# Insights from Soil Engineering Reliability-Based Design to Improve Partial Factor Design Approach

B. K. Low

School of Civil and Environmental Engineering.
Nanyang Technological University, Singapore.
E-mail: BKLow@alum.mit.edu

**Abstract:** Insights from reliability-based design (RBD) are presented for three soil engineering problems, namely (i) a spread footing subjected to both vertical and horizontal loads, (ii) an anchored sheet pile wall in which the load effects and resistance are nonlinear functions of soil shear strength, unit weight, geometry, and parametric correlations, and (iii) laterally loaded piles with depth-dependent nonlinear p-y curves and different cantilever lengths. The aim is to show how RBD via the first-order reliability method (FORM) can overcome some of the limitations and ambiguities of the partial factor design approach when applied to soil engineering. The reliability analysis and RBD are discussed with respect to parametric correlations and sensitivity information revealed by the design point in FORM. The differences and similarities between the design point in RBD and that in Eurocode 7 are discussed. The ability of RBD-via-FORM to provide interesting information at its design point and to automatically reflect parametric uncertainties, correlations, and case-specific sensitivities are demonstrated. It is shown that reliability analysis and RBD-via-FORM can improve partial factor design approach when (a) partial factors for less commonly used soil engineering parameters are not yet covered in Eurocode 7 or the Load and Resistance Factor Design (LRFD) approach; (b) the sensitivities of parameters vary from case to case; (c) realistic considerations warrant correlation among parameters; (d) a resistance or action parameter possesses stabilizing-destabilizing duality; (e) different target reliability index values are aimed at to reflect different consequence of failure.

Keywords: Reliability-based design; first-order reliability method; spread footing; retaining wall; laterally loaded pile; Eurocode 7; LRFD.

## 1 Introduction

This paper studies the similarities and differences between the design point of the first-order reliability method (Fig. 1(a)) and that of the partial factor design approach (e.g. Eurocode 7 or EC7, Load and Resistance Factor design or LRFD), Fig. 1(b). Reliability analysis and reliability-based design (RBD) via the first-order reliability method (FORM) is used in this paper. It can overcome some limitations and ambiguities in the EC7 and LRFD approaches, for example in situations where loads contribute not only destabilizing but also stabilizing effects, situations where load and resistance sensitivities vary due to different geometric and other details, situations with spatial variability and parametric correlations, and situations with different targets of probability of failure. Examples of RBD are presented to show that under these circumstances RBD can offer interesting insights apart from providing valuable guidance and caveats on the partial factors in the evolving EC7 and LRFD. Specifically, this study investigates RBD of (i) a shallow footing subjected to both vertical and horizontal loads, (ii) an

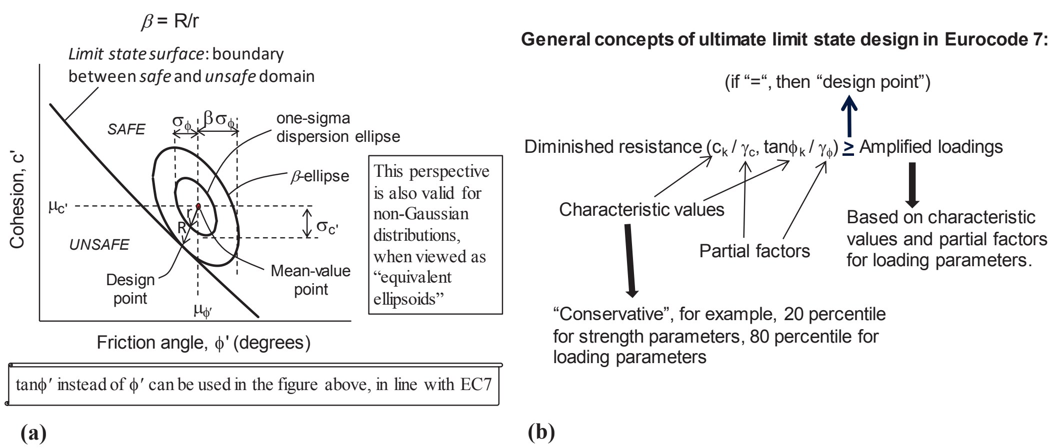



**Figure 1.** (a) FORM design point reflects parametric correlations and context-sensitivities; (b) EC 7 design point based on conservative characteristic values and code-specified partial factors.

*Proceedings of the 7th International Symposium on Geotechnical Safety and Risk (ISGSR)*
*Editors:* Jianye Ching, Dian-Qing Li and Jie Zhang

*Published by* Research Publishing, Singapore.
ISBN: 978-981-11-2725-0; doi:10.3850/978-981-11-2725-0_IS1-14-cd

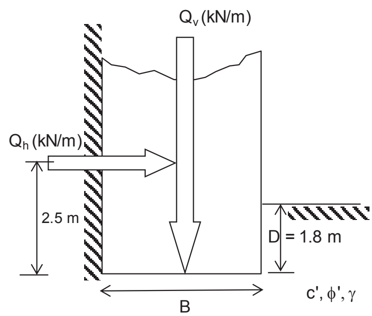
anchored sheet pile wall in which the load effects and resistance are nonlinear functions of soil shear strength, unit weight, geometry and parametric correlations, and (iii) laterally loaded piles with depth dependent nonlinear p-y curves and different cantilever lengths. It is shown that RBD can automatically reveal critical design scenarios and obtain design values of loads and resistance which reflect different parametric sensitivities across different scenarios, and resolves stabilizing-destabilizing duality. Focus is on insights and how RBD can complement EC7 and LRFD.

The reliability approach used here is the first-order reliability method (FORM) which can deal with correlated non-Gaussian random variables. A special case of FORM is the earlier Hasofer-Lind index for correlated Gaussian random variables. The classical mathematically intricate u-space approach for the FORM and the Hasofer-Lind method is described in Ang and Tang (1984), Haldar and Mahadevan (1999), Baecher and Christian (2003), for example. In addition, Low and Tang (2007) presented practical procedures for FORM-based reliability analysis and reliability-based design (RBD), which obtain the same results as the classical u-space approach but are much more transparent than the latter. Response surface method can be used to bridge standalone numerical software with FORM analysis, e.g. Chan and Low (2012).

The useful insights and information from FORM-based RBD may not be obtainable in other probabilistic approaches like the first-order second-moment (FOSM) method and the Monte Carlo simulation method.

## 2 Stabilizing-Destabilizing Duality and Context-Sensitivity Automatically Resolved in RBD-via-FORM

The retaining wall in Fig. 2 is similar to the deterministic Example 2.2 in Tomlinson (2001), which computed a $F_s$ of 3.0 against general shear failure of the soil below the base of the wall when base width B is 5 m. For the RBD-via-FORM in this section, Tomlinson's deterministic inputs for c′, tanϕ′, $Q_h$ and $Q_v$, namely 15 kPa, 0.47, 300 kN/m and 1100 kN/m, respectively, are used as mean values. For illustrative purpose, the coefficients of variation of c', tanϕ′, $Q_h$ and $Q_v$ are 0.2, 0.1, 0.15 and 0.1, respectively, yielding the standard deviations as shown in the figure. The $Q_h$ and $Q_v$ are assumed to be uncorrelated, as befitting the horizontal earth thrust and




$$q_u = cN_c s_c d_c i_c + p_o N_q s_q d_q i_q + \frac{B'}{2}\gamma N_\gamma s_\gamma d_\gamma i_\gamma$$

| Distr.Name | | x* | μ mean | σ StDev | n | Correlation matrix R | | | | $u = L^{-1}n$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Lognormal | c′ | 15.03 | 15 | 3 | 0.108 | 1 | -0.5 | 0 | 0 | 0.11 |
| Lognormal | tanϕ′ | 0.408 | 0.47 | 0.047 | -1.375 | -0.5 | 1 | 0 | 0 | -1.52 |
| Lognormal | $Q_h$/m | 429.8 | 300 | 45 | 2.485 | 0 | 0 | 1 | 0 | 2.49 |
| Lognormal | $Q_v$/m | 1020 | 1100 | 110 | -0.71 | 0 | 0 | 0 | 1 | -0.71 |

| B | L | D | γ |
|---|---|---|---|
| 4.55 | 25 | 1.8 | 21 |

| $c_a$ | $p_o$ | Ω |
|---|---|---|
| 12.02 | 37.8 | 22.86 |

| $e_B$ | B' | $e_L$ | L' | q | $q_u(x^*)$ |
|---|---|---|---|---|---|
| 1.054 | 2.442 | 0 | 25 | 417.5 | 417.5 |

| | |
|---|---|
| $N_q$ | 7.967 |
| $N_c$ | 17.09 |
| $N_\gamma$ | 7.313 |

| $s_q$ | $s_c$ | $s_\gamma$ | $i_q$ | $i_c$ | $i_\gamma$ |
|---|---|---|---|---|---|
| 1.021 | 1.024 | 0.987 | 0.578 | 0.518 | 0.324 |

| $d_q$ | $d_c$ | $d_\gamma$ |
|---|---|---|
| 1.125 | 1.158 | 1 |

| PerFunc g(**x**) | β |
|---|---|
| 0.00 | 3.00 |

**Figure 2.** RBD-via-FORM for the width B of a spread footing. The vertical load $Q_v$ possesses stabilizing-destabilizing duality when horizontal load $Q_h$ is acting.

the applied vertical load. The uncertainty in tanϕ′ is modelled instead of ϕ′, in line with EC7 which applies partial factor to tanϕ′. Negative correlation between c' and tanϕ′ is modelled, as shown by $\rho_{c,\tan\phi} = -0.5$ in the correlation matrix.

For the statistical inputs shown, a base width B of 4.55 m is required to achieve a target reliability index of β = 3.0 against bearing capacity failure. The design value of c′, 15.03 kPa, is slightly above the c′ mean value of 15 kPa, due to negative correlation coefficient of -0.5 between c′ and tanϕ′. For the case in hand, the design is much more sensitive to $Q_h$ than to $Q_v$, with n values 2.49 versus –0.71, and much more sensitive to tanϕ′ than c′, with n values -1.38 versus 0.11, where $n = (x^* - \mu^N)/\sigma^N$, in which superscript N denotes equivalent normal mean and equivalent normal standard deviation of lognormal distributions. One may note in Fig. 2 the ultimate bearing capacity $q_u$ consist of three components, with c′ affecting the first component only, but with ϕ′ (via $N_c$, $N_q$, $N_\gamma$, $s_q$, $i_q$, $i_\gamma$, $d_q$) affecting all the three components. Hence it is not surprising that for this case the sensitivity to ϕ′ is much more than the sensitivity to c′. RBD-via-FORM is able to reflect case-specific sensitivity (i.e., context-sensitivity).

When $Q_h = 0$, the vertical load $Q_v$ is an unfavorable action without ambiguity. However, when $Q_h$ is acting and of significant magnitude relative to $Q_v$, the latter possesses action-resistance duality, because load inclination and eccentricity decreases with increasing $Q_v$. RBD automatically takes this action-resistance (or unfavorable-favorable) duality into account in locating the design point. Interestingly, RBD reveals that the design value of $Q_v$ (1020 kN/m) is about 7.3% lower than its mean value of 1100 kN/m, thereby revealing the action-resistance (or stabilizing-destabilizing) duality of $Q_v$ when $Q_h$ is acting. It might be difficult for partial factor design approaches (e.g., EC7 and LRFD) to deal with a parameter that possesses action-resistance (unfavorable-favorable, destabilizing-stabilizing) duality, such as the vertical load $Q_v$ in the presence of horizontal load $Q_h$.

Note that the bearing capacity equation is approximate, even for idealized conditions. Also, several expressions for $N_\gamma$ exist. The $N_\gamma$ used here is attributed to Vesić in Bowles (1996). The nine factors $s_j$, $d_j$ and $i_j$ account for the shape and depth effects of foundation and the inclination effect of the applied load. The formulas for these factors are based on Tables 4.5a and 4.5b of Bowles (1996), which may differ from those in EC7.

RBD can be done for cases with multiple failure modes (ultimate limit states, or ULS) and serviceability limit states (SLS), as illustrated for a laterally loaded pile in Section 4.

## 3 RBD-via-FORM Resolves Action-Resistance Duality of An Anchored Sheet Pile Wall

In Fig. 3(a), free-earth support method was used, with $K_a$ based on Coulomb formula, and $K_p$ based on Kerisel-Absi chart. For the statistical inputs shown, RBD for a target β =3.0 results in design H (= 6.4 + z + d) of 12.31 m, and $P_f \approx \Phi(-\beta) = 0.13\%$. For comparison, Monte Carlo simulations (with 200,000 realizations each) gives average $P_f = 0.14\%$. For the statistical inputs shown, tanϕ′ and z are sensitive parameters, as indicated by their **n** values. The **n** values of tanδ and γ are due largely to correlations with tanϕ′, revealed if uncorrelated analysis is done. The design value of γ, 16.13 kN/m³, is lower than its mean value of 17 kN/m³, an apparent paradox which can be understood due to the logical positive correlation of γ to $\gamma_{sat}$ and tanϕ′ which both have design values below their respective mean values. Soil on either side is assumed to be same source, hence the same $\gamma_{sat}$ must be used. Reliability analysis yields $\gamma_{sat}^* = 17.32$ kN/m³, which is less than the mean $\gamma_{sat}$ of 19 kN/m³. This is useful to EC7 design, because even though the partial factor for soil unit weight is 1.0 in the current EC7, one still needs to use a conservative characteristic value of the soil saturated unit weight $\gamma_{sat}$, which affects (via $\gamma' = \gamma_{sat} - \gamma_w$) not only action (active earth pressure) but also resistance (passive soil pressure). That is, the unit weights possess action-resistance duality, and in EC7 design it may not be clear whether *conservative* characteristic values of unit weights are below or above their mean values. The mean embedment depth d = 12.31 – 6.4 – 2.4 = 3.51 m. The design embedment depth $d^* = 2.99$ m, i.e., “overdig” = 0.52m, which is determined automatically as a by-product of RBD.

### 3.1 Comparison with EC7 DA1b design of sheet pile total height H

Figure 3(b) shows the EC7 DA1 Combination 2 (referred to as DA1b hereafter) design for the anchored sheet pile wall, with no consideration of parametric correlations. EC7 has an “unforeseen overdig” allowance for z, to account for the uncertainty of the dredge level. The design value of z is obtained from $\mu_z$ + 0.5 m = 2.4 + 0.5 = 2.9 m, where 0.5 m is the “overdig”. In the current version of EC7, the partial factor of soil unit weight is specified to be 1.0, but conservative characteristic values of γ and $\gamma_{sat}$ still need to be estimated, and if originating from the same source, it is not logical to increase the unit weight on the active side and decrease the unit weight on the passive side. Also, assuming 5/95 percentiles for characteristic values leads to $\gamma^* > \gamma_{sat}^*$, which violates soil physics.

With characteristic values at 30/70 percentiles and EC7 partial factors from DA1b, one obtains a design H of 12.87 m, close to the RBD design H of 12.31 m for a target β of 3.0. A less critical design H of 12.64 m is obtained if one wrongly set the characteristic value of $\gamma_{sat}$ at the 70 percentile value (19.52 kN/m$^3$) instead of at the 30 percentile value (18.48 kN/m$^3$).

FORM reliability analysis based on the H from EC7 design will give different β index depending on whether correlations are modeled in the correlation matrix (Fig. 3(a)) or not. EC7 design cannot model parametric correlations even when such correlations are justified by physical considerations.

Even though partial factors are specified, EC7 does not produce a unique design, but depends on how conservative the characteristic values are determined. This is not objectionable, for it allows flexibility in design to match the consequence of failure; in the same way that target reliability index can be higher or lower depending on the consequence of failure. Analogous situation exists for LRFD's nominal values and load and resistance factors.

| | | x* | μ | σ | $n = \frac{x^* - \mu}{\sigma}$ |
|---|---|---|---|---|---|
| Normal | γ | 16.13 | 17 | 0.9 | -0.971 |
| Normal | $\gamma_{sat}$ | 17.32 | 19 | 1 | -1.677 |
| Normal | $q_s$ | 10.28 | 10 | 2 | 0.140 |
| Normal | tanφ' | 0.663 | 0.78 | 0.05 | -2.336 |
| Normal | tanδ | 0.310 | 0.35 | 0.023 | -1.760 |
| Normal | z | 2.921 | 2.4 | 0.3 | 1.736 |

Correlation matrix **R**

| | γ | $\gamma_{sat}$ | $q_s$ | tanφ' | tanδ | z |
|---|---|---|---|---|---|---|
| γ | 1 | 0.5 | 0 | 0.5 | 0 | 0 |
| $\gamma_{sat}$ | 0.5 | 1 | 0 | 0.5 | 0 | 0 |
| $q_s$ | 0 | 0 | 1 | 0 | 0 | 0 |
| tanφ' | 0.5 | 0.5 | 0 | 1 | 0.8 | 0 |
| tanδ | 0 | 0 | 0 | 0.8 | 1 | 0 |
| z | 0 | 0 | 0 | 0 | 0 | 1 |

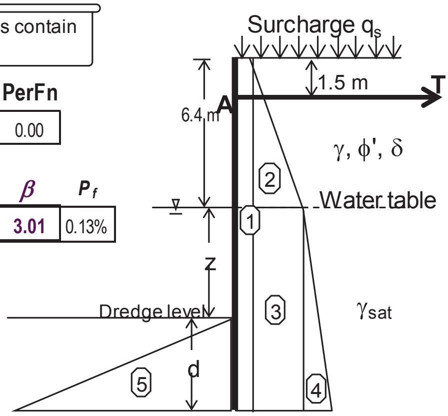
d* = H – 6.4 – z*

Boxed cells contain equations

| $K_a$ | $K_{ah}$ | $K_p$ | $K_{ph}$ | H | d* | PerFn |
|---|---|---|---|---|---|---|
| 0.261 | 0.2492 | 5.897 | 5.6333 | 12.31 | 2.989 | 0.00 |

| | Forces (kN/m) | Lever arm (m) | Moments (kN-m/m) |
|---|---|---|---|
| 1 | -31.54 | 4.655 | -146.81 |
| 2 | -82.31 | 2.767 | -227.73 |
| 3 | -152 | 7.855 | -1194.1 |
| 4 | -32.75 | 8.840 | -289.47 |
| 5 | 189.34 | 9.81 | 1858.11 |

| β | $P_f$ |
|---|---|
| 3.01 | 0.13% |




(a)

EC7 DA1b, **5/95** percentiles for $x_k$

| | $x_k$ | Partial F | x* |
|---|---|---|---|
| γ | 18.48 | 1.00 | 18.48 |
| $\gamma_{sat}$ | 17.36 | 1.00 | 17.36 |
| $q_s$ | 13.29 | 1.30 | 17.28 |
| tanφ' | 0.698 | 1.25 | 0.558 |
| tanδ | 0.312 | 1.25 | 0.250 |
| z | NA | NA | 2.90 |

$x_k$ = Characteristic value
x* = Design value
Computed EC7 design H = 13.95 m
Note: Unrealistic that γ* > $\gamma_{sat}$*

EC7 DA1b, **30/70** percentiles for $x_k$

| | $x_k$ | Partial F | x* |
|---|---|---|---|
| γ | 17.47 | 1.00 | 17.47 |
| $\gamma_{sat}$ | 18.48 | 1.00 | 18.48 |
| $q_s$ | 11.05 | 1.30 | 14.36 |
| tanφ' | 0.754 | 1.25 | 0.603 |
| tanδ | 0.338 | 1.25 | 0.270 |
| z | NA | NA | 2.90 |

Computed EC7 design H = 12.87 m.
Computed EC7 design H = 12.64 m if $\gamma_{sat,k}$ wrongly set at 70 percentile value (19.52) instead of the 30 percentile value (18.48).

EC7 DA1b, **30/70** percentiles

| | $x_k$ | Partial F | x* |
|---|---|---|---|
| γ | 17.47 | 1 | 17.472 |
| $\gamma_{sat}$ | 19.52 | 1 | 19.524 |
| $q_s$ | 11.05 | 1.3 | 14.362 |
| tanφ' | 0.75 | 1.25 | 0.603 |
| tanδ | 0.34 | 1.25 | 0.270 |
| z | NA | NA | 2.9 |

Computed EC7 design H = 12.64 m

(b)

**Figure 3.** (a) RBD of sheet pile total length H via FORM, (b) Eurocode 7 DA1b design of sheet pile total length H.

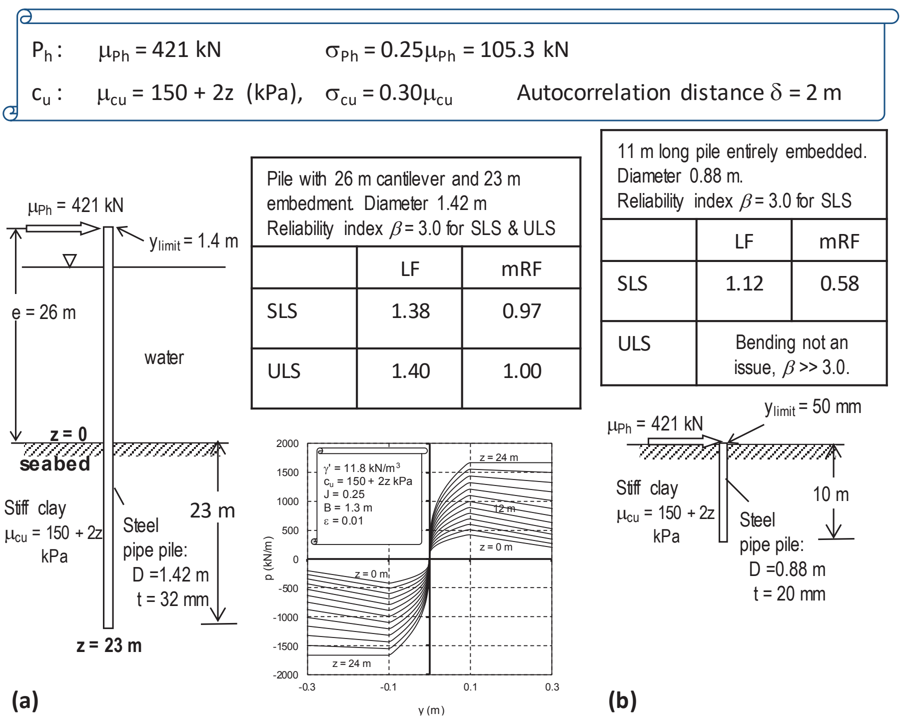
## 4 Case-Specific Parametric Sensitivity in RBD of Laterally Loaded Cantilever Piles

Figure 4(a) shows a steel tubular pile in a breasting dolphin, which was analysed deterministically in Tomlinson (1994). This laterally loaded pile with embedment depth of 23 m below seabed and a cantilever length *e* of 26 m above seabed is analysed probabilistically here with RBD-via-FORM, including investigation on the effect of cantilever length. Soil-pile interaction was based on the nonlinear and strain-softening Matlock p-y curves. At the mean input values of $P_H$ and undrained shear strength $c_u$, the pile deflection y is 0.06 m at seabed, and 1 m at pile head. For reliability analysis, the $P_H$ was assumed to be normally distributed, with mean value 421 kN and a coefficient of variation of 25%. The mean $c_u$ trend is $\mu_{cu} = 150 + 2z$, kPa, with a coefficient of variation of 30%. Spatial autocorrelation was modelled for the $c_u$ values below seabed by $\rho_{ij} = \exp\left(-\left|z_i - z_j\right|/\delta\right)$, with an autocorrelation distance $\delta$ of 2 m. The $\beta$ index obtained was 1.514 with respect to yielding at the outer edge of the annular steel cross section. The sensitivities of $P_H$ and $c_u$ change with the cantilever length *e*. The different sensitivities from case to case are automatically revealed in reliability analysis and RBD, but will be difficult to consider in codes based on partial factors.

A target β of 3.0 can be achieved in RBD for both ULS (bending) and SLS (assuming $y_{Limit} = 1.4$ m) using steel wall thickness t = 32 mm and external diameter d = 1.42 m.

The laterally loaded pile example of Fig. 4(a) is one of a group of piles in a breasting dolphin, with 23 m embedment length below seabed and 26 m cantilever length in sea water. For both the pile bending failure ULS and the pile head deflection SLS, the design point in RBD shows decreasing sensitivity of $c_u$ with depth, i.e., decreasing $(c_u^* - \mu_{cu})/\sigma_{cu}$ with depth, where $c_u^*$ are the design undrained shear strength values at various depths obtained in RBD. One may wonder how partial factor design approaches like EC7 and LRFD would determine the nominal (or characteristic) values of the undrained shear strength at different depths? Assuming uniform conservatism with depth in determining the characteristic $c_u$ values do not accord well with the different sensitivities of $c_u$ with depth as revealed by RBD, and may even alter the behavior of the pile at ULS and SLS.

For ULS design (e.g. bending of pile), having obtained the conservative $c_u$ nominal/characteristic values, it may not be consistent for a partial factor design approach to recommend applying the same partial factor for $c_u$ uniformly across the entire embedded portion of the pile given that different sensitivities are revealed in RBD-via-FORM.




SLS = Serviceability limit state, pile head deflection $y_{limit}$
ULS = Ultimate limit state, defined by bending-induced yield in steel pipe.
LF = Back-calculated load factor = Design load / Nominal load, where nominal load = $\mu_{load} + \sigma_{load}$
mRF = Back-calculated resistance factor w.r.t. mean resistance = Design resistance / mean resistance of SLS and ULS.

**Figure 4**. RBD-via-FORM reveals context-sensitivity of loads and resistance for laterally loaded pile (a) with 26 m cantilever length and (b) fully embedded pile with zero cantilever length.

RBD-via-FORM does not require estimated conservative nominal (characteristic) values and partial factors. Nevertheless, it is possible to back-calculate the partial factors (or load and resistance factors) from the design point of FORM. This was done in Low (2017) for a laterally loaded pile in sea with 26 m cantilever length (Fig. 4(a)), and a laterally loaded pile on land with zero cantilever length (Fig. 4(b)). The spatially autocorrelated soil properties and statistical inputs are the same for both cases. The back-calculated load factors (LF) and resistance factors (mRF) are tabulated in Fig. 4, for the case with 26 m cantilever length (Fig. 4(a)), and for the case with zero cantilever length (i.e. fully embedded), Fig. 4(b), for pile head displacement SLS and pile bending ULS. The case-specific and non-intrinsic sensitivities of the loads and resistance are revealed. Deeper discussions for this case are in Low (2017). Other examples are presented in Low et al. (2017), which is Chapter 4 of the TC205/TC305 joint report.

## 5 Summary and Conclusions

Reliability analyses and reliability-based designs were conducted for three soil engineering problems, namely a spread footing with vertical and horizontal loads, an anchored sheet pile wall, and laterally loaded piles with different cantilever lengths. The aim is to show how RBD via the first-order reliability method (FORM) can overcome some of the limitations and ambiguities of Eurocode 7 and LRFD. Among the merits of FORM is the information contained in its design point (the most probable point of failure) where an expanding dispersion ellipsoid (or equivalent ellipsoid if non-Gaussian distributions are involved) just grazes the limit state surface. The differences and similarities between the design point in RBD and those in EC7 and LRFD are discussed. The ability of RBD-via-FORM to provide interesting and useful information at its design point and to automatically reflect parametric uncertainties, correlations, and case-specific sensitivities are emphasized, and the need for caveats in implementing partial factor design is demonstrated. It may be concluded that reliability analysis and RBD-via-FORM can provide insights and guidance to the evolving EC7 design approach when (i) Partial factors for the myriad soil engineering parameters are not yet fully covered in EC7 or LRFD; (ii) The sensitivities of parameters vary from case to case; (iii) Physical considerations justify modelling of parametric correlations; (iv) A resistance/action parameter possesses stabilizing-destabilizing duality; (v) different target reliability index values are aimed at to reflect different consequence of failure.

It is suggested that RBD-via-FORM can be used in the following two ways to provide insights and guidance to the evolving EC7 and LRFD, and to detect potential pitfalls/inconsistencies (e.g. stabilizing/destabilizing duality) in EC7 and LRFD:

1. Estimate statistical inputs, and conduct FORM analysis on a design that derives from EC7 or LRFD, to estimate the reliability index and the probability of failure, and to compare the design point of FORM with that from EC7 or LRFD. If desired, partial factors can be back-calculated from the design point of FORM, for comparison with those specified in EC7 or LRFD. One should note that RBD, like EC7 and LRFD, aims at a sufficiently safe design, not a design with a precise probability of failure.
2. Estimate statistical inputs, then obtain a design based on FORM target $\beta$ (e.g. 2.5), for comparison with the design from EC7 or LRFD which is based on applying specified partial factors to conservative characteristic/nominal values. Parametric correlations should be modelled in RBD-via-FORM if justified by physical considerations.

## References


Ang, H.S. and Tang, W.H. (1984). *Probability Concepts in Engineering Planning and Design, vol. 2-Decision, Risk, and Reliability*, John Wiley, New York.

Baecher, G.B. and Christian, J.T. (2003). *Reliability and Statistics in Geotechnical Engineering*, J. Wiley.

Bowles, J. E. (1996). *Foundation Analysis and Design*, 5th edition, McGraw-Hill, New York.

Chan, C.L. and Low, B.K. (2012). Probabilistic analysis of laterally loaded piles using response surface and neural network approaches. *Computers and Geotechnics*, 43, 101-110.

Haldar, A. and Mahadevan, S. (1999). *Probability, Reliability and Statistical Methods in Engineering Design*, John Wiley, N.Y.

Low, B.K. (2017). Insights from reliability-based design to complement load and resistance factor design approach. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 143(11), 04017089.

Low, B.K. and Tang, Wilson H. (2007). Efficient spreadsheet algorithm for first-order reliability method. *Journal of Engineering Mechanics*, ASCE, 133(12), 1378-1387.

Low, B.K. et al. (2017). EXCEL-based direct reliability analysis and its potential role to complement Eurocodes. *Proc. TC205 Workshop of 19th International Conf. on Soil Mechanics and Geotechnical Engineering (19ICSMGE)*, Seoul, Korea, 17-22 Sept, 2017. (Chapter 4 of the *TC205-TC304 joint report*, http://alum.mit.edu/www/bklow, /Book chapters).

TC205/TC304 Working Group on "Discussion of statistical/reliability methods for Eurocodes" (2017). https://www.icsmge2017.org/download/19th%20ICSMGE_Workshop_TC205&304.pdf

Tomlinson M. J. (1994). *Pile Design and Construction Practice*, 4th ed, London: E & FN, Spon.

Tomlinson M. J. (2001). *Foundation Design and Construction*, 7th ed, Longman Scientific.