Resonance Avoidance in Seismic Design

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SUMMARY In seismic design, the minimization of structural damage from an earthquake is assisted if positive steps for resonance avoidance for buildings and building elements are taken. Resonance avoidance refers to lack of coincidence between the dominant ground frequencies and the natural frquencies of building elements. This paper examines the assessment of dominant ground frequencies and natural frequencies of buidings, foundations, floors, beams and columns. Some approaches to modifying the natural frequencies of buildings and building elements to avoid possible resonance are discussed.

1. INTRODUCTION

Damage to buildings or building components from earthquakes is caused when the materials of construction cannot withstand the stresses imposed by the ground motions. A necessary but clearly insufficient condition that must be satisfied if damage is to be minimised is that the natural frequencies of buildings or building components should be separated from the dominant ground motion frequencies in the earthquake.

An increasing amount of information is now becoming available regarding the magnitudes of the dominant ground frequencies at particular sites. For many sites the dominant frequencies lie between 1 and 5 hertz. With overlying alluvium it is possible, in the absence of records, to estimate the dominant ground frequencies from a knowledge of the stiffness and depth of the alluvium. Calculations indicate that the fundamental natural frequency of the ground in cases where the alluvium becomes stiffer with depth is likely to be greater than 1 hertz. Lower natural ground frequencies may occur with soft alluvium of constant stiffness.

Expressions for natural frequencies of whole buildings in terms of building height show that it is likely that coincidence between the natural frequencies of some buildings and the ground will occur. If this information can be made site specific then useful guidance on undesirable building heights may be obtained.

With raft and footing foundations resonance effects may be encountered particularly with soft ground. The foundation natural frequency rises as the ground becomes stiffer and it may be varied by altering the foundation dimensions and foundation loads.

Natural frequencies of floors may also fall within the range of commonly encountered dominant ground frequencies from earthquakes. Some control may be exercised over the natural frequencies of floors by varying the floor dimensions and support conditions at the edges.

Compared with other building components, beams and columns are less likely to suffer from resonance effects since the natural frequencies of these components are often much greater than the dominant ground frequencies.

2. DOMINANT GROUND FREQUENCIES

In the absence of records the dominant ground frequency at a site may be estimated from the calculated natural frequency of ground response resulting from the deep seated movement of the underlying strata. for the case of a homogeneous stratum overlying the basal rock the fundamental natural frequency (f_n) is given by (Richart et al, 1970)

$$f_n = v_s / 4L \text{ hertz} \qquad (1)$$

where v_s = shear wave velocity of the stratum
L = stratum thickness

Fig. 1 illustrates that natural frequencies may be less than 1 hertz for large layer thicknesses of soft to medium ground overlying the rock. For non-homogeneous ground for which the shear modulus increases with depth, Moore (1987) has shown that the fundamental natural frequency depends dominantly on layer thickness (L) and rate of increase of shear modulus with depth (m). The shear modulus (G_z) at any depth (z) is represented by

$$G_z = G_o + mz \qquad (2)$$

where G_o = shear modulus at ground surface

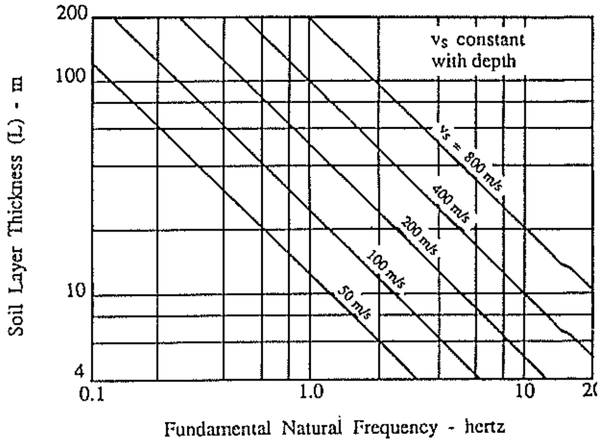


Fig. 1 Natural Frequency for Constant Shear Modulus Case

Fig. 2 indicates the effect of parameter m on the natural frequency. It may also be shown for particular values of G_0 and m that the natural frequency increases as the layer thickness decreases.

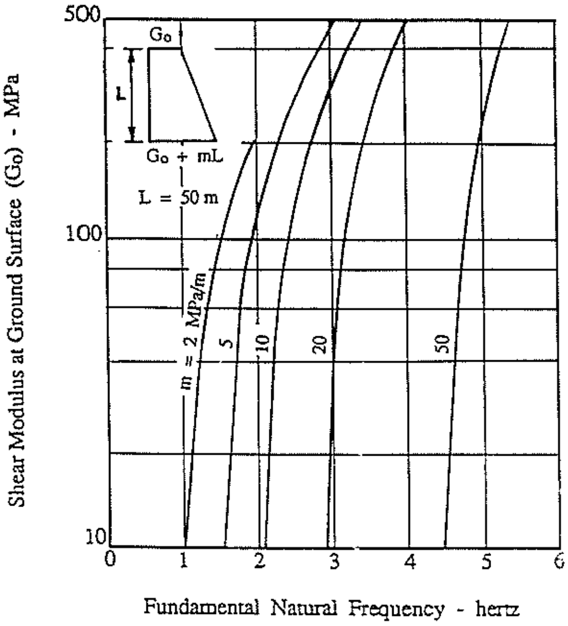


Fig. 2 Effect of Shear Modulus on Natural Frequency

3. NATURAL FREQUENCIES OF BUILDINGS

A number of simple empirical formulae have been proposed for the estimation of the natural frequency of a building for the fundamental transverse mode (mode 1) of vibration. Anderson et al (1952) have examined the popular formula for the fundamental period (T) of vibration

$$T = K\, H/D^{1/2} \text{ sec} \qquad (3)$$

where H = building height (m), and
D = depth of building (m) parallel to the direction of vibration

Based upon a large number of observations by the authors and by Salvadori and Heer (1960) it appears that the constant K should be approximately 0.1. Housner and Brady (1963) examined a number of formulae, among the simplest being

$$T = 0.1\, N \text{ sec} \qquad (4)$$

for rigid frame buildings, and

$$T = (0.5N^{1/2} - 0.4) \text{ sec} \qquad (5)$$

for steel frame buildings, where N = number of floors in the building.

More recently, Ellis (1980) has examined the accuracy of some of these simple formulae and he proposed that a slightly better expression for natural frequency (f_n) is

$$f_n = 46/H \text{ hertz} \qquad (6)$$

While errors of ±50% were not uncommon in the prediction of natural frequency, he found that simple formulae, such as equation (6), were likely to be as accurate as computer based predictions.

4. RESPONSE OF FOUNDATIONS

In examining the response of shallow footing and raft foundations to horizontal ground vibrations the ground is often idealized as an elastic solid with the stiffness being given as (Bycroft, 1956)

$$k = 32\,(1-\upsilon)\, Gr/(7-8\upsilon) \qquad (7)$$

where G and υ are the shear modulus and Poisson's ratio for the ground
r is the radius of the footing

An alternative and perhaps more relevant expression for rectangular footings is that quoted by Barkan (1962)

$$k = 2(1+\upsilon)\, G\beta_x\, (BL)^{1/2} \qquad (8)$$

where β_x is a factor depending on foundation shape.
B and L are footing dimensions

For square footings (B x B) equation (8) simplifies to

$$k = 2.4\, GB \qquad (9)$$

for a Poisson's ratio of 0.25

From equation (9) the natural frequency of horizontal vibration for the footing may be derived to yield

$$f_n = 0.8\, (G/\sigma B)^{1/2} \qquad (10)$$

where σ is the average bearing pressure acting on the footing.

Fig. 3 illustrates the way in which the natural frequency decreases as the bearing pressure increases or as the footing size increases. If normal foundation engineering design practice is followed then the bearing pressure should increase as the ground becomes stiffer. This can be allowed for by specifying a linear relationship between G and σ. If this is incorporated into equation (10) then it is obvious that the natural frequency becomes dependent on the width of the foundation only. This is shown in Fig. 4 for a typical range of foundation soils that are likely to be encountered. Fig. 4 indicates that the likely range of natural frequencies for shallow foundations is somewhat narrower than might be inferred from a study of Fig. 3.

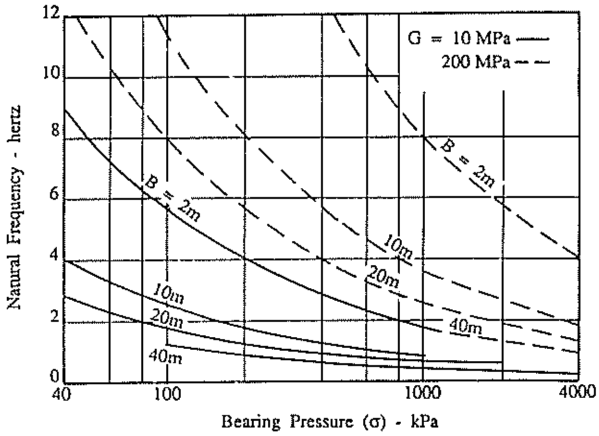


Fig. 3 Effect of Bearing Pressure and Footing Width on Natural Frequency

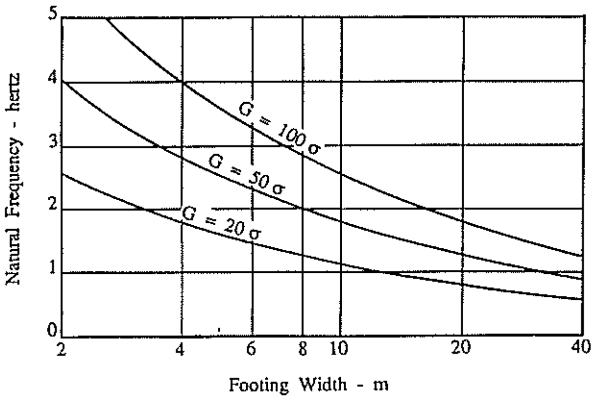


Fig. 4 Effect of Footing Width on Natural Frequency
- Shear Modulus/Bearing Pressure Relationships Included

For lightly loaded stiffened slabs, common in residential development, the use of equation (8) to evaluate horiontal soil stiffness may lead to a significant inderestimate of the natural frequency of the footing. As for vertical and torsional modes of vibration the horizontal stiffness increases as the depth to a rigid stratum underlying the elastic layer decreases.

Fig. 5, which was derived from a three-dimensinal finite element analysis, shows the effect of layer thickness on the natural frequency for a slab designed in accordance with Australian Standard 2870: Residential Slabs and Footings - the slab type chosen was for articulated full masonry construction placed on a moderetely reactive clay site.

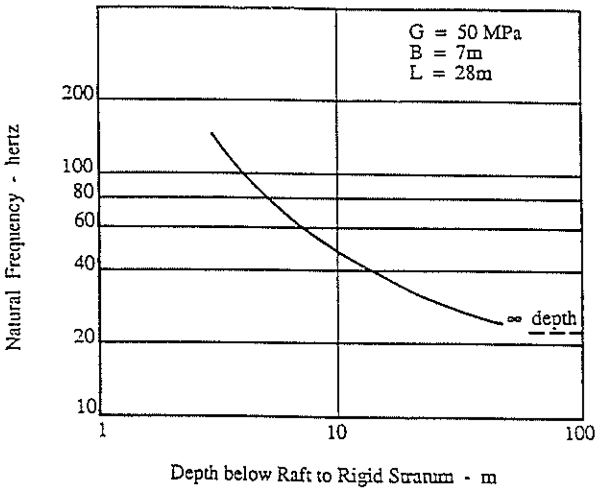


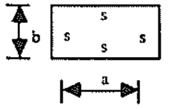
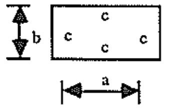
Fig. 5 Influence of Depth to Rigid Stratum
on Natural Frequency of Raft Footing

5. RESPONSE OF SLABS, BEAMS AND COLUMNS

Where a simple element is to be analysed or designed, tabulations of natural frequency for various support conditions and geometry are available for both beams and plates (eg. Stokey, 1976). An example of such a tabulation is given in Table 1. Where more information than the frequency of the first mode of vibration for a beam is required, approximate solutions, precise solutions involving frequency functions (e.g. Kohoutek, 1985) or finite element approaches need to be considered.

TABLE 1

FUNDAMENTAL NATURAL FREQUENCIES
FOR RECTANGULAR PLATES (from Stokey, 1976)

	b/a	1.0	2.0	∞
	$\omega_n/ (D/\rho h a^4)^{1/2}$	19.74	12.34	9.87
	b/a	1.0	2.0	∞
	$\omega_n/(D/\rho h a^4)^{1/2}$	35.98	24.57	22.37

$D = Eh^3/12(1-\upsilon^2)$
h = plate thickness
ρ = plate density
s = simply supported edge
c = clampled edge
a,b = length, width of plate

A complete analysis for natural frequencies and related deformations of a multi-storey building will usually require a three-dimensional finite element study. Factors which need to be considered include:

(a) Aspect Ratio (length : width) - particularly when large, dominant deformation under seismic loading is in the length direction;

(b) Column/Beam Panel in-fill material - if rigid in-fill (brick) is integrated with the beams/columns, natural frequency of the building increases;

(c) Rigidity of connexions;

(d) Rigidity of base and rotational stiffness of foundation;

(e) Disposition of lift and stair cores - particularly if eccentric will promote a torsional rather than lateral sway mode of vibration.

The response of a five-storey reinforced concrete framed building is given as an example of the stiffening effect of brick in-fill.

Overall Dimensions (L x W x H) (m)	54.1	22.2	18.4
Floors cast integrally with frame			
All connexions assumed rigid			
Natural Frequency (f_{n1}, f_{n2}, f_{n3}) (H_z)			
Considering Brick In-Fill	2.24,	2.45,	2.62
Ignoring Brick In-Fill	1.91,	2.19,	2.55

6. CONCLUDING REMARKS

For shallow footing and raft foundations located on deep deposits of soft to medium stiff soil, the natural frequency of the footing may coincide with the fundamental frequency of the soil deposit. The natural frequency of the footing decreases as the footing width increases.

It is unlikely that the natural frequencies of individual beams and columns in a building would coincide with dominant natural frequencies experienced in an earthquake. However, for some buildings, coincidence between the natural frequency for the building as a whole and the ground motion will occur. The fundamental natural frequency for whole buildings can be estimated approximately using simple empirical formulae.

7. REFERENCES

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