Numerical Simulation of the Time-Dependent Stratified Viscoelastic Soil Medium with Cracks

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SUMMARY The paper contains the numerical analysis of viscoelastic stratified half space with cracks filled out by air. The cracks are of different type, size and location. The employed constitutive model assumes the proportionality relationship between the spherical components of stress and strain tensor and viscoelasticity law between deviators. With respect to time variables the recurrent integral method is applied, while with respect to spatial variables – the finite element is employed.
In order to analyze the effect of cracks on the displacement and stress field, the computer program was prepared and some numerical examples are investigated. The conclusions connected with risk induced by the variably located cracks are drawn on the basis of numerical results.

1. INTRODUCTION

It is well known fact, that granular materials like soil exhibit time dependent behaviour. This property can be analyzed in the scope of the theory of viscoelasticity. The most common way of the investigation of the temporal properties of the medium is through the application of the time dependent material characteristics like Young modulus E, bulk modulus K or shear modulus G.
In this paper the viscoelastic homogeneous and nonhomogeneous half space of layered type with cracks are subjected to the analysis. The applied load is uniformly distributed over the circular area and simulates the load transferred by the wheel of the heavy vehicle.

2. FORMULATION OF THE PROBLEM

We will consider the nonhomogeneous medium of layered type with cracks filled by air. Each layer of the medium is assumed to be isotropic. This fact enables one to split the stress–strain relationship into two independent parts, that is volumetric and deviatoric. It is commonly recognized fact in analysis of granular materials to relate the spherical components of stress–strain tensor to the proportionality law. Mathematically, it can be expressed as:

$$\sigma_{kk} = 3K\epsilon_{kk} \qquad (1)$$

where

K –is the bulk modulus,

σ_{kk}, ϵ_{kk} –stand for spherical components of stress and strain tensor respectively.

In the paper, it is assumed that the bulk modulus is constant. The time dependent properties of the medium are connected to the deviatoric components of stress and strain tensor. At the constitutive level the integral approach available in the framework of the theory of viscoelasticity is employed. Thus the following type of relationship between deviators is taken to the analysis:

$$t_{ij}(t) = \int_0^t E_1\,(t-\tau)\,\dot{e}_{ij}\,(\tau)\,d\tau \qquad (2)$$

where

t_{ij}, $\dot{e}_{ij}$ –are the stress and strain rate deviator tensors respectively,

E_1 –denotes the relaxation function.

Dot over e_{ij} means the differentiation with respect to time. It is assumed, that time dependent behaviour of granular materials [1,2,3,5,6,8] will be simulated by means of the three parametric standard model. In consequence the explicit form of the relaxation described by eq. (2) is taken as:

$$t_{ij}\,(t) = 2\,\{[E_L + (E_S - E_L)e^{-\beta t}]e_{ij}(0) + \int_0^t (E_L + (E_S - E_L)e^{-\beta(t-\tau)}\,\dot{e}_{ij}(\tau)d\tau] \qquad (3)$$

where the meaning of E_L and E_S is indicated in Fig. 1 and β is the inverse of the retardation time.
The function which appears in equation (3) is the relaxation function and according to the definition represents the time dependent stress induced by uniformly applied state of deformation. The inverted relation to the equation (3) gives the creep function.

$$e_{ij}\,(t) = \frac{1}{2E_L} + [\frac{1}{2E_S} - \frac{1}{2E_L}]e^{\beta\,\frac{E_L}{E_S}\,t}\,t_{ij}\,(0) +$$

$$\int_0^t (\frac{1}{2E_L} + (\frac{1}{2E_S} + \frac{1}{2E_L})e^{\beta\,\frac{E_L}{E_S}\,(t-\tau)}\dot{t}_{ij}\,(0)d\tau \qquad (4)$$

where $J_1(t)$ shown in Fig. 1 is the creep function which

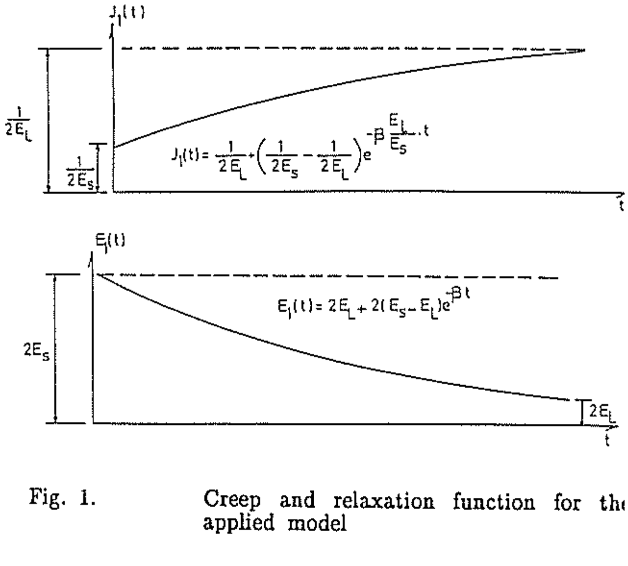

Fig. 1. Creep and relaxation function for the applied model

by definition describes the time dependent deformation induced by the uniformly applied state of stress. In analogy to the theory of elasticity, the relations (2) and (3) involve the time dependent shear modulus, while the temporal function of equation (4) represents the compliance of shear modulus. The analyzed functions connected with the applied model are shown in Fig. 1.

From Fig. 1, it is seen, that constitutive model has finite value of shear modulus at t = + 0 and decreases to finite value at t = infinity. It means that the applied model preserves finite value of stress never reaching zero. The analysis of deformation reveals, that with elapsing time, the deformation tends to the finite, stable value. Consequently, it means that the applied model involved to the behaviour of deviatoric stress-strain components has features of solid.
Combining the equation (1) with (3), after some regrouping, the following constitutive equation is used to the further analysis.

$$\sigma_{ij}(t) = K\, \epsilon_{kk}\, \delta_{ij} + 2\{[E_L + (E_S - E_L)\, e^{-\beta t}]e_{ij}(0) + \int_0^t [E_L + (E_S - E_L)e^{-\beta(t-\tau)}]\dot{e}_{ij}(\tau)\, d\tau\} \quad (5)$$

where

δ_{ij} –is the Kronecker symbol,

σ_{ij} –stands for the stress components.

3. THE DISCRETE FORM OF THE TEMPORAL BEHAVIOUR OF THE MODEL

Since the time dependent relationship is connected with the deviators of stress and strain, we will refer to the equation (3), which for arbitrary time instant t_n is written as follows:

$$t_{ij}(t_n) = 2[E_L\, e_{ij}(t_n) + (E_S - E_L)e^{-\beta t_n}\, e_{ij}(0) + \int_0^{t_n} (E_S - E_L)e^{-\beta(t_n-\tau)}\dot{e}_{ij}(\tau)\, d\tau] \quad (6)$$

According to recurrent formulation, assuming the correctness of the relationship (6) – the analogous relation for the next time instant t_{n+1} has the form:

$$t_{ij}(t_{n+1}) = 2\{[E_L\, e_{ij}(t_{n+1}) + (E_S - E_L)e^{-\beta t_{n+1}}\, e_{ij}(0) + \int_0^{t_n} (E_S - E_L)e^{-\beta(t_{n+1}-\tau)}\dot{e}_{ij}(\tau)\, d\tau\}\, d\tau \quad (7)$$

where t_{n+1} is translated by Δt with respect to the time instant t_n.

Comparing the relationships (6) and (7), we wish to formulate the temporally discrete formula for arbitrary time instant which involves:

a) previous time instant,
b) current time instant,
c) increment of time.

Thus, after some regrouping, the equation (7) can be written as follows:

$$t_{ij}(t_{n+1}) = 2\{E_L\, e_{ij}(t_{n+1}) + e^{-\beta\Delta t}[e_{ij}(0)\,(E_S - E_L)e^{-\beta t_n} + \int_0^{t_n} (E_S - E_L)e^{-\beta(t_n-\tau)}\,\dot{e}_{ij}(\tau)d\tau] + \int_{t_n}^{t_{n+1}} (E_S - E_L)e^{-\beta(t_{n+1}-\tau)}\,\dot{e}_{ij}(\tau)\, d\tau\} \quad (8)$$

It is worth to notice, that the last integral has finite value and is equal to:

$$\Delta I_n = (E_S - E_L)\,\frac{1 - e^{-\beta\Delta t}}{\beta(t_{n+1} - t_n)}\,[e_{ij}(t_{n+1}) - e_{ij}(t_n)]$$

The glance at the equation (8) enables one to notice terms which have some physical interpretation. Namely, the first term represents the current deformation, the second one contained within square bracket denotes the history of deformation, and the last term of finite value defines the increment of history of deformation.

The first integral from zero to t_{n+1} can be precisely evaluated on the basis of formula (8) in the following way:

$$t_{ij}(t_{n+1}) = 2[E_L\, e_{ij}(t_{n+1}) + I_{n+1}] \quad (9)$$

where

$$I_{n+1} = e^{-\beta\Delta t}\, I_n + \Delta I_n \quad (10)$$

The above described algorithm combined with equation (1), gives the following formula which constitutes the discrete form of the applied constitutive equation.

$$\sigma_{ij}(t_{n+1}) = \sigma_{kk}\, K\, \delta_{ij} + 2\{E_L\, e_{ij}(t_{n+1}) + (E_S - E_L)\,\frac{1 - e^{-\beta\Delta t}}{\beta(t_{n+1} - t_n)}\,(e_{ij}(t_{n+1}) - e_{ij}(t_n) + e^{-\beta\Delta t}\,(e_{ij}(0)\,(E_S - E_L)e^{-\beta t_n} + \int_0^{t_n} (E_S - E_L)e^{-\beta(t_n-\tau)}\dot{e}_{ij}(\tau)\, d\tau\} \quad (11)$$

4. NUMERICAL FORMULATION WITH RESPECT TO SPATIAL VARIABLES

To analyze the described problem with respect to spatial variables the finite element method (FEM) is implemented [4,7,9]. Combination of the recurrent integral method with respect to temporal variables and FEM with respect to spatial variables leads to the following discrete matrix equation valid for arbitrary type of the finite element:

$$\{[K]^E + [K]^V\,[E_L + (E_S - E_L)\,\frac{1 - e^{-\beta\Delta t}}{\beta(t_{n+1} - t_n)}]\}\,\{V_{n+1}\} = \{P_{n+1}\} - [K]^V\{e^{-\beta\Delta t}\, I_n + (E_S - E_L)\,\frac{1 - e^{-\beta\Delta t}}{\beta(t_{n+1} - t_n)}\}\{V_n\} \quad (12)$$

where

$[K]^E$, $[K]^V$ –are the stiffness matrices connected with elastic and viscous properties respectively,

$\{V_{n+1}\}$, $\{V_n\}$ –are the displacement vectors related to t_{n+1} and t_n time instants respectively.

After determination of the displacement field, the stress components are computed in accordance with the following formula:

$$\sigma(t_{n+1}) = [D]^E\,[B]\,\{V_{n+1}\} + [D]^V\,[B]\,[E_L\{V_{n+1}\} + e^{-\beta\Delta t}I_n + (E_S - E_L)\,\frac{1 - e^{-\beta\Delta t}}{\beta(t_{n+1} - t_n)}\,[\{V_{n+1}\} - \{V_n\}]] \quad (13)$$

where

$[D]^E$, $[D]^V$ – are the material matrices connected with elastic and viscous properties respectively,

$[B]$ – stands for the deformation matrix.

5. NUMERICAL EXAMPLES

The described constitutive model which employs with respect to time variables the recurrent integral method and with respect to spatial variables the FEM is used in the analysis of some problems of practical importance.

The geometry, boundary conditions, location of cracks are shown in Fig. 2. The finite element mesh consists of the eight noded isoparametric quadrilateral elements.

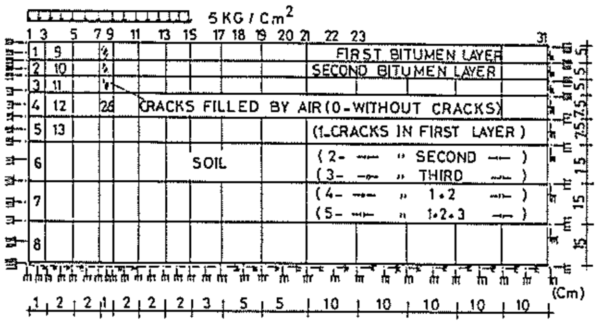

Fig. 2. Geometry, finite element mesh, load and boundary conditions.

The time dependent analysis involves also two special cases, that is the time instant t = +0 and t = infinity. Regarding the effect of the location of the cracks on the temporal displacement and stress fields the following cases are analyzed:

1) –homogeneous medium with cracks of different location (notation of schemes 4H0 and 8H0),
2) –homogeneous medium with cracks of different location (notation of schemes 8H1 ÷ 8H5),
3) –nonhomogeneous medium of layered type without cracks (one layer of bitumen and soil – notation of schemes – 8N0),
4) –nonhomogeneous medium of layered type with cracks (one layer of bitumen and soil – notation of schemes 8NS1 ÷ 8NS5),
5) –nonhomogenous medium of layered type without cracks (double layer of bitumen and soil – notation of schemes 8N0D)
6) –nonhomogeneous medium of layered type with cracks (double layer of bitumen and soil – notation of schemes – 8ND1 ÷ 8ND5)

The first number in the notation of schemes defines the number of nodes connected with the FE used in the analysis. The location of the cracks in the scheme is denoted by the last number in the notation of schemes. That is: single crack in the top layer – as 1, single crack in the second layer (from the top) – as 2, single crack in the third layer – as 3, double crack, in the top two layers – as 4, the triple crack in the top three layers – as 5. The location of the cracks is indicated in Fig. 2. For all cases described above, the common value of the time step Δt = 100 is used in numerical computations.
In order to compare the accuracy of the results, the four noded isoparametric element is used. The comparative investigations are done for the homogeneous medium in the absence of cracks and in presence of cracks. The obtained results in terms of the displacement and stress fields almost coincide. Some of the results in terms of the temporal displacement field are presented in Figs. (3 ÷ 10). The comparison of the displacements for different media for the first and the tenth time steps are shown in Fig. 11. The temporal diagrams of stresses for different analyzed cases are shown in Figs. (12 ÷ 14). The temporal variability of stress components is computed for the element denoted as 28 located at the tip of the crack (below the crack).

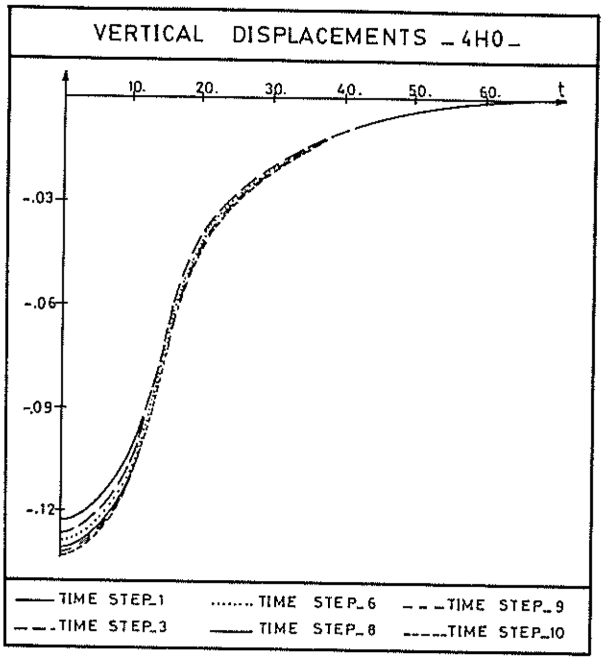

Fig. 3. The vertical displacements for the homogeneous medium without cracks for indicated time steps (4 noded FE).

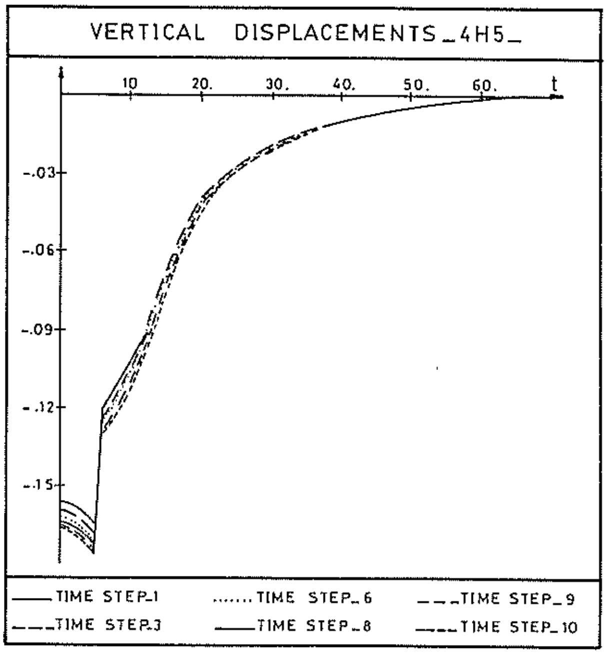

Fig. 4. The vertical displacements for the homogeneous medium with cracks located in top three layers for indicated time steps (4 noded FE).

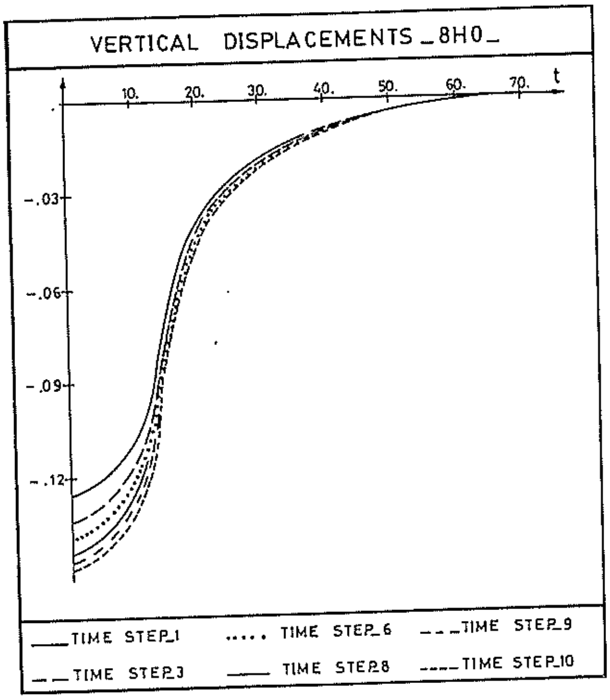

Fig. 5. The vertical displacements of homogeneous medium without cracks for indicated time steps (8 noded FE).

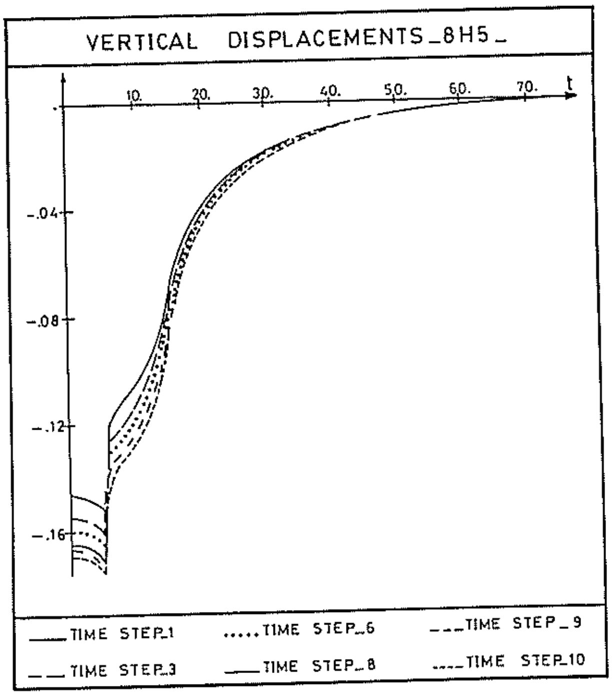

Fig. 6. The vertical displacements of homogeneous medium with cracks located in the top three layers for indicated time steps (8 noded FE).

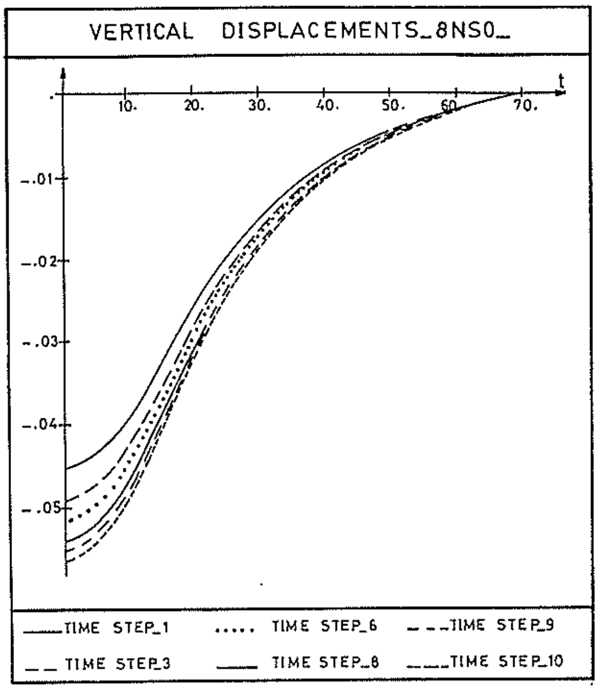

Fig. 7. The vertical displacements for nonhomogeneous medium (one layer of bitumen and soil) without cracks for indicated time steps (8 noded FE).

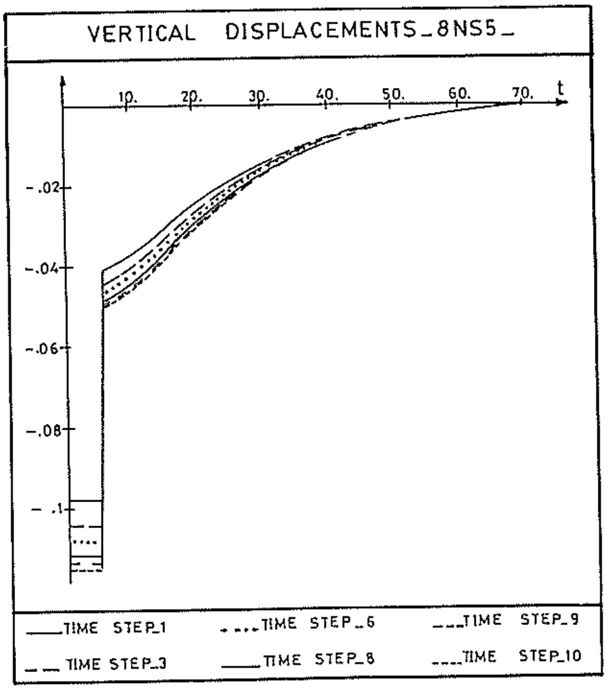

Fig. 8. The vertical displacements for nonhomogeneous medium (one layer of bitumen and soil) with cracks in the top three layers for indicated time steps (8 noded FE).

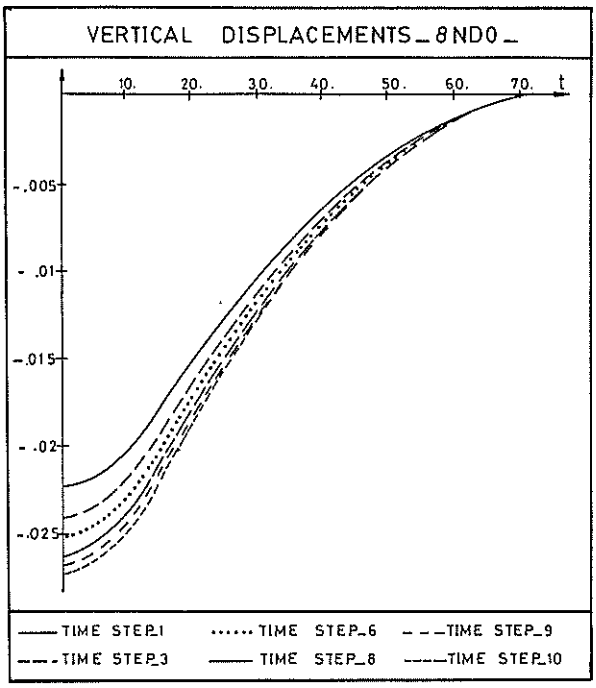

Fig. 9. The vertical displacements for nonhomogeneous medium (two layer of bitumen and soil) without cracks for indicated time steps (8 noded FE).

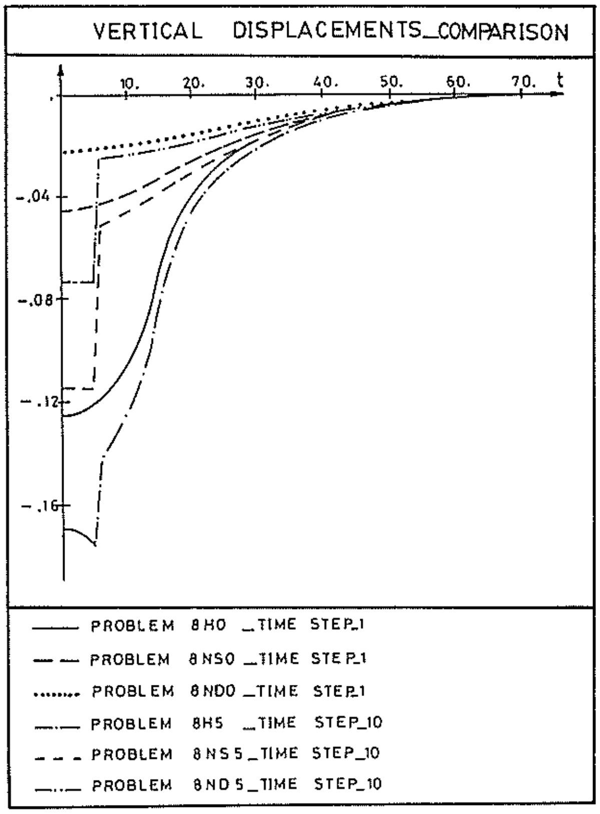

Fig. 11. The comparative distributions o vertical displacements for indicatec time steps (8 noded FE).

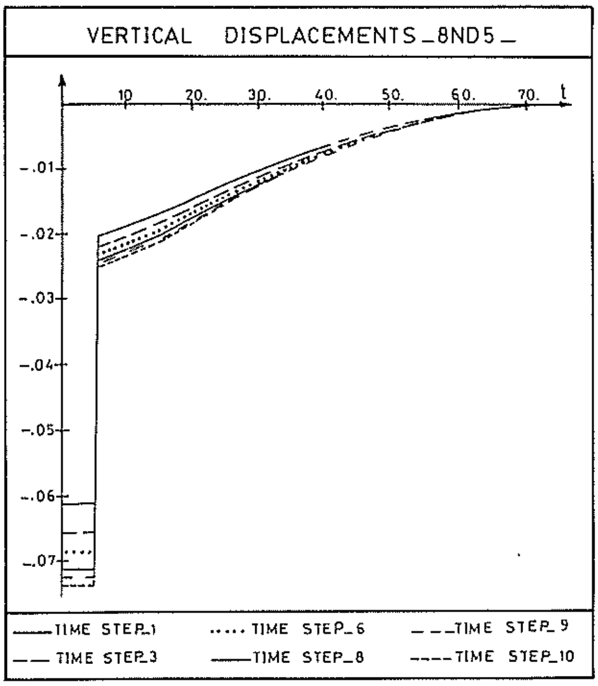

Fig. 10. The vertical displacements for nonhomogeneous medium (two layers of bitumen and soil) with cracks in the top three layers for indicated time steps (8 noded FE).

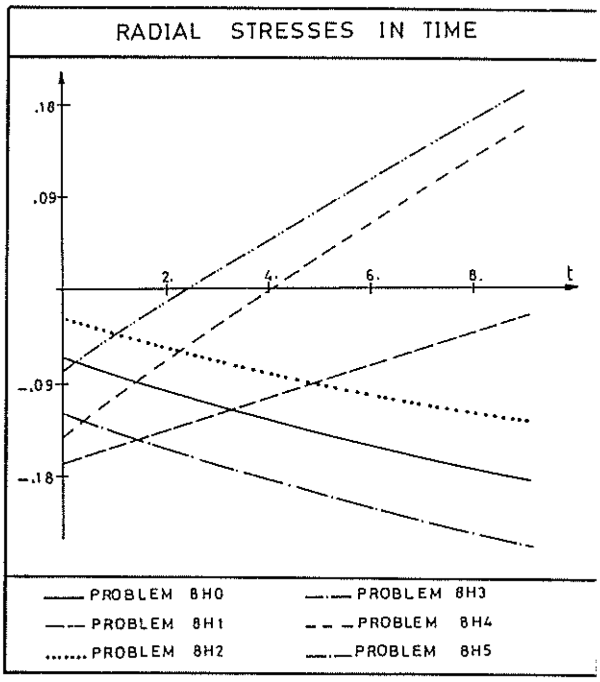

Fig. 12. The temporal radial stress distributioı for indicated cases of the analysiı (el.nr. 28, 8 noded FE – middl Gaussian point).

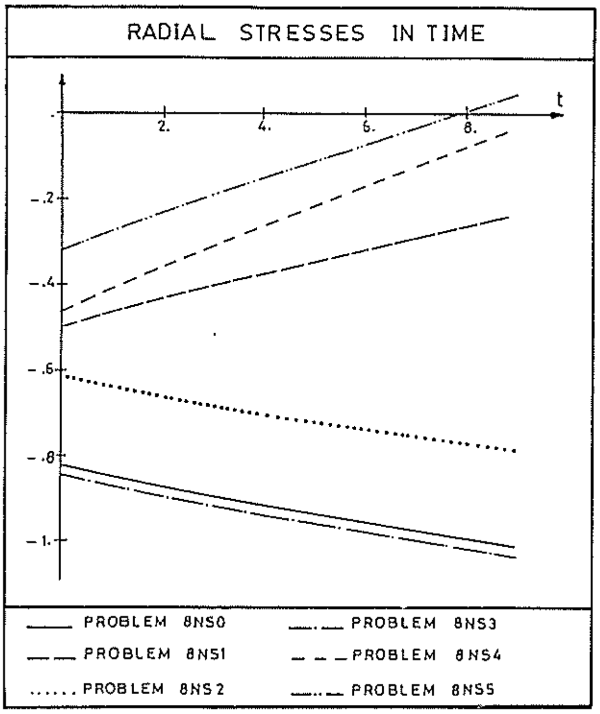

Fig. 13. The temporal radial stress distribution for indicated cases of the analysis (el.nr.28, 8 noded FE – middle Gaussian point).

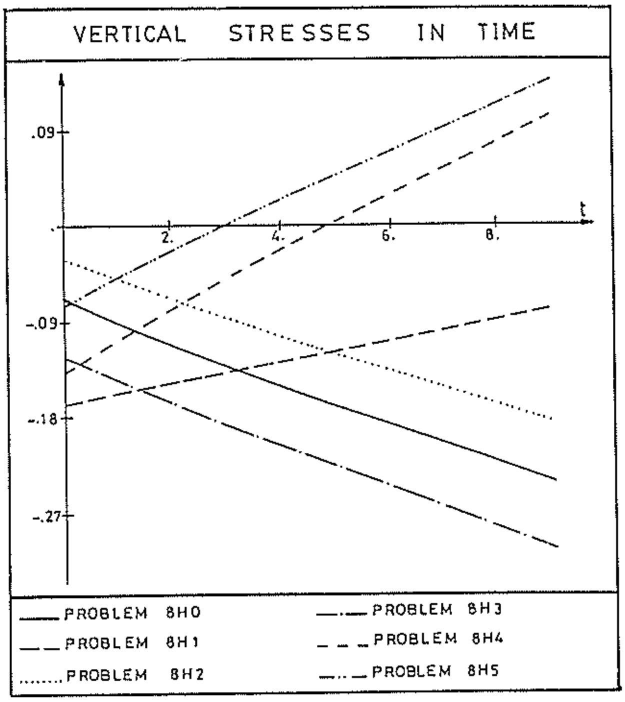

Fig. 14. The temporal vertical stress distribution for indicated cases of the analysis (el.nr.28, 8 noded FE – middle Gaussian point).

6. CONCLUSIONS AND FINAL REMARKS

In the paper the viscoelastic homogeneous and nonhomogeneous medium with variably located cracks are subjected to the analysis. The problem employs the simple type of constitutive model for which the volumetric components of stress and strain tensor are related to the proportionality law, while the time dependent properties are connected to deviatoric components. The temporal discrete analysis employs the recurrent integral method, while with respect to spatial variables the FEM is used. The applicability of the described model to the investigation of the effect of cracks on the displacement and stress field is analyzed on the basis of some examples of practical importance. The obtained results form the basis to the following conclusions:

1) The main factor which effects significantly the magnitude of the vertical displacements is due to the thickness of the bitumen layers.
For comparison: The equivalent displacements of the homogeneous medium are three times larger than the analogous displacements of nonhomogeneous medium which consists of single layer of bitumen and the soil medium. The equivalent displacements of the homogeneous medium are six times larger than the analogous displacements of the nonhomogeneous medium which consists of double layer of bitumen and the soil medium.
2) The presence of the inner cracks (covered by the top layer) – located, in second and third layer has no substantial influence on the magnitudes and the shapes of the deflection surface.
3) A basic change in the shape as well as magnitude of the deflection of the top surface takes place when the cracks start to appear from the top surface. In this case a sharp jump producing discontinuity of the deflection surface is observed in the closest vicinity of the cracks. The magnitude of the displacement jump depends heavily on the depth of the cracks. From this fact it follows, that the application of the load in the vicinity of the top cracks highly increases the risk of failure of the medium through the large discontinuous deformations.
4) The analysis of stress field for investigated cases indicates that for homogeneous materials all stress components increase with time. For the media with cracks the radial σ_x and vertical σ_z stress components have tendency to decrease.

7. ACKNOWLEDGEMENTS

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