Geotechnical Risk and the Use of Grey Extrapolation Technique

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SUMMARY This paper is concerned with an aspect of the assessment of geotechnical risk with particular reference to observational data and its interpretation. The use of a grey prediction model (grey extrapolation technique) is discussed. This is part of what is known as the grey system theory which is concerned with systems subject to uncertainty. An illustrative example concerned with data on pore pressure equilibrium in an excavated slope of overconsolidated clay has been presented.

1. ASSESSMENT OF GEOTECHNICAL RISK

Assessment of geotechnical risk is often an important task for engineers involved in a project. Such an assessment is often made before the project is undertaken. During the planning and design stages, geotechnical and other data must be collected, organised and analysed carefully to develop an appreciation of the factors which influence geotechnical performance and the adequacy of that performance during the life of the project. A judgement can then be made about geotechnical risk on the basis of available information, completed analyses and on the basis of previous experience on similar projects or on other projects with some similar aspects to the new proposed project.

Judgements made during the planning and design stages are not final. Considerable information may become available during the construction phase of a project. Based on this new or additional information, it is very useful to update the judgements made previously. In particular, observational data about stresses, strains, displacements or pore water pressures may highlight some gaps in analyses made previously. It may also be necessary to revise some of the assumptions on which the analyses were based. More improtantly, observational data may either validate or invalidate the geotechnical models adopted for different elements of the project or for different stages of the same element of the project. Consequently, assessment of geotechnical risk must continue during the execution of the project.

2. OBSERVATIONAL PROCEDURES BEYOND CONSTRUCTION STAGE

In significant geotechnical projects it is considered necessary to continue the observational approach beyond the construction stage. There are two main reasons for this. Firstly, it is an acknowledgement of the uncertainties in relation to geotechnical prediction and performance. Both systematic and random uncertainties are associated with the adopted values of geotechnical parameters such as shear strength and deformation parameters, pore water pressures and in situ stresses. In particular, spatial variability of these parameters is often not known except perhaps qualitatively. There are also uncertainties concerning loads and, in particular, dynamic loads associated with random events such as earthquakes. The influence of such events on geotechnical parameters also involves considerable uncertainty. Finally, there are uncertainties related to geotechnical models which have been adopted for analyses. Considering all these factors, it is only logical that geotechnical engineers should invest in instrumentation and other observational procedures for performance monitoring beyond the construction stage of a project.

The second main reason for continued observational approaches on a project after its completion is that long-term performance is often quite different from the end-of-construction performance. For example, as is now well recognised, excavated slopes in saturated overconsolidated clays usually have their highest factor of safety at the end of construction. This is because the unloading process of excavation generates excess negative porewater pressures in the zone adjacent to an excavation. Consequently, there is an increase in shear strength which partly compensates for the increased shear stresses associated with the excavation process. However, these negative excess pore water pressures are 'transient' and must, therefore, dissipate in the long-term. The long-term pore water pressures are significantly higher and the shear strength associated with these equilibrium pore water pressures is, therefore, significantly lower than that at the end of construction. On the other hand, the shear stresses are more or less the same as at the end of construction. Therefore, the factor of safety in the long-term can be significantly lower than that in the short-term. Many years or decades may pass before the pore water pressures rise to equilibrium values. The modelling of the rate of pore water pressure equilibration is not easy and, in any case, significant uncertainty would be associated with a predictive process. Consequently, the installation of permanent devices such as piezometers is advisable. Based on the actually measured pore water pressures, the geotechnical risk associated with an excavation can be updated. Moreover, the rate of change of risk can also be interpreted on the basis of such observational data.

3. AIM AND SCOPE OF THIS PAPER

The main aim of this paper is to introduce briefly an approach which can be used for interpretation of observational data as well as its extrapolation. This approach is part of what has been called "Grey System Theory" and may be called the "Grey Extrapolation Technique". After a brief introduction to this technique, its relevance to geotechnical risk assessment is considered. for this purpose an illustrative example is considered which relates to long-term pore water pressure equilibration after an excavation is made in overconsolidated soil. This is the type of problem which has already been introduced in the previous section of this paper.

The application of the grey system theory to settlement prediction was considered in a previous paper and excellent performance of the method was shown for that type of problem (Zhang et al, 1991).

The grey extrapolation technique may also be used for problems involving the variation of more than one parameter. There are many geotechnical situations in which it is desirable to consider the variation of multiple parameters. However, the use of the grey extrapolation technique for such problems is outside the scope of this paper. The research work is not so well advanced at this stage.

Analysis of data relating to a major case history of progressive slope failure associated with reservoir filling and rainfall infiltration has just been undertaken and the results will be published at a subsequent date.

3.1 Grey System Theory

The grey system theory was proposed by Deng (1982, 1985) and is concerned with the analysis and interpretation of uncertain information. A system which is completely unknown may be denoted as a 'black' system and one which is fully known as a 'white' system. Most systems are, however, neither fully unknown nor fully known and, therefore, may be denoted as 'grey' systems. In general, all geotechnical engineering systems can be regarded as 'grey' since the quality of data is never perfect, the quantity of data may be insufficient and since the relationships between parameters or between parameters and performance are never known exactly or completely.

Grey system theory includes the development of system models and system analysis, grey forecasting, grey decision-making and grey control. From the original data, a generating data model is established. System analysis involves the analysis of correlation between various factors influencing a problem and the identification of the importance of each factor . The tendency of one or more factors to vary together is assessed in grey system theory. Forecasting may include the prediction of the time of occurrence of a particular value of a variable and the occurrence of an unusual or catastrophic event. Decision-making is concerned with the identification of an optimal strategy based on given criteria. The execution of a strategy is called 'control' and grey control is primarily concerned with the control of grey parameters in a system.

Grey system theory can be applied to diverse fields such as economics and meteorology and its relevance to geomechanics must be considered on the basis of experience. In an earlier paper (Zhang et al, 1991) settlement prediction based as grey system theory was shown to be both reliable and accurate. In the present paper, the method is used for prediction of pore water pressures with time based on observational data.

As in the preious paper (Zhang et al, 1991) only the prediction technique of the methodology is used in the present paper.

Grey prediction technique is based on the accumulation trend of random data. The intrinsic characteristics of a grey system can, in general, be modelled using differential equations. However, in this paper only a simple one-parameter random model (or one-dimensional random model) is used.

4. TECHNIQUES FOR ENHANCING DATA INTERPRETATION

4.1 Accumulation Operations

The first step is to determine the accumulation trend of random variables. Considering m random variables x(1), x(2) .... x(m). The first order accumulation operation is performed as follows:

$$\begin{aligned} x^1(1) &= x(1) \\ x^1(2) &= x^1(1) + x(2) \\ x^1(3) &= x^1(2) + x(3) \\ x^1(m) &= x^1(m-1) + x(m) \end{aligned} \qquad (1)$$

The variables x(1), x(2), x(3) .... etc could also be the values of a single parameter at different times. In many cases it is found to be useful to consider the incremental changes in a parameter which varies with time. This is also discussed in a subsequent section.

This is an accumulation operation of the first order. An accumulation operation of the nth order will relate $x^n(1)$ to $x^{n-1}(1)$, $x^n(2)$ to $x^n(1)$ and $x^{n-1}(2)$ and so on as shown below:

$$\begin{aligned} x^n(1) &= x^{n-1}(1) \\ x^n(2) &= x^n(1) + x^{n-1}(2) \\ &\dots\dots \\ &\dots\dots \\ x^n(m) &= x^n(m-1) + x^{n-1}(m) \end{aligned} \qquad (2)$$

Initially the data may be quite random. With the first accumulation operation, the randomness of the data is reduced and with each subsequent accumulation operation randomness decreases further. Thus the cumulatively generated data can be plotted as a smooth curve with a clearly evident trend. Often one or two accumulation operations may be sufficient and higher order operations are, therefore, unnecessary.

4.2 Inverse Accumulation Operations

Another type of operation is the inverse accumulation operation. For example, starting from the accumulation data of nth order given in Eq. (2), the first order inverse accumulation operation is shown below:

$$\begin{aligned} Inv^1\,[x^n(1)] &= x^{n-1}(1) \\ Inv^1\,[x^n(2)] &= x^n(2) - x^n(1) = x^{n-1}(2) \\ &\dots\dots \\ &\dots\dots \\ Inv^1\,[x^n(m)] &= x^n(m) - x^n(m-1) = x^{n-1}(m) \end{aligned} \qquad (3)$$

If the inverse accumulation operation is carried out to the nth order, the original data are obtained. Thus

$$\begin{aligned} Inv^1\,[x^n(1)] &= x(1) \\ Inv^1\,[x^n(2)] &= x(2) \end{aligned} \qquad (4)$$

5. GREY PREDICTION MODEL

A simple differential equation to model the accumulation trend of Eq. (1) is

$$\frac{dx^1}{dt} + Ax^1 = u \qquad (5)$$

in which the variable x^1 is a function of time t; A and u are constants to be determined.

The incremental changes in the variable are first obtained from the raw data. The accumulation operations are carried out from this incremental data. A variable like x^1 is the value obtained after the first such operation.

A solution may easily be obtained for this differential equation considering a continuous case. Similarly, a solution can be obtained for a discrete case.

For a continuous case the solution is

$$x^1(t) = [x(1) - u/A]\, e^{-At} + u/A \qquad (6)$$

For a higher order accumulation operation (nth order) the equation becomes.:

$$x^n(t) = [x^{n-1}(1) - u/A]\, e^{-At} + u/A \qquad (7)$$

The constant A and u are determined from a matrix operation as discussed elsewhere (Zhang et al, 1991). The elements of the component matrices are the original data and the data generated from accumulation operations.

Table 1
Variation of r_u with time

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Time (years)	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	6	7	8	9	10	11	12	13	14	15
r_u	0.05	0.11	0.17	0.21	0.24	0.26	0.27	0.28	0.28	0.28	0.29	0.28	0.29	0.28	0.29	0.29	0.29	0.30	0.29	0.30

6. ILLUSTRATIVE EAMPLE

6.1 Statement of the Problem

At the site of an excavated slope pore water pressures have been monitored over a period of 15 years. The value of r_u, the pore water pressure ratio, at a significant location is shown in Table 1.

The following calculations are required

(a) Based on observed data over the first four years (8 data points), what would have been the predicted value of r_u after the passage of 15, 27 and 35 years. What would have been the final equilibrium value?

(b) Based on observed data over the first six years (11 data points), what would have been the predicted value of r_u after the passage of 15, 27 and 35 years? What would have been the final equilibrium value.

(c) Considering all the data, what is the predicted value of r_u after 27 years? What are the predicted values after 35 and 65 years? What is the final equilibrium value?

6.2 Results of Analyses

The grey extrapolation technique was based on the grey prediction model discussed in the previous section. The incremental approach worked very well and the results are shown in Tables 2, 3 and 4.

Table 2
Calculated Values of r_u based on the first eight data points (4 years data)

Year	Calculated Value	Observed Value
15	0.3182	0.30
27	0.3183	-
35	0.3183	-
Final	0.3183	-

Table 3
Calculated Values of r_u based on the first eleven data points (6 years)

Year	Calculated Value	Observed Value
15	0.30067	0.30
27	0.30070	-
35	0.30070	-
Final	0.30070	-

Table 4
Calculated Values of r_u based on the whole data set, twenty data points (15 years)

Year	Calculated Value of r_u
27	0.2976
35	0.2976
65	0.2976
Final	0.2976

It should be noted that the particular model chosen in this case works only with the use of incremental changes in the observed variable. These incremental changes should be used as the basis for the accumulation operations. Some calculations were, of course, made with the raw data. These preliminary calculations showed that the calculated value of r_u increases significantly with time without levelling out at any time. Such results are obviously unreasonable and incorrect for the type of problem under consideration.

The calculation procedure of the model was developed in such a way that it is not necessary to use equal time steps. Therefore, data concerning a variable recorded at irregular time intervals can be used for analysis.

7. DISCUSSION AND CONCLUSIONS

For pore water pressures is an excavated slope of saturated overconsolidated clay, the main trend is always an increase in the pore water pressure. It is well known that the rate of this movement towards equilibrium of pore water pressure decreases with time. There are, however, many other factors and mainly climatic factors which may influence this trend. Therefore, the changes occurring may not appear to follow a regular and smooth curve during the equilibration period. In this type of problem, the use of a numerical technique for extrapolation is of significant interest. Purely theoretical predictions are not easy to make and may not provide accurate estimates because of the various uncertainties involved. In the present case, the use of grey extrapolation technique based on incremental changes in the observed data proved to be successful. The use of the technique based on the raw data proved to be unsuccessful.

It is interesting to note that even with only 8 data points (covering a 4 year period) the predicted value is reasonably close to the observed value after 15 years. Moreover, there is insignificant change after the 15 year period which appears to be quite reasonable on the basis of the available data.

As the number of data points considered in the prediction model increases, so does the accuracy of the prediction. With 11 data points (covering 6 years) the value of r_u after 15 years is almost identical to the observed value.

The results are encouraging and indicate that more complex applications of grey systems theory should be considered in geomechanics. The geotechnical risk associated with major projects should be considered using the grey prediction technique to analyse observational data. Such an approach would indeed facilitate the assessment of risk associated with the reactivation of major landslides and with the development of major dam-reservoir systems. At present, the authors are investigating the development of the technique for geotechnical risk involving multiple factors. In particular, a case history involving a major project is being considered for which significant observational data are available.

8. REFERENCES

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