The paper was published in the proceedings of the 7th International Symposium on Geotechnical Safety and Risk (ISGSR 2019) and was edited by Jianye Ching, Dian-Qing Li and Jie Zhang. The conference was held in Taipei, Taiwan 11-13 December 2019.

Reliability Analysis of an Infinite Slope

A.T. Siacara[1], A.T. Beck[2], and M.M. Futai[3]

[1]Department of Geotechnic and Structural Engineering, University of Sao Paulo, Av. Professor Almeida Prado, travessa 2 no 83, Sala S-25, Cidade Universitária – SP, Brazil.
E-mail: adriantorricosiacara@gmail.com
[2]Department Structural Engineering, University of Sao Paulo, Brazil.
E-mail: atbeck@sc.usp.br
[3]Department of Geotechnic and Structural Engineering, University of Sao Paulo, Brazil.
E-mail: futai@usp.br

Abstract: This paper addresses the probability of failure (P_f) and reliability index (β) of infinite slopes of undrained clay, dry slopes and saturated slope with seepage conditions. To this end, the infinite slope is analyzed in different conditions, following the Mohr-Coulomb failure criterion. Parameters such as soil strength, slope geometry and pore pressures are generated using random variables, with the StRAnD v2.00 program. Within the limitations of infinite slope assumptions, the paper clearly demonstrates the importance of reliability analysis in random materials with the use of First order second moment (FORM), Second order reliability method (SORM), "Crude" Monte Carlo Simulation (MCS) and importance Monte Carlo Simulation (IMCS) in the design of infinite slopes.

Keywords**:** Infinite Slope; reliability analysis; stability; probability of failure.

1 Introduction

The infinite slope method is widely used as the geotechnical component of geomorphic and landscape evolution models. Its assumption that shallow landslides are infinitely long is usually considered valid for natural landslides on the basis that they are generally long relative to their depth (Baecher and Christian 2003). Research activity in the mechanics of landslides has led to renewed interest in the infinite slope equations, and the need for a more general framework for giving insight into the probability of failure of long slopes involving non-homogeneous vertical soil profiles and variable groundwater conditions (Silva et al. 2008; Zolfaghari and Heath 2008; Griffiths et al. 2011).

Infinite slope methods analyze the translational slope stability. The assessment of reliability and probability of failure for a slope should be considered as complementary to the usual deterministic analyses (Glade et al. 2005). The superficial sliding layer is thin compared to the slope's height, and it is assumed to extend upslope and downslope infinitely (Briaud 2013; Duncan et al. 2014).

The values of minimum *FS* found in specifications and in the literature tend to be fairly consistent, even for different types of slope, regardless of (1) the degree of uncertainty associated with loads inducing instability of the slope and the resistance against the loads and (2) the geometry of the slope (Rackwitz 2000; Salgado and Kim 2014).

The analyses of infinite slope stability depend on saturation and seepage conditions. The analyses objective is to produce estimates of the probability of infinite slope failure, as opposed to the factor of safety. Consider a typical slice of the infinite slope shown in Fig. 1.

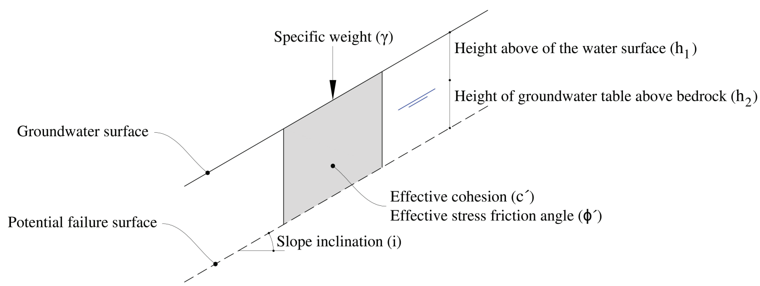

Figure 1. Infinite slope configuration.

Proceedings of the 7th International Symposium on Geotechnical Safety and Risk (ISGSR)
Editors: Jianye Ching, Dian-Qing Li and Jie Zhang

Published by Research Publishing, Singapore.
ISBN: 978-981-11-2725-0; doi:10.3850/978-981-11-2725-0_IS1-17-cd

2 Problem Setting

2.1 Performance function

The structural reliability problem is formulated in terms of a random variable vector, $\mathbf{X}= \{X_1, X_2,\ldots, X_n\}$, representing a set of random geotechnical parameters. The probability of failure is given by:

$$P_f = P[g(\mathbf{X}) \leq 0] = \int_{g(\mathbf{x})\leq 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{1}$$

where $f_X(\mathbf{x})$ represents the joint probability density function of $\mathbf{x}$ and the integral is carried out over the failure domain. In these cases, the minimal distance of the limit state function g($\mathbf{y}$)=0 to the origin is the so-called reliability index (β), and the point over the limit state with minimal distance to the origin is called "design point". In standard Gaussian space, the limit state function is approximated by:

$$P_f \approx \Phi(-\beta) \tag{2}$$

where Φ(.) is the standard Gaussian cumulative distribution function (Melchers and Beck, 2018). The limit state function in slope stability problems is generally expressed as (Phoon 2008):

$$\mathrm{g}(\mathbf{X}) = \mathrm{FS}(\mathrm{x}_1, \mathrm{x}_2, \ldots\ldots, \mathrm{x}_n) - 1.0 \tag{3}$$

where *FS* is the safety factor.

2.2 Reliability Methods

In this study, approximate methods such as First-Order Reliability Method (FORM) and Second-Order Reliability Method (SORM) are employed, and compared with exact methods as "Crude" Monte Carlo Simulation (MCS) and Importance Sampling Monte Carlo Simulation (iMCS) in different infinite slopes conditions. A complete explanation of these reliability methods is found in (Melchers and Beck 2018).

2.3 Application of the problem and boundary conditions

The slope of the study consists of a homogenous earth slope. The uncertain stability analysis is characterized by uncertain effective cohesion (c'), undrained shear strength (Cu), specific weight (γ), saturated specific weight (γ_{sat}), slope inclination (i) and effective stress friction angle (ϕ'). The geometry is considered deterministic and change in function of the problem ($f_{problem}$), with variables height above of the water surface (h_1), height of the groundwater table above bedrock (h_2) and depth soil above bedrock (H). Consider a typical slice of the infinite slope, as shown in Fig. 1. The infinite slope equation for the factor of safety (FS) of a homogeneous soil in this case (Phoon 2008; Griffiths et al. 2011) is given by

$$FS = \frac{c?+(h_1\gamma + h_2\gamma')cos^2(i)tan(\phi')}{(h_1\gamma + h_2\gamma_{sat})sin(i)cos(i)} \tag{4}$$

All the parameters are shown in Figure 1. The uncertain stability parameters are assumed to have Lognormal (LN) distribution, according to the situation of the analysis. The mean (μ) and the Coefficient of Variation (COV) of all the parameter of the slope are shown in Table 1.

Table 1. Soil parameters of the slope problem.

Number	Variable	Unit	μ	*COV* (%)
X(1)	Effective cohesion (c')	kPa	10.0	50.0
X(2)	Undrained shear strength (Cu)	kPa	25.0	20.0
X(3)	Specific weight (γ)	kN/m^3	17.0	7.00
X(4)	Saturated specific weight (γ_{sat})	kN/m^3	20.0	7.00
	Height above of the water surface (h_1)	m	$f_{problem}$	-
	Height of groundwater table above bedrock (h_2)	m	$f_{problem}$	-
	Depth soil above bedrock (H)	m	$f_{problem}$	-
X(5)	Slope inclination (i)	°	30.0	-
X(6)	Effective stress friction angle (ϕ')	°	30.0	20.0

The numerical calculations were implemented in the StRAnD v2.00 program (Beck 2008), developed in Fortran language.

3 Examples of Infinite Slopes

In this section, reliability analysis is applied for three cases of infinite slope: undrained clay, dry slopes and saturated slope with seepage. The limit state functions for the three examples are derived from Eq. (4). The examples differ in the number of random variables, but also depth soil above bedrock (H).

3.1 Example A: undrained clay

The first infinite slope example considers two random variables (C_u and γ), and Eq. (4) therefore simplifies to:

$$FS = \frac{2C_u}{\gamma H \sin(2i)} \tag{5}$$

Using the mean values (μ) of the parameters, safety factors are found as 3.40, 1.70 and 1.13, for heights H of 1.0, 2.0 and 3.0 meters, respectively.

3.1.1 Results of deterministic and reliability analysis

In order to compute the P_f with FORM, SORM, MCS and IMCS, we must assume a Lognormal (*LN*) distribution of the two random variables (C_u and γ), hence the probability of failure is given by the limit state function Eq. (5). The P_f and the β are shown for 3 cases of the deterministic parameter H in Table 2.

Table 2. Results of the reliability analysis with FORM, SORM, MCS and IMCS.

Analysis	H (m)	FORM		SORM		MCS		IMCS	
		P_f	β	P_f	β	P_f	β	P_f	β
A1	1.0	$4.79*10^{-9}$	5.74	$4.79*10^{-9}$	5.74	$5.23*10^{-9}$	5.82	$5.05*10^{-9}$	5.73
A2	2.0	$7.35*10^{-3}$	2.44	$7.35*10^{-3}$	2.44	$6.60*10^{-3}$	2.48	$7.68*10^{-3}$	2.42
A3	3.0	$3.05*10^{-1}$	0.51	$3.05*10^{-1}$	0.51	$3.04*10^{-1}$	0.51	$3.10*10^{-1}$	0.49

An advantage of the FORM method is the possibility of carrying out sensitivity analyses, through the direction cosines at the design point (α^2). Figure 2 shows sensitivity factors α_i^2 for limit state function of this example. Again, it is observed that undrained shear strength (*Cu*) is the most relevant random variable, and the specific weight (γ) has less relevance.

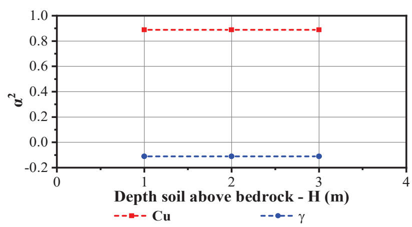

Figure 2. FORM sensitivity coefficients α_i^2 for limit state function of the analysis A.

Results for reliability analysis were computed by two Monte Carlo simulation techniques: crude Monte Carlo and importance sampling using design points. Fig. 3 shows converge plots for crude and importance sampling Monte Carlo, in terms of a number of samples n_S, for the case with medium failure probability (H=2.0 m); in addition, Fig. 3 shows the confidence intervals (P_f Upper and P_f Lower curves).

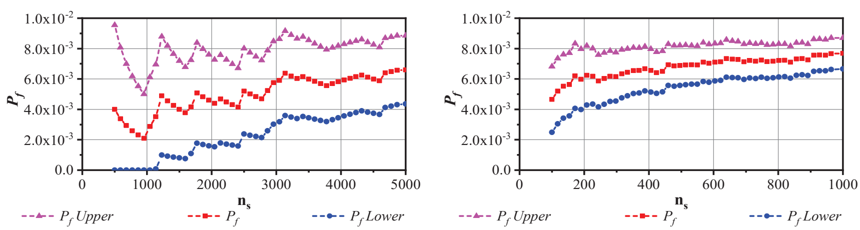

Figure 3. Convergence of crude (left) and importance sampling (right) of the analysis A2.

The upper and lower probability of failure approximate 95% confidence limits for the mean flow of the P_f. In 95% of cases the true value lies in the confidence interval. In 5% of cases it is outside the interval. For the MCS the P_f Upper and Lower has a large variation between both of them, in turn, the IMCS has a closely fluctuation of the P_f (Figure 3).

Acceptable failure probabilities for slopes are given for temporary structures as 0.1, and for general slopes as 0.0001 (Santamarina 1992; Salgado and Kim 2014). A maximum value of P_f of natural slopes in a range of 0.001-0.15 is suggested by (Chowdhury and Flentje 2003), changing with potential failure modes and consequences of slope failure. (Christian et al. 1994) suggested a P_f of 0.001 would be a reasonable number to use in design. (Loehr et al. 2005) set the range of P_f from 0.001 to 0.01 for slopes with high potential risk and low potential risk, respectively. (Christian 2004, 2013) has done characterization to express the acceptable probability of failure as a function of a number of fatalities.

Depending on the importance of the slope, the P_f of 30 % in a slope with H of 3 meters should be taken with care. According to authors above, this slope is unacceptable. It is very important to determine the correct depth in an infinite slope reliability analysis. A variation of more than 100 % of β between the analysis A1 to A2 or A3 shows that the parameter of the geometry H is a very important parameter. The analyses A1 and A2 show an acceptable P_f in these conditions; therefore, the design of the slope is possible until 2 meters above the intact rock.

It is important to keep track of the computational efficiency of the solutions: first-order methods (FOSM and FORM) applied to the infinite slope problem resulted in accurate results, with much smaller computational cost, in comparison with simulation methods (MCS and IMCS).

3.2 Example B: Dry Slopes

The second infinite slope example in dry condition considers three random variables (c', γ and ϕ'); hence Eq. (4) therefore simplifies to:

$$FS = \frac{2c'}{\gamma H \, sin(2i)} + \frac{tan(\phi')}{tan(i)} \tag{6}$$

Using the mean values (μ) of the parameters, safety factors are found as 2.36, 1.45 and 1.27 for heights H of 1.0, 3.0 and 5.0 meters, respectively.

3.2.1 Results of deterministic and reliability analysis

In order to compute the probability of failure with the FORM, SORM, MCS and IMCS, we assume a Lognormal (LN) distribution of the three random variables (c', γ and ϕ'). The probability of failure is evaluated considering Eq. (6) as limit state function. The P_f and the β are shown for 3 cases of the deterministic parameter H in Table 3.

Table 3. Results of reliability analysis with FORM, SORM, MCS and IMCS.

Analysis	H (m)	FORM		SORM		MCS		IMCS	
		P_f	β	P_f	β	P_f	β	P_f	β
B1	1.0	$7.69*10^{-4}$	3.17	$7.69*10^{-4}$	3.17	$6.00*10^{-4}$	3.24	$5.21*10^{-4}$	3.27
B2	3.0	$6.29*10^{-2}$	1.53	$6.29*10^{-2}$	1.53	$5.32*10^{-2}$	1.61	$5.11*10^{-2}$	1.63
B3	5.0	$1.69*10^{-1}$	0.96	$1.69*10^{-1}$	0.96	$1.46*10^{-1}$	1.05	$1.49*10^{-1}$	1.04

Figure 4 shows sensitivity factors α_i^2 for limit state functions of this example. Again, it is observed that effective cohesion (c') and the effective stress friction angle (ϕ') are the most relevant random variables, and the specific weight (γ) has less relevance. For 1 meter of depth, the cohesion is the most important parameter, but between 1 and 3 meters of depth the situation change and the friction angle become the most relevant parameter.

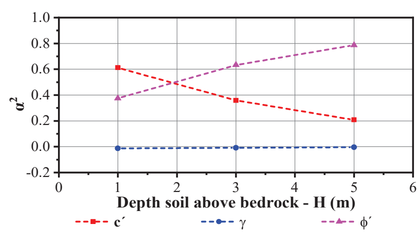

Figure 4. FORM sensitivity coefficients α_i^2 for limit state function of the analysis B.

The most critical situation is the infinite slope with 5 meters of soil above the intact rock with P_f of 16.9%. According to the authors (Santamarina 1992; Christian et al. 1994; Chowdhury and Flentje 2003; Christian 2004, 2013; Loehr et al. 2005; Salgado and Kim 2014), this slope is unacceptable. As shown in example A, the variation of more than 100 % of the β between the different analysis shows that the geometry parameter H is very important. The analysis B1 and B2 show an acceptable P_f in these conditions; therefore, the design of the slope is possible until 3 meters above the intact rock.

The MCS for the different analysis shows that the number of simulations is adequate in order to find the P_f and β. Results confirm that FORM and SORM are accurate enough for this problem, in comparison with MCS and IMCS, with errors between 2 and 10%.

3.3 Example C: Saturated Slope with Seepage

The third infinite slope example considers four random variables (c', γ, ϕ' and γ_{sat}). In this case, Eq. (4) simplifies to:

$$FS = \frac{2c'}{\gamma_{sat} H \sin(2i)} + \frac{\gamma' \tan(\varphi')}{\gamma_{sat} \tan(i)} \tag{7}$$

Using the mean values (μ) of the parameters, the safety factors are found as 2.82, 1.66 and 1.09 for heights H of 0.5, 1.0 and 2.0 meters, respectively.

3.3.1 Results of deterministic and reliability analysis

In order to compute the probability of failure with the FORM, SORM, MCS and IMCS, we assume Lognormal (*LN*) distributions for the four random variables (c', γ, ϕ' and γ_{sat}). The limit state function is Eq. (7). The P_f and the β are shown for 3 cases of the deterministic parameter H in Table 4.

Table 4. Results of the reliability analysis with the FORM, SORM, MCS and IMCS.

Analysis	H (m)	FORM		SORM		MCS		IMCS	
		P_f	β	P_f	β	P_f	β	P_f	β
C1	0.5	$3.45*10^{-3}$	2.70	$3.45*10^{-3}$	2.70	$4.20*10^{-3}$	2.64	$2.91*10^{-3}$	2.76
C2	1.0	$8.76*10^{-2}$	1.36	$8.76*10^{-2}$	1.36	$7.28*10^{-2}$	1.46	$8.26*10^{-2}$	1.39
C3	2.0	$4.78*10^{-1}$	0.05	$4.78*10^{-1}$	0.05	$4.41*10^{-1}$	0.15	$4.44*10^{-1}$	0.14

Figure 5 shows sensitivity factors α_i^2 for limit state functions of this example. Again, it is observed that effective cohesion (c') is the most relevant random variable, and the others parameters have less relevance.

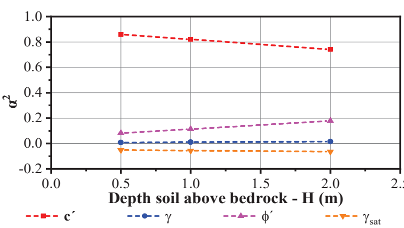

Figure 5. FORM sensitivity coefficients α_i^2 for limit state function of the analysis C.

The most critical situation is the infinite slope with 2 meters of soil above the intact rock with 47.8% P_f and according to different authors (Santamarina 1992; Christian et al. 1994; Chowdhury and Flentje 2003; Christian 2004, 2013; Loehr et al. 2005; Salgado and Kim 2014) this slope is unacceptable. As is shown in the examples A and B, the variation of more than 100 % of β between the different analysis shows that geometry parameter H is very important. The analyses C1 and C2 show an acceptable P_f in these conditions; therefore, the design of the slope is possible until 1 meter above the intact rock.

The MCS for the different analysis shown that the number of simulations is adequate in order to find the P_f and β. Comparing MCS and IMCS with the FORM and SORM method, the variations are from 2 % to 60 % that shown some variations in specially when the slope has more P_f.

4 Conclusions

In this paper, we addressed reliability analysis for three cases of infinite slope: undrained clay, dry slope and saturated slope with seepage. In all cases studied, slope reliability was found to change significantly with the depth of soil above bedrock (H). For larger values of H, slope reliability drops below internationally accepted values. For undrained clay in example A, it was observed that undrained shear strength (Cu) is the most relevant random variable, and the specific weight (γ) has less relevance in the reliability analysis. For a dry slope in example B, effective cohesion (c') and the effective stress friction angle (ϕ') are the most relevant random variables, and the specific weight (γ) has less relevance in slope reliability. For a saturated slope with seepage (example C), it was observed that effective cohesion (c') is the most relevant random variable; whereas specific weight (γ), effective stress friction angle (ϕ') and saturated specific weight (γ_{sat}) have less relevance in the reliability analysis. In all cases, results obtained by FORM and SORM agreed with results obtained by Monte Carlo simulation, showing that the infinite slope reliability problem is not excessively non-linear. This includes the linearity of the limit state functions, and the transformation of log-normal variables to standard normal space. Taking into account the reliability analysis, the design of slopes shown different acceptable height of infinite slopes between the P_f and FS.

Acknowledgments

The first author would like to thank the financial support of the Coordination for the Improvement of Higher Education Personnel (CAPES) for research funding (Grant number nº 88882.145758/2017-01) and the Brazilian Research Council (CNPq).

References

Baecher, G.B. and Christian, J.T. (2003). *Reliability and Statistics in Geotechnical Engineering*, John Wiley & Sons Ltd, University of Michigan.

Beck, A.T. (2008). Structural risk analysis and design - StRAnD. Release 1. *São Carlos, Brazil: Departamento de Engenharia de Estruturas*, Escola de Engenharia de São Carlos, Universidade de São Paulo.

Briaud, J-L. (2013). *Geotechnical Engineering: Unsaturated and Saturated Soils, Vol. 12*, John Wiley & Sons Ltd, New Jersey.

Chowdhury, R. and Flentje, P. (2003). Role of slope reliability analysis in landslide risk management. *Bull Eng Geol*, 62, 41–46.

Christian, J.T. (2004). Geotechnical engineering reliability: How well do we know what we are doing? *J Geotech Geoenvironmental Eng*, 130(10), 985–1003.

Christian, J.T. (2013). Issues of reliability in stability of slopes. *Geo-Congress 2013*, 2246–2261.

Christian, J.T., Ladd, C.C., and Baecher, G.B. (1994). Reliability applied to slope stability analysis. *J Geotech Eng*, 120(12), 2180–2207.

Duncan, J.M., Wright, S.G., and Brandon, T.L. (2014). *Soil Strength and Slope Stability*.

Glade, T., Anderson, M., and Crozier, M.J. (2005). *Landslide Hazard and Risk*.

Griffiths, D.V., Huang, J., and Fenton, G.A. (2011). Probabilistic infinite slope analysis. *Comput Geotech*, 38(4), 577–584.

Loehr, J.E., Finley, C.A., and Huaco, D. (2005). *Procedures for Design of Earth Slopes Using LRFD*, Vol. OR 06-010.

Melchers, R.E. and Beck, A.T. (2018). *Structural Reliability Analysis and Prediction, 3er Editio*, John Wiley & Sons Ltd, Chichester and UK.

Phoon, K-K. (2008). *Reliability-Based Design in Geotechnical Engineering: Computations and Applications*, Taylor & Francis Group, London and New York.

Rackwitz, R. (2000). Reviewing probabilistic soils modelling. *Comput Geotech*, 26(3–4), 199–223.

Salgado, R. and Kim, D. (2014). Reliability analysis of load and resistance factor design of slopes. *J Geotech Geoenvironmental Eng*, 140(1), 57–73.

Santamarina, J. (1992). Reliability of slopes: Incorporating qualitative information (ABRIDGMENT). *Transp Res Board*.

Silva, F., Lambe, T.W., and Marr, W.A. (2008). Probability and risk of slope failure. *J Geotech Geoenvironmental Eng*, 134(12), 1691–1699.

Zolfaghari, A. and Heath, A.C. (2008). A GIS application for assessing landslide hazard over a large area. *Comput Geotech*, 35(2), 278–285.