The Effect of Auto-correlation on the Probability of Failure of Slopes

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SUMMARY Probabilistic analyses of slopes are generally based on treating only the most significant variables as random variables. Usually simplifications are made regarding the correlation structure of these variables. This is especially so with respect to the correlation of each variable with itself in space (i.e. auto-correlation). Most analyses ignore this auto-correlation. The paper presents several cases from the literature for which probabilities of failure have been determined where auto-correlation has been ignored. These are re-analysed and the sensitivity of the computed probability of failure to various assumptions regarding the auto-correlation structure is determined. It is found that, in general, realistic assumptions regarding auto-correlation result in very significant reductions in the computed probabilities of failure for most civil engineering slopes. Thus most methods of probabilistic slope analysis can at best be regarded as producing only an index of slope stability.

1 INTRODUCTION

In the last twenty years many methods have been developed to determine the probability of failure of a slope, many of these are discussed in Mostyn and Small (1), but there is no single method that has attained the degree of acceptance as, say, Bishop's method has for deterministic analysis of slopes. For this reason it is difficult to assign acceptable limits to probabilities of failure to assist with decision making. This paper presents the results of an investigation into the effect of auto-correlation on the computed probabilities of failure for some slopes that have been analysed by various probabilistic methods.

2 PROBABILISTIC METHODS OF SLOPE ANALYSIS

The conventional method to assess the stability of a slope is to determine the factor of safety, FOS, by one of several generally accepted methods, such methods are described in almost every geotechnical engineering textbook and do not need to be further described here. The factor of safety has several drawbacks which are described by Li and Lumb (2).

An alternate method of analysis of a slope is to determine the probability of failure and use this to assess the stability or decide on further action. Acceptance of probabilistic methods has been slow, Chowdhury (3) suggests three reasons why this may be so. First, the results are not readily used by engineers; second, extra data is required to statistically describe the soil; and third, there is no agreement as to what probability of failure constitutes an unsafe slope.

Most of the methods in use to determine the probability of failure, P_f, of a slope treat the slope material as layers with given means, μ, and standard deviations, σ, for the strength parameters, generally the cohesion, c, and angle of friction, ϕ. While sometimes the correlation of c with ϕ is taken into account, it is very rare for the correlation of each of these parameters with itself in space to be taken into account, this property is called auto-correlation. Thus most methods of probabilistic slope analysis assume that a random variable, e.g. ϕ, is perfectly correlated with itself over infinite distances, so that each realisation consists of layers of soil of fixed strength.

Auto-correlation is reflected in the fact that the property of a soil within a particular layer will vary in space, but the value of this property at two points close together is likely to be similar, while the values for points remote from each other are likely to be independent of each other. Probabilistic analyses are often undertaken with the unstated assumption of perfect auto-correlation, this would only be acceptable if the resulting probabilities were similar to those obtained from analyses taking the actual auto-correlation structure into consideration.

The assumption of perfect auto-correlation allows the material property to be sometimes either very high or low throughout the layer, auto-correlation on a realistic scale acts to reduce the variance and thus these homogenous outriders are less likely.

There are several methods of describing auto-correlation, one method is to add a third parameter, the scale of fluctuation, δ, to the statistical description of the layer. This parameter is a measure of the distance within which the soil property shows relatively strong correlation with itself. In other words, the values of the parameter lying within a distance δ of each other are both likely to be either above or below the mean. Thus a small δ implies rapid fluctuations about the mean and a large reduction in variance over any failure plane; this results in a small "spread" of the performance function and for factors of safety greater than unity a smaller P_f. Conversely a large δ means much longer variations about the mean.

The concept can be extended to three dimensions, in which case δ_x, δ_y and δ_z are used. Li and White (4) have examined the literature and indicate that δ_x and δ_y are often about an order of magnitude larger than δ_z and are often of the order of 3 to 30m.

Various funtions have been used to model auto-correlation, these are generally of an exponential form and four have been described by Li and White (4).

3 PROGRAM PROBSN

Li has written a program, PROBSN, which uses a first order second moment, FOSM, method to determine the probability of failure of a slope using the generalised procedure of slices, GPS, method of analysis. Non circular and circular surfaces can be analysed and the method is rigorous in that it satisfies both force and moment equilibrium. The performance function adopted in the analysis is not the factor of safety but the safety margin, as this is more likely to be Normally distributed. The reliability of the critical surface is determined as the Hasofer-Lind reliability index, β_{HL}, described by Hasofer and Lind (5). PROBSN was ported to personal computers by Waddell (6) and Furrer & Steele (7). The resulting code contained several errors that were detected and corrected by Soo (8).

4 CASE STUDIES

In order to examine the influence of autocorrelation on the values determined for the probability of failure, several cases that had been analysed and then published in the literature were re-analysed taking into account auto-correlation. The first two cases also formed a check on the accuracy of PROBSN.

4.1 Case 1: Catalan & Cornell

Catalan and Cornell (9) used a level crossing method to determine the probability of failure of a slope, the analyses was based on a circular failure surface using the simplified Bishop method and did take account of auto-correlation.

The slope analysed was a uniform slope over a rigid base, 11 m high with a face angle of approximately 26 degrees. The slope was comprised of material with an undrained shear strength of 33.5 kPa and unit weight of 16.5 kN/m^3. The reported value of the FOS was 1.53 and equalled that obtained by PROBSN. The analyses were completed with coefficients of variation (V_s, i.e. standard deviation divided by the mean) on the undrained shear strength of 0.2 and 0.4.

The results of the analyses run by Catalan and Cornell and those obtained using PROBSN are given on Table I.

TABLE I
P_f FOR CASE 1

δ_x	δ_y	P_f PROBSN	P_f Reported
$V_s = 0.2$			
10000	100	3.9E-2	**
91.5	3.05	1.6E-3	7.0E-3
30.5	1.53	2.0E-4	1.0E-3
3.05	1.53	1.0E-6	1.0E-4
1.0	0.01	4.0E-8	**
$V_s = 0.4$			
10000	100	1.9E-1	**
100	1.0	3.2E-2	**
30.5	1.53	3.7E-2	6.0E-2
3.05	1.53	1.5E-2	9.0E-3
1.0	0.01	3.5E-3	**

** Results not reported

The differences between the values of P_f derived by PROBSN and those reported by Catalan and Cornell are probably due to the fact that the relationship between the scale of fluctuation, δ, in PROBSN and the correlation distances used in the refernce had to be estimated. Equally parts of the slope geometry were estimated. As expected these matters and assumptions regarding the distribution of the performance function can have a large effect on estimation of small failure probabilities. Notwithstanding these matters the broad agreement for the larger values of P_f is reasonable. In addition the rapid reduction in P_f with a decrease in δ is very apparent.

4.2 Case 2: Alonso

Alonso (10) analysed the Green Creek slide and included auto-correlation in his analysis. The slope analysed is shown on Figure 1, the critical circle was the conventional minimum FOS circle obtained using Bishop's method. The correlation distance was assumed to be the same in all directions. Slope material properties were (mean & standard deviation): cohesion of 30 & 9 kPa, co-efficient of friction of 0.34 & 0.052, density of 16 and 0.4 kN/m^3. Alonso obtained an FOS of 1.40 compared with 1.38 from PROBSN. The results of the analyses completed by Alonso and those obtained using PROBSN are given on Table II.

TABLE II
RESULTS FOR CASE 2

δ	c = 30 kPa		c = 15 kPa
	P_f PROBSN	P_f Reported	P_f PROBSN
10000	0.11	**	0.77
1000	0.084	**	0.78
100	0.067	**	0.82
10	0.0056	**	0.92
8	0.0043	0.008	0.93
4	0.0023	0.004	0.94
1	0.0009	0.001	0.95
0.1	0.0008	0.001	0.95
0.01	0.0008	0.001	0.95

** Not reported in reference

As can be seen from Table II there is reasonable agreement between the estimates of P_f for the two methods. In addition P_f reduces rapidly with reductions in δ, this behaviour is shown on Figure 2.

Cases 1 and 2 provide some support for the accuracy of PROBSN in that there is broad agreement with the few reported cases from the literature that have included auto-correlation in probabilistic slope analysis. The discrepancies at small P_f would be expected with almost any probabilistic analysis, but the broad agreement at higher P_f is encouraging.

All the cases analysed in the literature had estimated probabilities of failure less than 0.5 (i.e. FOS greater than

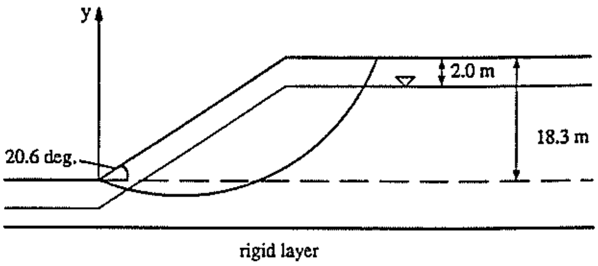

Figure 1 Slope adopted in Case 2

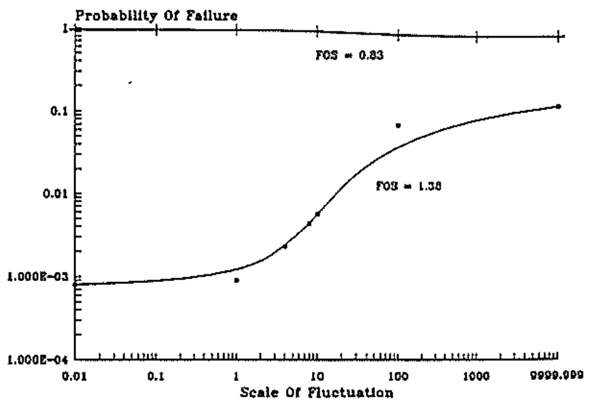

Figure 2 P_f versus δ for Case 2

one). In order to investigate the effect of autocorrelation on high values of P_f the cohesion in Case 2 was reduced to 15 kPa, this resulted in a reduction in the FOS to 0.83 and the P_f values are contained in the fourth column of Table II. It can be seen that P_f increases with a reduction in δ, as the resulting reduction in variance now acts to draw more of the realisations to failure conditions.

4.3 Case 3: Chowdhury (11)

Chowdhury (11) analysed a slope as shown in Figure 3 comprised of materials with (mean & coefficient of variation) a cohesion of 30 kPa and 30%, angle of friction of 20 degrees and 10% and unit weight of 18.8 kN/m^3 and zero. The analysis was based on circular failure surfaces and the simplified Bishop method with the critical circle being that with the minimum FOS. Auto-correlation was not considered by Chowdhury. The reported value of FOS was 1.36 with a P_f of 2.3E-2, this should be compared with the FOS obtained from PROBSN of 1.40 and the probabilities of failure given in Table III as a function of the scales of fluctuation.

TABLE III
P_f FOR CASE 3

δ_x (m)	δ_y (m)	P_f
10000	10000	1.9E-2
100	10	1.3E-3
10	10	6.6E-5
10	1	2.0E-5
1	1	8.4E-6
0.001	0.001	6.0E-6

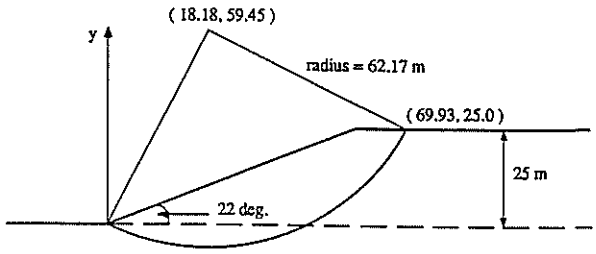

Figure 3 Slope adopted in Case 3

Here it can be seen that there is good agreement between the output of PROBSN for δ equal to 10000m and the results from the literature which were obtained ignoring auto-correlation but these P_f are one to three orders of magnitude too high if real scales of fluctuations of between 1 and 100 m in the horizontal plane are taken into account.

4.4 Case 4: Chowdhury (12)

Chowdhury (12) developed a probabilistic analysis of progressive failure, the slope analysed is shown in Figure 4 comprised of materials with a mean cohesion of 23.94 kPa and a co-efficient of variation of 50%, an angle of friction of 38 degrees and unit weight of 18.8 kN/m^3. Analyses were completed with the coefficient of variation of the angle of friction (V_ϕ) varying from 0.1 to 0.5. The analysis was based on circular failure surfaces and the simplified Bishop method. Auto-correlation was not considered. The FOS obtained from PROBSN was 2.41 for circle A. The probabilities of failure as functions of V_ϕ and δ are given in Table IV.

TABLE IV
P_f FOR CASE 4

δ_x	δ_y	P_f for $V_\phi = 0.10$	P_f for $V_\phi = 0.50$
Chowdhury			
∞	∞	8.4E-3	3.3E-2
PROBSN			
10000	10000	8.9E-3	2.7E-2
10000	100	7.8E-3	2.6E-2
100	10	1.9E-3	1.2E-2
100	1	8.2E-7	1.2E-4
10	1	2.4E-7	6.0E-5
0.001	0.001	~ 0	6.0E-8

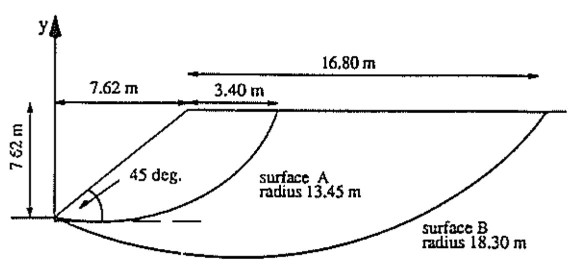

Figure 4 Slope adopted in Case 4

It can be seen that there is good agreement between Chowdhury and PROBSN when δ is large relative to the slope but as δ approaches realistic values then P_f reduces by several orders of magnitude.

4.5 Case 5: D'Andrea & Sangrey

D'Andrea and Sangrey (13) analysed the slope shown on Figure 5 using conventional critical slip circle, simplified Bishop analysis. Auto-correlation was not considered. The slope was comprised of material with a mean cohesion of 24.75 kPa and unit weight of 16.33 kn/m^3. The coefficient of variation of the cohesion (V_c) was varied between 0.05 and 0.35, a comparison of the results obtained by PROBSN with the referenced paper for "no auto-correlation" is given in Table V.

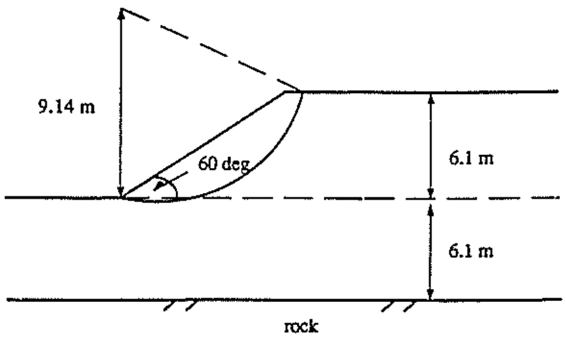

Figure 5 Slope adopted in Case 5

TABLE V
COMPARISON OF P_f FOR CASE 5

V_c	P_f PROBSN *	P_f Reported
0.05	0.021	0.054
0.15	0.112	0.136
0.25	0.210	0.221
0.35	0.277	0.281

* Determined with $\delta_x = \delta_y = 10000$

These results are in good agreement, especially for the larger P_f.

The effects of auto-correlation on the probabilities of failure as functions of V_c and δ are given in Table VI.

TABLE VI
P_f FOR CASE 5

δ_x	δ_y	P_f for $V_c = 0.05$	P_f for $V_c = 0.35$
10000	100	0.021	0.27
1000	10	0.018	0.24
100	10	0.016	0.24
10	1	0.007	0.12
1	0.01	0.004	0.055
0.001	0.001	0.004	0.052

As seen above there is good agreement between D'Andrea & Sangrey and PROBSN when δ is large relative to the slope but as δ approaches realistic values then P_f reduces five fold. This is a smaller drop than for some of the other cases as P_f is larger and the greatest effects are in the tails of the probability curves. It should also be noted that this slope has a FOS of 1.3, but using the common methods of probabilistic slope analysis (i.e. ignoring auto-correlation) has a P_f of between 2 and 27% depending on the variance of the cohesion. These are large P_f for an "apparently" stable slope.

If the strength is reduced so that the FOS falls below unity then the effect of δ on P_f is much reduced. For a cohesion of 18 kPa, the FOS is 0.93 and the P_f varies between 57 and 71 percent depending on auto-correlation and the coefficient of variation for the strength.

5 CONCLUSIONS

A summary of the comparisons contained in the previous sections is presented on Figure 6. The horizontal axis is the P_f obtained by PROBSN assuming perfect auto-correlation (i.e. δ=10000m) for the different slopes. The vertical axis gives the P_f for the same slope when δ_x is set at 10m, some of these had to be interpolated from other values of δ_x. PROBSN was used even for the perfect auto-correlation cases to eliminate any variation between different analyses.

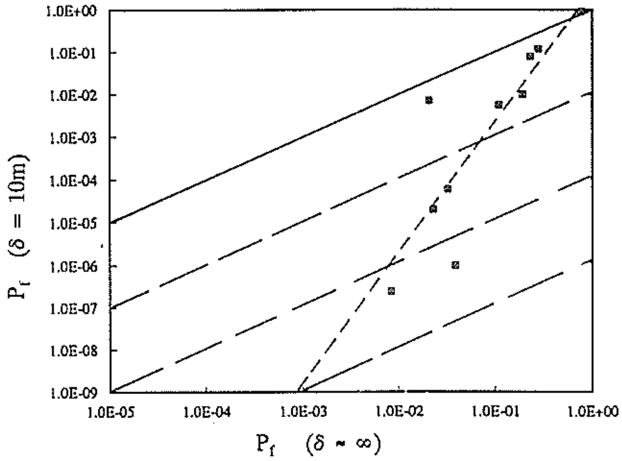

Figure 6 Effect of auto-correlation on P_f

A number of features evident on this figure are:

- The solid line represents equality between the two analyses, the broken lines below this are contours at ratios of two orders of magnitude between the answers obtained with and without auto-correlation.
- For a P_f of 50% there is no difference between the answers as a reduction in variance does not change P_f but only a change in the mean of the performance function would.
- Above a P_f of 50% ignoring auto-correlation is not conservative, this is not really significant as P_f values of this magnitude will always present a problem.
- As the P_f decreases the ratio between the two results increases rapidly, from unity at P_f equal to 50% to 10^5 for P_f ($\delta=\infty$) equal to 1%.
- A line of best fit to the logrithmic data is shown as a dashed line. It can be seen from this that for realistic values of P_f ($\delta=\infty$) for civil engineering structures, say 10 to 0.1%, there is between two and

six orders of magnitude difference between the results generally reported and the more realistic results obtained taking account of auto-correlation. These artificially high values of P_f are likely to explain some practising engineers reluctance to adopt probabilistic analyses.

* The results are monotonically increasing preserving rank of P_f on both axes, thus results of analysis based on perfect auto-correlation can be used as an index of stability (and may be better than FOS).
* Even though the results obtained ignoring auto-correlation are conservative, the fact that they are several orders of magnitude in error indicates that they should not be used in economic analysis but only as an index.

It should be noted that for P_f lower than about 1% other factors such as assumed distribution of the performance function, method of analysis, etc can also have large effects on the derived P_f.

6 ACKNOWLEDGEMENTS

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