3. FINITE ELEMENT METHODS

The finite element method is now accepted as a valuable analysis tool in geotechnical engineering. However, its use is still confined to research tasks and the more challenging and sophisticated technical problems in the geotechnical profession, and perhaps that is as it should be. The method has the advantage of being able to model the complex geometry and the variety of different materials which occur in real engineering problems, as well as having the ability to incorporate the complex constitutive behaviour of soils and rocks, and being able to model, at least approximately, the actual construction process and loading history.

A review of the literature over the recent past indicates that the solution of large three-dimensional problems has become much more common in geomechanics, e.g. Brown and Shie (1990a, 1990b), Lee and Rowe (1990a, 1990b), Lai and Booker (1991), obviously reflecting the availability of relatively cheap computing power. The economies to be gained by coupling together finite element and boundary element schemes have also been exploited for problems of this nature, e.g. Duddeck (1991), Beer (1983). A number of papers on this theme may be found in the

TABLE I
CLASSIFICATION OF SESSION PAPERS

Authors	Method					Application					
	Deter- ministic	Proba- bilistic	Grey/ Fuzzy	Expert System	New	Slopes	Dams	Foot- ings	Piles	Explo- ration	Seismic
Assadi and Sloan	✓					✓					
Budkowska & Grzesiak	✓							✓			
Chen	✓					✓					
Styles, Mooré & Gupta	✓										✓
Li		✓				✓					
Lo, Li & Lee		✓							✓		
St. George		✓				✓					
Williams & Zou		✓				✓					
Chowdhury, Zhang & Li			✓			?					
Xiao & Carter			✓			✓					
Goh				✓						✓	
Hudson, Sheng & Arnold					✓		✓				

proceedings of the Sixth International Conference on Numerical Methods in Geomechanics (Beer, Booker and Carter, 1991), held in Cairns in May 1991.

Of the papers reviewed in this session, only those presented by Assadi and Sloan (1992), Budkowska and Grzesiak (1992) and Williams and Zou (1992) make use of the finite element technique.

3.1 Deterministic Finite Element Analysis

The paper by Sloan and Assadi describes a relatively new finite element formulation that can be used to obtain rigorous limit solutions for collapse loads in geotechnical problems. This represents a significant advance and presents the profession with a valuable new computational tool. Reliable estimates of collapse loads may now be obtained to problems involving very complicated geometry and boundary conditions. In their paper the authors have described the procedure and presented limit solutions for the collapse of slopes in cohesive-frictional soils, i.e. in materials that obey the Mohr-Coulomb strength criterion and deform plastically according to an associated flow rule. This analysis demonstrates the power and versatility of the finite element technique, and illustrates how it can help to obtain very useful solutions to practical problems. No doubt the limit analysis has many applications in the geotechnical field, as well as in other branches of engineering mechanics (e.g. in the yield line theory of structural analysis).

Presumably the finite element procedure presented in the paper by Assadi and Sloan could also be applied to materials obeying other failure criteria, e.g. the Hoek-Brown strength criterion for intact and jointed rock, as long as a suitable multi-linear approximation to the strength criterion can be found. It is also noted that their method, based on the bound theorems of plasticity, is strictly applicable only to materials deforming with an associated plastic flow rule. For many frictional soils the

associated flow rule overestimates the influence of dilatancy and therefore the volume changes that will occur in the soil as it fails. Perhaps it may be useful if some guidance could be given about the validity and applicability of these solutions to obtaining approximate estimates of collapse loads in materials deforming with a non-associated flow rule, e.g. a frictional soil at critical state deforming without volume change.

A specific question that the authors may wish to address concerns Figure 9 of their paper. This figure shows stability bounds for a 45 degree slope in purely cohesive material. Two extreme points on each bound have been joined by a straight line. An explanation of why linear interpolation between these extremes is valid would be helpful. Furthermore, in Figures 10 and 11 of their paper the upper and lower bounds diverge increasingly as the friction angle increases, and therefore the true collapse load is more difficult to estimate. Why does this occur, and what would be required to bracket more tightly the collapse loads at higher friction angles? Finally, can the same technique be used to analyse the stability of jointed rock masses, where the strength of the discontinuities may have an overriding influence on the strength of the mass?

In the paper by Budkowska and Grzesiak a scheme is presented for the finite element simulation of the time dependent behaviour of a pavement structure containing cracks. The time dependence enters the problem because the soil and pavement courses have been modelled as viscoelastic materials. A relatively simplistic viscoelasticity has been assumed, in which only deviatoric creep occurs, i.e. there is no volumetric creep. Some detail of the mathematical formulation is provided. While the approach for this idealised problem is valid, in the writer's opinion the solution of the problem using Laplace transforms together with the finite layer techniques, e.g. Booker and Small (1985), would have been more economical.

In the paper some significance is given to the role of cracks in the pavement and underlying layers in determining the overall response to surface loading. No detail is given about how the cracks were modelled. As the analysis was also axi-symmetric, presumably so also were the cracks, i.e. they are ring cracks concentric with the centre of the circular loading on the surface. Surely this is an extreme idealisation and one that would rarely be encountered in practice.

In Figures 3 to 10 of this paper the horizontal axis is labelled as 't' for time. It is likely that this axis is mislabelled and should actually be radial distance from the centre of the loaded region. This point requires clarification.

3.2 Probabilistic Finite Element Analysis

The stochastic finite element method can be used to determine the range of responses of soil or rock masses where some of the input parameters are uncertain and have been assigned assumed distributions. This procedure is being used increasingly in geotechnical engineering where much uncertainty accompanies the selection of typical soil parameters and loads, e.g. Drumm et al, 1990.

One of the concerns of the paper by Williams and Zou is the application of the stochastic finite element method to the analysis of the stability of coal mine spoil piles. In particular, the finite element procedure has been used to make a prediction of the stresses in the spoil pile. These predictions have then been used as input to a stability analysis based on the approximate limit equilibrium techniques. Specifically, the finite element results have been used to identify the probable location of the critical slip surface. Because the stress predictions are based on soil properties with specified statistics, the computed stresses also have associated probability distributions.

The use of the finite element method in this way begs certain questions. Because the analysis presented by Williams and Zou allowed for non-linear material behaviour, why was it necessary subsequently to analyse the stability of the spoil pile using limit equilibrium techniques? Could the finite element solutions themselves indicate the degree of risk or even the overall factor of safety for the slope? On the question of the finite element analysis, it is unclear from the paper how the loading was applied to the spoil pile. Was gravity "switched on" in the analysis, or was the appropriate sequence of spoil pile construction modelled. This question may be important, as the final response of the spoil pile will be dependent, to some degree, on the loading sequence, and therefore it may be important to follow this sequence as accurately as possible. The paper by Williams and Zou deals with wider issues than just finite element analysis, and these will be addressed later in this review.

4. BOUNDARY ELEMENT METHODS

The boundary element method has grown in stature during the past decade. It now rivals the finite element method for use in geomechanics. Its major application to date has been in the analysis of underground openings and thus it has received much attention by practitioners in the field of rock mechanics. It also has particular application to the analysis of piles and pile groups, e.g. Chin and Poulos (1991), the dynamic behaviour of foundations, e.g. Alarcon

Cano and Dominguez (1989), and the analysis of excavations in anisotropic, jointed rock, e.g. Carter and Alehossein (1990).

The past five years or so has seen the merger of the finite and boundary element techniques in the one computational procedure. There are now several software packages available commercially for coupled BE-FE analysis in geomechanics. Since this is a topical subject, and one in which there is considerable research effort currently being expended, it is surprising that there are no papers in this session that concern the boundary element methods.

5. DISTINCT ELEMENT METHODS

Research into the development of the distinct element technique and its application in geomechanics has accelerated over the last few years. It is now feasible to solve three dimensional problems with this technique, e.g. Ghaboussi and Barbosa (1990). It is interesting that there are no papers on this topic submitted to this session of the conference as several software packages employing the distinct element methods are now available commercially.

6. EXPERT SYSTEMS

Until recently the majority of computer applications that have been developed in the geotechnical field have been numeric and deterministic. Their main function has been to relieve engineers of the need for carrying out manually repetitive, and sometimes quite sophisticated mathematical calculations.

It has been recognised for some time that much of geotechnical practice involves the application of deductive reasoning, often in the absence of precise calculation. Sometimes important design decisions have to be based on knowledge gained by experience.

With the emergence of expert system technology, geotechnical engineers now face the prospect of capturing this empirical-type knowledge in a suitable knowledge base, from where it may be retrieved by appropriate interrogation procedures.

Recently, the development of expert systems designed specifically for use in geotechnical practice have been described in the literature, e.g. Wong, Poulos and Thorne (1989). In this conference the paper by Goh (1992) describes an expert system for sub-surface exploration. This system has been designed to provide "advice on a sub-surface exploration program for a particular site". Embedded in the knowledge base is information from selected textbooks, codes of practice, design manuals, and information gleaned from interviews with "expert" practitioners.

Several questions arise in relation to expert systems in general and these are relevant to the system described in Goh's paper. Much effort is obviously required to develop this type of design tool. However, the user should also be concerned with establishing the validity of the system's conclusions and recommendations. As far as the writer is aware, there exist no well established procedures for validating particular expert systems. This is an area that requires further exploration.

With regard to the expert system described by Goh for site exploration, it would appear that some information about the soil profile is required *a priori*, before a recommendation can be made by the system about the depth of planned exploration. This feature of the system requires clarification.

7. FUZZY MATHEMATICS AND GREY SYSTEM THEORY

In the last few decades theoretical methods have been developed to deal with uncertainty in real systems. The use of stochastic techniques in geotechnical engineering is now well established. However, there are alternatives to probabilistic methods such as the grey system theory and fuzzy mathematics. Special mathematical theories have been developed to describe systems which are inexact or indistinct, e.g. Zadeh (1965, 1975). Much of the adaptation of these theories to geotechnical problems has taken place recently, e.g. Nguyen (1985), Boissonnade (1986), Xiao and Zhou (1987), Sakurai and Shimizu (1987), and Xiao and Yu (1990). Some of these methods are directly applicable to the issue of risk assessment in geotechnical works.

In this conference, papers by Xiao, Carter and Yu (1992) and Chowdhury, Zhang and Li (1992) fall into this category. Xiao *et al* describe the use of fuzzy mathematics for the description of typical geotechnical parameters and then demonstrate the use of the theory for the prediction of the stability factor for planar sliding of rock slopes. Detail is provided on how to describe fundamental properties as fuzzy numbers, e.g. the strength parameters of a rock mass. This includes a description of how to assign appropriate weights to different measures of the same parameter, according to the perceived levels of reliability that should be associated with each measure. It would appear that the major attraction of this technique is that it is unnecessary to assume that the uncertainty of the parameter must be described by a normal or some other statistical distribution. The use of fuzzy mathematics is akin to the use of linguistic variables to describe the physical system and its stability.

In the paper by Chowdhury, Zhang and Li the use of grey system theory to interpret and then to extrapolate field observations is described. The authors point out that a procedure such as this is useful for updating risk assessment during construction, i.e. as some real data becomes available. Such a process would help planning and redesign where necessary. An example problem is considered (presumably a hypothetical one), in which measurements of pore water pressures in a slope are observed, and then the theory is applied to produce an extrapolation in time, i.e. a prediction of the subsequent pore water pressures based on the observations to date. The paper contains insufficient detail of the method to allow the reader to follow the exact sequence of calculations required for the example problem. It is also unclear whether the example problem may have an inbuilt trend to which this extrapolation technique is well suited. It would be worthwhile knowing for what problems this method is well suited and what are its limitations. Furthermore, the application to a real problem would also provide a better test of the technique. Nevertheless, it would seem that the technique proposed in this paper merits further investigation.

8. COUPLED PROBLEMS

One of the great benefits of the increased computing power that has become readily available is that researchers are now turning to some important problems in geotechnics, problems that in previous times had proved too challenging and dependent on much computing power for their solution. One important class of problems that has come under scrutiny recently is that which involves the coupling together of two or more physical and perhaps chemical processes.

Of course, coupled problems are not new in geomechanics. One problem studied extensively since the beginnings of soil mechanics has been the consolidation of soil, in which a solid deformation process is coupled to the flow of a pore fluid.

Recently, problems such as the coupling of heat flow, stress-deformation and even pore fluid flow have been examined and solutions to practical problems have been obtained, e.g. Savvidou and Booker (1989), Booker and Savvidou (1985), Britto, *et al* (1989), Smith (1990). The basic formulation for these problems and the solutions to various boundary value problems have application to the storage of hot radioactive waste in "stable" geological deposits, and also to mining at depth in hot rocks.

Another problem of this type involves the analysis of contaminant migration through soil and fissured rock, e.g. Rowe and Booker (1989, 1990). These problems involve fluid flow, advection and dispersion of pollutants throughout the medium. The new solutions that are being obtained are invaluable for designers of landfills and other domestic and hazardous waste storage facilities. Of course, information about the basic physical (and chemical) properties of the soils and rocks is an essential input to these mathematical models. Comprehensive and complementary experimental studies are also underway at present to measure these key parameters, e.g. Quigley *et al* (1987).

9. CONSTITUTIVE MODELLING

Research work has been underway for some time now into the development of appropriate constitutive models describing the stress-strain behaviour of soils and rocks. Numerous conferences and publications have been devoted to this topic. There are many reported instances in the literature where quite sophisticated non-linear, elastoplastic soil models have been applied to practical problems.

The topics in this category that have received recent interest include the analysis of shear band formation, e.g. Vardoulakis *et al* (1988), and the application of Cosserat theory to soil and rock problems, e.g. Mühlhaus (1986). The full potential of these theories has yet to be realised but they may help improve our understanding of deformation and collapse mechanisms in soil and rock.

10. SLOPE STABILITY ANALYSIS

Judging by the number of papers in this conference dealing with slope stability, it can be inferred that this is still a topical subject, and one with a number of unresolved issues.

The perennial (and deterministic) problem of reliably locating the critical failure surface in soil slopes continues to receive attention. The paper by **Chen** (1992) in this conference is concerned with this problem. Chen describes his experience in this area and discusses several key issues including: the importance of using a non-linear failure envelope for cohesionless soils, the determination of the global minimum factor of safety by a random search approach, comparison of results obtained using the simplified Bishop method and the Morgenstern-Price method, and the important issue of the verification of the results of limit equilibrium techniques with closed form solutions. The paper ends with some useful conclusions including the comforting recommendation that for slopes where geological heterogeneity and discontinuities do not dominate, the simplified Bishop method generally produces satisfactory results.

The paper by **Williams and Zou** has been discussed previously in relation to stochastic finite element analysis. However, the paper also contains the results of wedge analysis of the spoil pile stability. These deterministic, limit equilibrium analyses have been carried using the techniques and computer codes developed by Donald and Giam (1989). The results have been compared to those obtained from the dynamic programming method coupled to the results of the stochastic finite element stress analysis. This comparison showed that good agreement between the methods was possible for the location of the critical slip surfaces. However, there was some disagreement in the minimum factors of safety predicted by the two methods. This begs the question of which method should be relied upon in design.

In the paper by **Williams and Zou** selected values of the strength parameters are provided. In particular, relatively large mean values have been adopted for the cohesion intercepts of the spoil pile material and the floor material. Presumably an effective stress analysis was performed. It would be instructive to learn to what physical source such large cohesions may be attributed.

10.1 Reliability Methods for Slopes

Most of the recent attention of researchers in the area of slope stability has focussed on replacing the classical and deterministic factor of safety approach by methods which hopefully can deal with uncertainty in a meaningful fashion. The concept of reliability analysis is now being pursued actively. The papers by **Li** (1992) and **St. George** (1992) to this conference address this issue.

Li has developed an efficient point estimate method (PEM) for calculating the reliability index of slopes. The paper also contains a useful discussion of the merits of the PEM and goes to some length to discredit the alternative single random variable (SRV) approach. The presentation of this argument is particularly noteworthy and it asserts that the use of the SRV approach, that apparently is common in practice, results in a gross overestimation of the probability of failure of slopes. This occurs largely because the SRV approach ignores the reduction in the variance of the key soil properties that will occur with spatial averaging, such as along a potential failure surface. An example involving the undrained failure of a slope in clay provides a useful insight into this problem. Authors claimed by **Li** to have used the "erroneous" SRV approach may wish to respond to these criticisms.

Li goes on to develop his point estimate method after noting the shortcomings of existing PEMs, including that due to **Rosenblueth** (1975, 1981). The reliability index for the slope is defined as:

$$\beta = \frac{\mu_F}{\sigma_F} \quad (1)$$

where μ_F and σ_F are respectively the mean and standard deviation of the factor of safety. The obvious strength of **Li's** method for determining β is the avoidance of a multitude of calculations of the factor of safety, corresponding to the large number of possible combinations of the random variables, such as the soil strength parameters and unit weights. **Li's** paper sets out the method but contains only one numerical example. The application of the PEM to more problems of practical interest will be welcome.

In the paper presented by **St. George**, a reliability approach has also been followed and the sensitivity of predictions to the input data has been examined. Unlike **Li**, **St. George** has selected the first order second moment method (FOSM) for the analysis of the reliability index for slopes. He presents examples taken from extensive case studies of failures in opencast mines in the United Kingdom. The limit equilibrium technique employed to compute the factors of safety was that due to **Janbu**.

A number of interesting observations are made in the paper by **St. George**. He warns that "it is unreasonable to expect the techniques of risk analysis and statistics to resolve problems which cannot be formally answered, and as such are not introduced directly into the analysis". He goes on to draw an important distinction between "subjective uncertainties" that arise from a lack of knowledge, and "objective uncertainties" that are related to measured statistical or probabilistic information. **St. George** states that the major uncertainties in slope stability analysis arise:

- i) because there may be natural heterogeneity in the shear strength of geological materials,
- ii) when structural features such as discontinuities control the failure,
- iii) because of the presence of water (and pore water pressures) in the slope, and
- iv) from uncertainties about the appropriateness of the limit equilibrium models.

These key issues are examined at length in the paper. The point is also made that these uncertainties are compounded by the sampling and testing techniques that are used to gather information about the geological materials. However, the point is well made that it is only possible to account for those uncertainties which are measurable, unless a judgement factor is also included in the reliability analysis.

The paper concludes that for two-dimensional problems the probability of failure is sensitive to changes in the modelling error and variations in the water table. Spatial and parameter correlations have less influence on the probability of failure. For three dimensional problems, the uncertainty of the resistance that can be provided at the side margins of the moving mass has an important influence on the probability of failure.

11. LIMIT STATE APPROACHES

Lo, Li and Lee (1992) present a review of the probabilistic theory for the limit state method, and give interesting background information about the use of the limit states approach in the new draft Australian code for piling. The paper includes a description of procedures recommended in the code for the determination of partial safety factors in the limit state format. This paper and the draft code form part of a concerted effort to bring codes of practice for geotechnical engineering into line with approaches used elsewhere, such as in structural engineering. The argument advanced in favour of the limit states approach is that it generally gives better control over the reliability of design.

This paper points out that the uncertainties involved in pile design are different to those involved in structural design, and therefore the approach used for the formulation of the limit state piling code is different from those used in structural codes.

One of the strengths of this paper is the clear exposition of the various levels of probabilistic analysis that are available and the discussion of which levels are suitable to piling practice. This information may be well known in structural engineering, but the writer suspects that generally this is not the case in geotechnical engineering, so that the present paper is both timely and informative. The paper reminds us that in piling practice the foundation engineer has some control over the estimates of reliability. For example, he can obtain more reliable estimates of the soil parameters by carrying out more extensive site investigation, or he can reduce uncertainty by carrying out pile load tests. How such measures can be incorporated into a quantitative risk assessment is explained clearly in the paper by the use of examples. The paper concludes that a consistent level of reliability is possible in pile design, provided the strength reduction factor is properly correlated with the confidence level of the design by a thorough statistical analysis of the available and relevant data. Many practitioners await the release of this draft standard.

12. SEISMIC RISK ANALYSIS

The premise of the paper to this conference by Styles, Moore and Gupta (1992) is that structural damage from an earthquake will be minimised if the natural frequencies of the elements of a building are separated from the frequency of the excitation, i.e. if resonance is not allowed to occur. The paper discusses the individual elements of a building, including its footing and the superstructure, and provides estimates of the natural frequencies of these components. Simple, but approximate closed-form expressions for the natural frequencies are provided.

While the issue of resonance is important, it should be borne in mind that damage to structures can also take place without resonance occurring, as the authors appear to acknowledge in their introduction. The prediction of other quantities, such as the dynamic response of the structure, is also important.

The issues addressed in this paper have been explored elsewhere, e.g. see Novak (1974a, 1974b), Novak and El Hifnawy (1983, 1984), Roesset (1980), where it has been shown that the natural frequencies of buildings are

affected by the type and stiffness of the foundation. The problem of predicting the overall response of the building to seismic excitation is complex, and should also include the influence of damping, both within the soil as radiation and material damping, and in the structure. The stiffness and damping constants for foundations actually depend on frequency, but it is often possible to choose constant values which are representative for the region of the dominant frequency. It has been demonstrated by Novak and El Hifnawy (1984) that the flexibility of the foundation may have a profound effect on the response of buildings to seismic excitation. Both the absolute and relative effect of different foundations vary with soil stiffness. Piles usually yield higher natural frequencies but lower damping than other types of foundations. The depth of the soil layer may also be important. With a finite soil layer, a significant loss of geometric damping occurs and this results in larger building displacements.

13. NEW DEVELOPMENTS

The remaining paper assigned to this section is by Hudson, Sheng and Arnold (1992) and it presents a new approach to the assessment of risk in rock engineering. The paper identifies the need for a coherent approach for the identification of parameters which are significant in any rock engineering activity. It then proposes one possible approach. This involves the use of a "rock mechanics interaction matrix" to identify the relative significance of factors such as: the overall environment including geology and climate, the quality of the intact rock, the nature of the discontinuities in the rock mass, the ground water conditions, the proposed construction procedures, etc. It is asserted that an important feature of the matrix approach is that the interactions between parameters can be used as an indication of which parameters and mechanisms are critical before, during and after the construction of a project. Unfortunately, no detail about how this is done is given in the paper. We are promised this detail in later publications. The paper goes on to describe rock mechanics aspects of the Fei-Tsui Dam in Taiwan. This includes a description of the measures used to improve the shear strength of the rock mass, which included cleaning of the discontinuities and replacement of the infill material with cement grout. The paper contains little in the way of mathematical analysis.

14. CONCLUSIONS

The availability of increased computing power continues to encourage the development of numerical tools in geotechnical engineering.

This paper has reviewed recent developments in the fields of analytical and probabilistic methods in geotechnical engineering, with particular reference to the papers presented under this heading in the conference.

It appears that there is a growing awareness of the need to use the analytical tools in the assessment of the reliability of geotechnical works. It is also evident that slope stability problems are still the focus of much attention in the literature. However, recent activity in this area seems to be aimed at risk assessment rather than deterministic methods of analysing slope stability problems.

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