Limit State Design of Pile Foundations: A Probabilistic Appraisal

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SUMMARY: A draft revised Australian piling code has recently been issued for public comment. In the draft piling code, a limit state approach is used for the design of pile foundations. The purpose of this paper is to review the probabilistic theory for the limit state method. The procedure for determining the partial safety factor in the limit state design format is described and the implementation of the limit state design format is discussed with reference to the draft piling code.

1. INTRODUCTION

The concept of overall factor of safety is traditionally used in pile design. Such a design philosophy was enshrined in Australia by the original Australian Piling Code AS2159-1978. However, there is a strongly supported trend towards the use of limit state design in which a series of partial safety factors are used in lieu of the overall factor of safety. It is generally believed that a well formulated limit state design code can give better control on the reliability of design. Most structural design codes are now based on limit state design.

One can argue that limit state procedures should also be used for pile design. Indeed, the draft unified European code on geotechnical design (Euro 7) has already adopted the limit state method for geotechnical design. A committee was appointed by Standards Australia in 1988 to revise the current Australian Piling Code and the draft consequent code, issued for public comment in August 1991, will hereafter be called the draft piling code. To improve the control on reliability level of pile designs and maintain consistency with the recently introduced structural design codes (AS3600, AS4100), a decision was made to adopt the limit state method in the draft revised piling code. The partial factors for loadings recommended in the draft code are based largely on the structural design code AS3600 whereas the partial factors for the resistance of the pile, expressed as strength reduction factors, were deduced mainly from calibration with the existing piling code AS2159.

The characteristics of the uncertainties involved in pile design are different from those involved in structural design. The approach used for the formulation of the limit state piling code may need to be different from those in structural codes. At present, study on the limit state design of pile foundations is very limited. More research is urgently needed for a probabilistic appraisal of the limit state approach.

The objective of this paper is to discuss the use of limit state method for pile design based on rigorous probability theory. Special reference is made to the draft Australian Piling Code. Parametric studies are performed to examine the effectiveness of the limit state method in the control of the reliability level of pile design.

2. LIMIT STATES AND PERFORMANCE FUNCTION

A pile foundation is designed to satisfy certain requirements such as the ability to support the structure without failure and excessive settlement. Each of these requirements is called a 'limit state'. A failure is interpreted as the violation of a limit state. Here, failure is interpreted in the most general sense. If the limit state is related to settlement, a failure is said to occur if the settlement is excessive. The paper will focus on the ultimate resistance of pile foundations.

The safety-failure state of a pile foundation can be described by a performance function $G(X)$ where X is the collection of input parameters. The performance function is defined in such a way that failure is implied by the condition of $G(X)<0$ and safety by $G(X)>0$. The hypersurface defined by $G(X)=0$ is termed the limit state boundary.

3. LEVELS OF PROBABILISTIC ANALYSIS

Probabilistic analysis can be classified into three different levels depending on the rigour and level of sophistication of the analysis. To discuss the difference between the three levels of analyses, we will use a single pile as an illustrative example. If the ultimate capacity of the pile is R and the applied loading (or actions) is S, the performance function can be written as

$$G(X)=R-S \tag{1}$$

It is assumed that R and S are independent random variables. The mean value, standard deviation and coefficient of variation (COV) of R are denoted respectively by μ_R, σ_R and V_R. Similar notations are used for S and all other random variables discussed later.

3.1 Level 3 methods

A Level 3 analysis is the most complete method of risk analysis in which all the random variables are represented by their joint probability density function and the failure probability is evaluated directly by integrating the probability density function over the failure domain $G(X)<0$ as indicated in Fig.1. In foundation engineering, the joint probability density function of the random input parameters is seldom known and a Level 3 analysis cannot be performed in practice.

3.2 Level 2 methods

Define the centroid of the random variables to be the point $\mu=(\mu_1,\mu_2,..,\mu_n)$ where μ_i is the mean value of the ith random variable. If the centroid is farther away from the limit state boundary, the probability of failure, which is numerically equal to the volume ABCD under the probability density function as shown in Fig.1, will become smaller. Therefore, the minimum distance between the centroid and the failure boundary can be used as a measure of the safety level of the design. Intuitively, the minimum distance can be defined in the parameter space X. However, such a definition will not give rise to a consistent measure of reliability. Consider two designs with the same minimum distance OP from the limit state boundary as shown in Fig.2(a) & (b). The variability of R and S is higher for the design in Fig.2(a) than for the second design in Fig.2(b). Obviously, the first design has a higher failure probability even though the centroids are both at the same distance from the failure boundary for two designs. To obtain a consistent measure of risk, Hasofer & Lind (1) suggested that the variables be transformed to the uncorrelated standardised parameter space Z as shown in Fig.2(c). In this case, the minimum distance OD in Fig.2(c) is independent of the variability of the input parameters and can therefore be used as a consistent measure of the safety level of the design. The distance OD is defined as the reliability index β and is now commonly used both in structural and geotechnical reliability analyses as a measure of the reliability level of a system. If the random variables are normally distributed, the failure probability P_f can be related to the reliability index by the following relationship (2).

$$P_f = \Phi(-\beta) \tag{2}$$

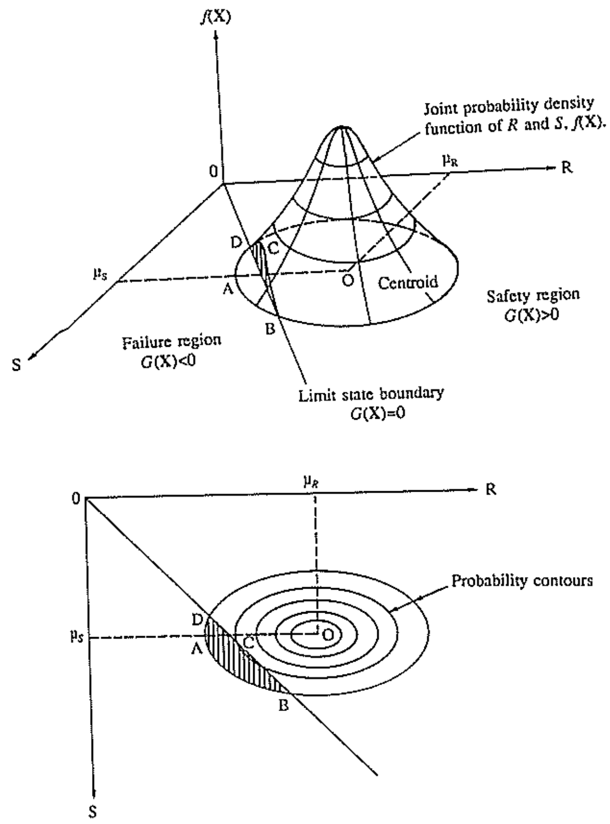

Fig.1 Joint probability density function of R and S

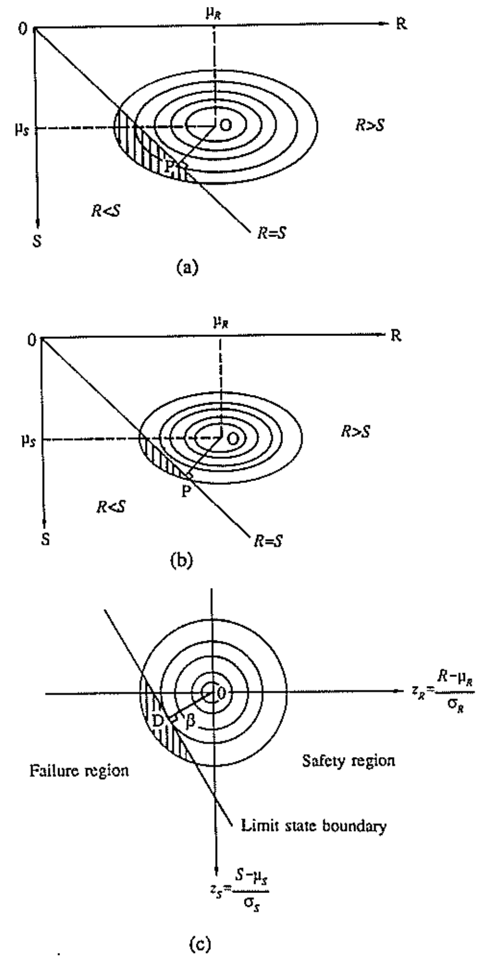

Fig.2 Level 2 design

A probabilistic analysis based on the concept of reliability index is referred to as a Level 2 method. The value of β can be calculated using the first- and second-order statistical moments (i.e. mean values, variances and covariances) of the input parameters. These quantities can be estimated from the soil data in pile designs.

3.3 Level 1 methods

Level 1 methods include the traditional overall factor-of-safety (FOS) approach and the limit state method. At present, geotechnical designs are based almost exclusively on the overall FOS approach. A pile design is deemed to be sufficiently safe if the factor of safety, defined by the following equation, is larger than a specified minimum FOS, F_{min}.

$$F = \frac{R'}{S'} \tag{3}$$

where R' and S' are the nominal ultimate capacity and applied load determined by the designer. The procedure is equivalent to checking that the factored or allowable ultimate capacity of the pile, $R_{all} = R'/F_{min}$, is greater than the nominal load S'. Conceptually, safety analysis is effected by ensuring that the checking point $X_d=(R_{all},S')$ satisfy the following inequality.

$$G(X_d)=G(R_{all},S')\geq 0 \quad (4)$$

The procedure for calculating the nominal load S' is usually well defined in structural codes. However, the procedure for determining the nominal value of R is less well defined in geotechnical codes. Risk-averse geotechnical engineers tend to choose a more conservative value of R'. The minimum overall factor of safety for pile design is sometimes stipulated in foundation codes. However, the choice of F_{min} is largely based on experience rather than rigorous statistical analysis.

Due to the limitations of the overall FOS approach, there is a trend towards the use of the limit state method which can produce a more uniform level of safety for geotechnical designs. In this method, the checking point has the form $X_d=(R^*,S^*)=(\gamma_R{\cdot}R',\gamma_S{\cdot}S')$ where R^* and S^* are called the design values of R and S; γ_R and γ_S are respectively the strength reduction factor and load factor and are collectively called the partial safety factors. Safety analysis is effected by checking that the following inequality is satisfied.

$$G(\gamma_R{\cdot}R',\gamma_S{\cdot}S')\geq 0 \text{ or } \gamma_R{\cdot}R'\geq\gamma_S{\cdot}S' \quad (5)$$

An alternative format of the form $\gamma_R{\cdot}R(x_i/\gamma_i)$ where x_i is an input soil parameter and γ_i is the partial safety factor on soil parameters can also be used for the design value R^*. However, the format in (5) is preferred because the relationship between the resistance R and the soil parameters is sometimes not known. An example of this is the skin friction factor 'F' for piles in sands which in an unknown function of many soil parameters.

Although the limit state method is not as effective as the Level 2 method in controlling the reliability level of foundation designs, the safety checking procedure is much simpler and welcomed by practising engineers. Furthermore, by carefully selecting the partial safety factors, the reliability level can be controlled to within reasonably narrow limits.

4. LIMIT STATE METHOD

In this section, the limit state method is examined in more detail. Referring to Fig.2(c), it can be noted that point D is a stationary point. If this point is chosen to be the checking point, the reliability index β will be least sensitive to small variations in the variability of the input parameters. Therefore, a suitable choice for the checking point X_d in the X-space will be the point corresponding to the stationary point D in the Z-space. The design values corresponding to this checking point are given as (2):

$$R^*=\gamma_R{\cdot}R'=\mu_R{\cdot}(1+\alpha_R{\cdot}\beta{\cdot}V_R) \quad (6)$$

$$S^*=\gamma_S{\cdot}S'=\mu_S{\cdot}(1+\alpha_S{\cdot}\beta{\cdot}V_S) \quad (7)$$

α_R and α_S are the direction cosines of checking point D in the Z-space. Re-writing the nominal values as $R'=\delta_R{\cdot}\mu_R$ and $S'=\delta_S{\cdot}\mu_S$ where δ_R and δ_S are coefficients, the partial safety factors can be obtained from the following equations according to (6) and (7).

$$\gamma_R=(1+\alpha_R{\cdot}\beta{\cdot}V_R)/\delta_R \quad (8)$$

$$\gamma_S=(1+\alpha_S{\cdot}\beta{\cdot}V_S)/\delta_S \quad (9)$$

Usually, the nominal ultimate capacity R' is chosen to be smaller than the mean value i.e. $\delta_R<1$ while a value higher than the mean value is adopted for the nominal load i.e. $\delta_S>1$. According to (8) and (9), the partial safety factors γ_R and γ_S depend on the variability of the input parameters if the reliability index has to be maintained at an absolutely constant level . To keep the limit state design format reasonably simple, some simplifications must be made. Two options are available. In the first option, the partial safety factors are maintained constant. In the second option, variable partial safety factors are used.

The first option is acceptable if the variability of input parameters is relatively constant for all design situations, as in structural designs. In pile designs, the foundation engineer usually has some control over the reliability of the input parameters. He can obtain more reliable estimates of the soil parameters by carrying out more extensive site investigation and soil testing. Alternatively, he can reduce the uncertainty of the design method by performing pile load tests. Therefore, V_R can vary over a range of values depending on the efforts spent in reducing the level of uncertainty in the design. Because of this, the use of a constant strength reduction factor will not be effective in controlling the reliability level of pile designs. The partial safety factors can be obtained by calibration. The procedure is best illustrated by examples.

4.1 Example 1

In this example, the partial safety factors in (6) and (7) are obtained by calibration using the following parameters.

$V_R=0.25$ $\quad V_S=0.2$ $\quad \beta=3$

A reliability index of 3 is consistent with the reliability level of structural design. The nominal values are taken to be the mean values of the random variables, i.e. $\delta_R=\delta_S=1$. Based on these information, the following results can be obtained using the procedure described elsewhere (3).

$\alpha_R=-0.983$ $\quad \alpha_S=0.186$ $\quad \gamma_R=0.263$ $\quad \gamma_S=1.112$

The reliability index based on the above two options are compared. In the first option, the partial safety factors obtained from the above calibration are maintained constant for all designs. In the second option, a variable strength reduction factor is used. As discussed earlier, the stationary point in the Z-space is not sensitive to small changes in the variability of the design parameters, the value of α_R can be assumed to be constant for practical purposes. Using the value of α_R obtained from the above calibration procedure, the following relationship based on (8) can be used to determine the strength reduction factor for any given value of V_R.

$$\phi=(1+\alpha_R{\cdot}\beta{\cdot}V_R)/\delta_R=1-2.95{\cdot}V_R \quad (10)$$

Table 1 Variation of reliability index with V_R

V_R	0.15	0.2	0.25	0.3
β	2.76 (4.85)	2.96 (3.71)	3.00* (3.00)*	2.98 (2.51)

* Results from calibration

Table 1 compares the results for the two options. The values in brackets are the results based on a constant strength reduction factor. It can be observed that the use of a variable strength reduction factor gives a much better control of the reliability level.

4.2 Example 2

In practice, R and S may consist of more than one component. The ultimate capacity is the sum of the shaft resistance R_F and bearing resistance R_B and the applied load may consist of dead load S_D and live load S_L. Therefore, a more general model for ultimate failure of a pile can be written as follows.

$$G(X)=R-S=R_F+R_B-S_D-S_L \tag{11}$$

The limit state format for the design loading can be expressed as $\gamma\cdot(S_D'+S_L')$ or $\gamma_D\cdot S_D'+\gamma_L S_L'$, where γ, γ_D and γ_L are load factors. The second format is commonly used in structural codes and it is logical to follow the same format in geotechnical codes. For a particular type of structures, the variability and relative magnitude of the different components of S usually vary within a fairly narrow range. Therefore, the use of constant load factors are usually adequate.

Similarly, the limit state format for the design resistance can be written as $\gamma_R\cdot(R_F'+R_B')$ or $\gamma_F\cdot R_F'+\gamma_B R_B'$ where γ, γ_F and γ_B are strength reduction factors. As discussed earlier, the variability of the resistance can vary over a range of values. Furthermore, the relative magnitude of R_F and R_B can also vary significantly. For instance, the resistance of floating piles in sand is dominated by R_F and that of piles founded on a hard stratum is controlled by R_B. A variable strength reduction factor is therefore required to control the reliability level to a reasonably constant level. The following two formats are examined in more detail.

Format A:

$$\gamma_R\cdot R'\geq\gamma_D\cdot S_D'+\gamma_L\cdot S_L' \quad \text{or} \quad \gamma_R\cdot(R_F'+R_B')\geq\gamma_D\cdot S_D'+\gamma_L\cdot S_L' \tag{12}$$

Format B:

$$\gamma_F\cdot R_F'+\gamma_B\cdot R_B'\geq\gamma_D\cdot S_D'+\gamma_L\cdot S_L' \tag{13}$$

To examine the effectiveness of these two formats in controlling the reliability level, a sensitivity analysis is performed. Both formats are calibrated using the following parameters.

V_F=0.15 $\quad$ V_B=0.3 $\quad$ V_D=0.05 $\quad$ V_L=0.3

$\mu_F/(\mu_F+\mu_B)$=0.5 $\quad$ $\mu_L/(\mu_D+\mu_L)$=0.3 $\quad$ β=3

$R_F'=\mu_F$ $\quad$ $R_B'=\mu_B$ $\quad$ $S_D'=\mu_D$ $\quad$ $S_L'=\mu_L$

where V_F and μ_F denotes respectively the COV and mean value of R_F and so on. Again, the stationary point in the Z-space is chosen to be the checking point in the calibration. Once the direction cosines of the checking point are determined, the variable strength reduction factor for Format A can be determined using an equation identical to (8). The COV of R can be calculated using the following relationship.

$$V_R=\sqrt{r^2\cdot V_F^2+(1-r)^2\cdot V_B^2} \tag{14}$$

where $r=\mu_F/(\mu_F+\mu_B)$. If Format B is used, the strength reduction factor are determined from the following equations.

$$\gamma_F=1+\alpha_F\cdot\beta\cdot V_F \tag{15}$$

$$\gamma_B=1+\alpha_B\cdot\beta\cdot V_B \tag{16}$$

where α_F and α_B are the direction cosines for R_F and R_B obtained from calibration.

Fig.3 shows the sensitivity of the reliability index to changes in the ratio of $r=\mu_F/(\mu_F+\mu_B)$. Format A tends to give a better control of the reliability than Format B. A sensitivity analysis has also been performed to examine the variation of β with changes in the variability of R_F and R_B and the results are shown in Fig.4 for the two extreme cases of r=0 and r=1. Again, Format A is less sensitive to changes in V_F and V_B. In summary, Format A outmatches Format B. The former is adopted in the draft piling code.

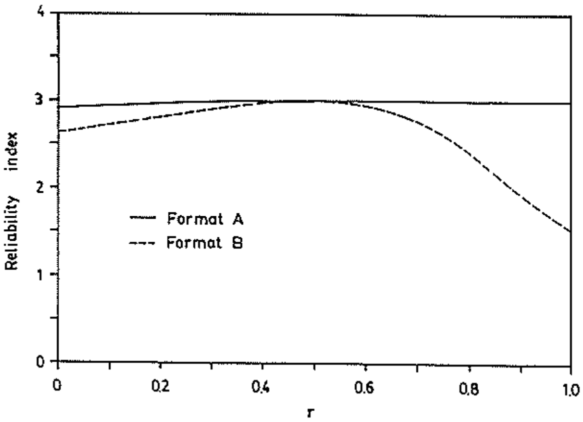

Fig.3 Variation of β with r

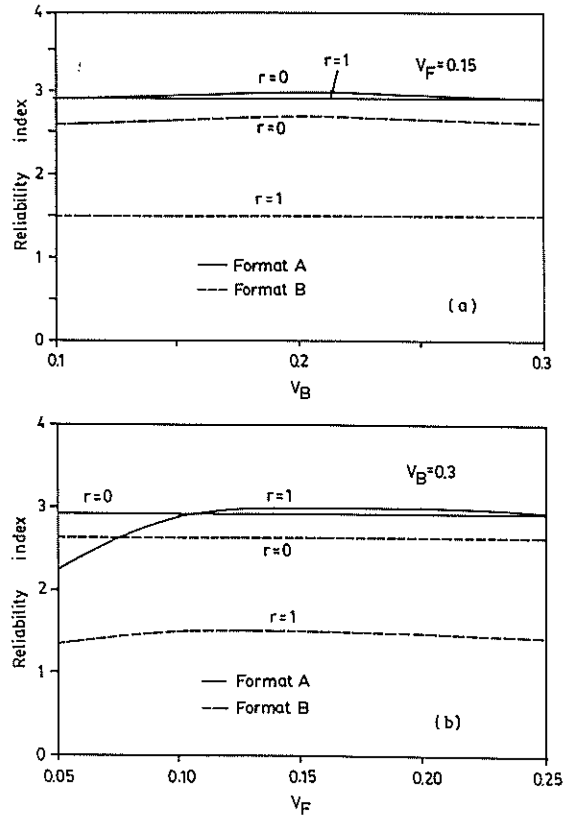

Fig.4 Variation of β with (a) V_B and (b) V_F

5. DISCUSSION

Since the above examples are based on assumed, although reasonable, values of input parameters, the partial safety factors presented in those examples should be taken as recommended values at this stage. However, the conclusions drawn from these examples should remain valid in relative terms.

The value of V_S=0.2 used in Example 1 is considered to be typical of conventional foundation loadings which are dominated by the dead load component and the value of V_R=0.25 also represents a typical level of confidence in pile design. The partial safety factors obtained from the calibration corresponds to an overall factor of safety of $F=\gamma_S/\gamma_R$=1.112/0.267=4.2. This value may seems to be high compared with that traditionally used. The discrepancy is mainly due to the fact that foundation designs are usually based on a nominal value of R' less than the mean value. For instance, the adhesion factor recommended in the current (1978) Australian piling code for piles in cohesive soils is based on more or less the lower bound values of the observed data. As a result, the nominal value R' based on this conservative estimate of adhesion factor will be significantly less than the mean value of R. δ_R is therefore less than unity and the minimum overall factor of safety based on a conservative nominal value will naturally be smaller in magnitude. This illustrates the important point that the partial safety factor will be influenced by the choice of nominal values.

Both Examples 1 and 2 clearly illustrate the need for using a variable strength reduction factor in pile design. This design philosophy has been stated although in an implicit manner in the current Australian piling code AS2159-1978 (and other foundation codes such as the British Code CP2004) because the designer is allowed to use a lower overall factor of safety depending on his judgement on the confidence level of the pile design. In the draft piling code, the designer is allowed to use a variable strength reduction factor depending on the level of certainty of the design parameters.

Example 2 suggests that it is not necessary and indeed not desirable to use separate strength reduction factors for the shaft and base resistance. The use of a single strength reduction factor is sufficient to give effective control on the reliability level provided a variable 'weighted' COV is used for V_R using (14). However, the requirement of linking the variable strength reduction factor to the variability of V_F, V_B and the ratio r is not evident in the draft piling code. From a theoretical standpoint, the level of confidence should ideally be quantified in terms of the variability of the ultimate resistance V_R. The probabilistic procedure for determining the variability of the shaft resistance V_F and that of the base resistance V_B is now well established (4),(5). The overall variability of the ultimate resistance can be obtained easily using (14). However, foundation engineers with little training in probabilistic analysis may still prefer to use a deterministic procedure for pile design. An alternative to expressing the level of confidence directly in terms of V_R will be to provide a series of tables from which the designer can obtain the confidence level for any particular type of pile design. For instance, using typical values of V_B and V_F for one type of pile design, the value of V_R and hence the confidence level can be directly related to the ratio of r using (14). A table can then be prepared to relate r to the confidence level for that particular type of pile design.

The current Australian piling code AS2159-1978 uses a lumped nominal load for design i.e. equal load factors are applied to various load components. For normal foundation loads which are dominated by the dead load, the use of equal load factors may be adequate. However, when the foundation loads are dominated by other environmental loads such as wind loads which tend to be more variable, the use of separately factored nominal loads (a format which is used in the draft piling code) will be necessary for good control of reliability level. Fig.5 shows the ratio of load factors for live load and dead load required to achieve a reliability index of 3 for different combinations of V_L and ratio of $\mu_L/(\mu_D+\mu_L)$. As the variability of live load and proportional of live load increase, a higher ratio of γ_L/γ_D is required. Hence, if γ_L/γ_D is to be maintained constant for simplicity, a 'average' ratio greater than unity is required.

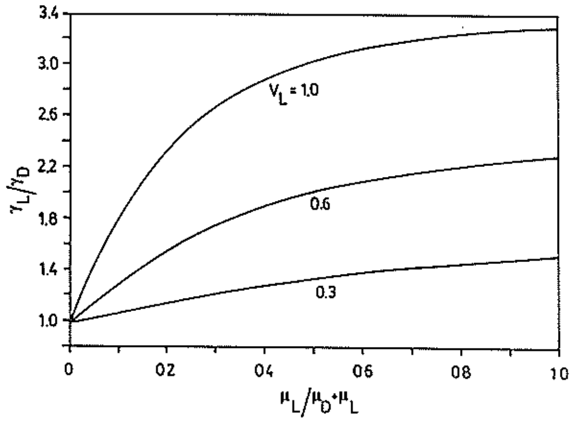

Fig.5 Variation of β with $\mu_L/(\mu_D+\mu_L)$

6. CONCLUSIONS

The probabilistic theory for the limit state method is reviewed. The partial safety factor in the limit state design can be deduced from a Level 2 probabilistic analysis using parameters that can be estimated from geotechnical data. It is observed that the use of a variable strength reduction factor is required to give effective control of the reliability level of pile designs. The limit state method provides formal procedure whereby the uncertainties involved in the design can be taken into account in the analysis. If more reliable data or design methods are used, the limit state method will allow a higher strength reduction factor and hence a less 'conservative' design.

The limit state format used by the draft piling code outmatches other limit state formats examined. A very consistent reliability level can be achieved provided that the strength reduction factor is properly correlated with the 'confidence level' of the design by a thorough statistical analysis of the relevant database. The range of dead to live load ratio for which a particular limit state format is applicable also needs to be specified.

To achieve a consistent reliability level, the confidence level of the pile capacity has to be assessed from the confidence levels of the shaft resistance and base resistance using a weighted averaging procedure as described mathematically by (14) which takes into account the relative magnitude of shaft resistance and base resistance. Such a procedure is not present in the draft piling code.

Further improvement in the control of reliability level can be achieved by using partial load factors which depend on the live to dead load ratio and characteristics of the live load.

The paper is a preliminary study on the limit state design of pile foundations. Further studies are needed and pursued by the authors.

7. ACKNOWLEDGMENT

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