Robust Geotechnical Design Based on Sensitivity Analysis of FORM Using Spreadsheet

Xiaohui Tan[1*], Mengmeng Niu[1], Fanchao Wang[1], Xiaole Dong[1], and Suozu Fei[1]

[1]School of Resources and Environmental Engineering, Hefei University of Technology, Hefei 230009, China.
E-mail: tanxh@hfut.edu.cn
E-mail: Niu_Mengm@163. com
E-mail: Fancy_wangg@163.com
E-mail: dongxiaole_dd@163.com
E-mail: fsz2017@mail.hfut.edu.cn

Abstract: Reliability-based design (RBD) is widely adopted in many countries, but the variation of input parameters' statistics will lead to the variation of computed system response of designs. Robust geotechnical design (RGD) aims to ensure a design insensitive to the variation of input parameters by maximizing the design robustness and minimizing the cost under safety constraint. Several robustness indexes have been employed to measure the design robustness, among which the feasibility robustness is theoretical rigorous and user-friendly. However, current RGD method uses first order reliability method (FORM) for the calculation of reliability index and uses point estimate method (PEM) for the calculation of the statistics of reliability index and the feasibility robustness. This method needs a large amount of computational effort, which hinders the application of the RGD method. To overcome this deficiency, we use a method for calculating the feasibility robustness based on the sensitivity of reliability index (SRI) to the distribution parameters of basic variables. Considering the SRI to the distribution parameters of basic variables is a by-product of the widely used FORM algorithm, this sensitivity analysis method for calculating the feasibility robustness can be readily integrated into the FORM algorithm. Therefore, the RGD using FORM implemented with SRI is computational efficient and easy to use. Moreover, the RGD using FORM implemented with SRI can be easily implemented in a spreadsheet program such as Excel. Through an example of shallow foundation, the implementation of this new RGD approach is described. The results show that the new RGD method is very accurate and efficient compared to the RGD using FORM implemented with PEM.

Keywords: Robust geotechnical design; sensitivity analysis; feasibility robustness; reliability-based design; spreadsheet.

1 Introduction

Uncertainties exist in all geotechnical engineering, and they must be considered in the design of geotechnical structures. Reliability-based design (RBD) is a rational design method which can consider the uncertainties of soil parameters (Wang et al. 2011). The input parameters of RBD are the statistics (e.g., means, coefficients of variation (COVs) or standard deviation) of basic variables whose uncertainties are considered. If the means and COVs of the basic variables can be accurately determined, the design of RBD is reasonable. However, the means and COVs of basic variables are usually estimated from limited number of test samples or estimated by experience, so they may not be so accurate, which will result in over or under designs of geotechnical structures (Juang and Wang 2013; Cho et al. 2016).

To overcome the shortcomings of the RBD, robustness geotechnical design (RGD) was proposed and developed during the last decade (Juang et al. 2013; Khoshnevisan et al. 2015; Gong et al. 2014; Gong et al. 2016). The RGD can consider structural safety, cost requirements and design robustness simultaneously. In the RGD, structural safety is expressed by reliability index or probability of safety, which is same with the RBD. A special characteristic of RGD is the concept of design robustness. A robust design is that a design is insensitive to the uncertainties of the statistics of input parameters.

To perform an RGD for geotechnical structures, the design robustness must be quantified. Several design robustness indices have been used in the literatures (Khoshnevisan et al. 2014; Gong et al. 2014, 2016), among which feasibility robustness index is widely used because it can reflect the design robustness directly. However, traditional method for computing the feasibility robustness index needs the combination of the first order reliability method (FORM) and the point estimate method (PEM), among which the FORM is used to calculate the reliability index (β) or failure probability (P_f), and the PEM is used to calculate the feasibility robustness index (β_β). Because the calculation of β_β needs several calculations of β, (i.e., one PEM computation contains several FORM computations), the traditional RGD using the FORM-PEM algorithm is computational inefficient.

In order to reduce the computational effort of the traditional RGD using the FORM-RSM algorithm, Tan et al. (2019) proposed a new RGD using SRI-based FORM algorithm. In the proposed RGD, the feasibility robustness index (β_β) can be easily calculated within the FORM algorithm, and no PEM computation is needed. Therefore, the new RGD has high computational efficiency compared to the traditional RGD using the FORM-PEM algorithm. However, the new RGD was implemented using the scientific programming language Matlab,

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designers should be familiar with this language. Considering the popularity and accessibility of spreadsheets such as Excel (Low 2014; Khoshnevisan et al. 2015), this paper performs the new RGD using the spreadsheet of Excel. The design of a shallow spread foundation is used to illustrate the advantage of this method.

2 Methodology for Robust Geotechnical Design

2.1 Traditional robust geotechnical design

A robust design is to optimize design parameters (***d***) so that the system response (reliability index (β) or failure probability (P_f)) of the design is robust against or insensitive to the variation of noise factors ($\boldsymbol{\theta}$). The noise factors are usually called basic variables ($\boldsymbol{X}$) in reliability analysis. Due to the uncertainties of noise factors ($\boldsymbol{\theta}$) or basic variables ($\boldsymbol{X}$), the system response will vary in a certain range. Feasibility robustness is the probability that the system response still meets design requirement even the statistics of basic variables are uncertain. The design requirement is usually expressed by $P_\mathrm{f}>P_\mathrm{f}^\mathrm{T}$ or $\beta<\beta^\mathrm{T}$, where P_f^T and β^T are pre-defined target failure probability and target reliability index, respectively. Consequently, the feasibility probability (P_c) is the probability of a safety requirement is satisfied even when the noise factors vary, and P_c can be expressed by Eqs. (1a) or (1b) (Juang and Wang 2013):

$$P_\mathrm{c} = P(P_\mathrm{f} - P_\mathrm{f}^\mathrm{T} \le 0) \tag{1a}$$

$$P_\mathrm{c} = P(\beta - \beta^\mathrm{T} \ge 0) = \Phi(\beta_\beta) \tag{1b}$$

where $P(\cdot)$ is the probability that the target failure probability or target reliability index can be satisfied; Φ is the cumulative Gaussian distribution function; β_β is an equivalent feasibility robustness index of the feasibility probability (P_c). The larger the value of P_c or β_β is, the more robust of a design is.

According to Juang and Wang (2013), the feasibility robustness index (β_β) can be computed using Eqs. (2a) and (2b) when the reliability index meets normal and lognormal distribution, respectively:

$$\beta_\beta = \left(\mu_\beta - \beta^\mathrm{T}\right) / \sigma_\beta \tag{2a}$$

$$\beta_\beta = \left[\mu_{\ln\beta} - \ln(\beta^\mathrm{T})\right] / \sigma_{\ln\beta} \tag{2b}$$

$$\mu_{\ln\beta} = \ln\left[\mu_\beta \middle/ \sqrt{1+\left(\sigma_\beta/\mu_\beta\right)^2}\right] \tag{3a}$$

$$\sigma_{\ln\beta} = \sqrt{\ln\left[1+\left(\sigma_\beta/\mu_\beta\right)^2\right]} \tag{3b}$$

where μ_β is the mean of reliability index (β), and σ_β is the standard deviation of β.

As can be seen in Eqs. (2a) and (2b), the mean (μ_β) and standard deviation (σ_β) of reliability index should be calculated for the calculation of feasibility robustness index (β_β). The value of μ_β can be easily obtained using the FORM, but the calculation of σ_β is relatively complicated. In the traditional RGD, PEM is adopted to calculate the value of σ_β (Juang and Wang 2013; Zhao and Ono 2001). Take the 7-point PEM for example, $7n_\mathrm{c}$ times of FORM algorithms are required to calculate σ_β, where n_c is the number of basic variables whose statistics are considered as random variables. Therefore, the traditional RGD using the FORM-PEM algorithm is computational inefficient because many times of FORM iterations are needed.

2.2 New robust geotechnical design

Similar to Juang et al. (2013), the COVs of basic variables (noise factors) are taken as random variables. To improve the computational efficiency of the traditional RGD using the FORM-PEM algorithm, Tan et al. (2019) developed a new RGD algorithm, in which the standard deviation of reliability index (σ_β) is calculated using Eq. (4):

$$\sigma_\beta = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial\beta}{\partial\sigma_i}\frac{\partial\beta}{\partial\sigma_j}\sigma_{\sigma_i}\sigma_{\sigma_j}\rho_{ij}\right)\right)^{\frac{1}{2}} \tag{4}$$

where $\partial\beta/\partial\sigma_i$ is the gradient of reliability index to the standard deviation of basic variable X_i; $\sigma_{\sigma i}$ is the standard deviation of σ_i (i.e., $\sigma_{\sigma i} = \mu_{\sigma i}\times\delta_{\sigma i}$, $\mu_{\sigma i}$ and $\delta_{\sigma i}$ are the mean and COV of σ_i, respectively).

In Eq. (4), the value of $\partial\beta/\partial\sigma_i$ can be easily calculated along with the FORM algorithm because $\partial\beta/\partial\sigma_i$ can be deemed as the sensitivity of reliability index (SRI) with respect to the distribution parameter of basic variables. Denoting the performance function of a geotechnical structure as $Z=g(\boldsymbol{X})$, the gradient $\partial\beta/\partial\sigma_i$ can be calculated as follows (Sudret and Kiureghian 2002):

$$\frac{\partial\beta}{\partial\sigma_i}=-\frac{\partial g(\boldsymbol{x}^*)}{\partial X_i}\cdot\frac{\partial Y_i}{\partial\sigma_i}\cdot\frac{\sigma_i}{\sigma_{\mathrm{g}}(\boldsymbol{x}^*)} \tag{5}$$

where $\partial g(\boldsymbol{x}^*)/\partial x_i$ and $\sigma_{\mathrm{g}}(\boldsymbol{x}^*)$ are the gradient and standard deviation of the performance function $g(\cdot)$ at design point $\boldsymbol{x}^*$; variable $\boldsymbol{Y}=\Phi^{-1}(F(\boldsymbol{X}))$ is a normal variable and it is the transformation of variable $\boldsymbol{X}$ (Φ is the cumulated normal distribution function for variable $\boldsymbol{Y}$, and F is the cumulated distribution function for variable $\boldsymbol{X}$).

If $\boldsymbol{X}$ is a normal variable whose mean vector is $\boldsymbol{\mu}$ and the diagonal matrix of standard deviation is $\boldsymbol{\sigma}$, then

$$\boldsymbol{Y}=\boldsymbol{\sigma}^{-1}(\boldsymbol{X}-\boldsymbol{\mu}) \tag{6}$$

and

$$\frac{\partial Y_i}{\partial\sigma_i}=-\frac{X_i-\mu_i}{\sigma_i^2} \tag{7}$$

If $\boldsymbol{X}$ is a lognormal variable, then

$$\boldsymbol{Y}=\boldsymbol{\xi}^{-1}(\ln\boldsymbol{X}-\boldsymbol{\lambda}) \tag{8}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\xi}$ are the mean vector and diagonal matrix of standard deviation of $\ln(\boldsymbol{X})$. The ith element of $\boldsymbol{\xi}$ and $\boldsymbol{\lambda}$ are as follows:

$$\xi_i=\sqrt{\ln(1+(\sigma_i/\mu_i)^2)} \tag{9a}$$

$$\lambda_i=\ln\left(\mu\Big/\sqrt{1+(\sigma_i/\mu_i)^2}\right) \tag{9b}$$

Then,

$$\frac{\partial Y_i}{\partial\sigma_i}=\frac{\partial Y_i}{\partial\lambda_i}\frac{\partial\lambda_i}{\partial\sigma_i}+\frac{\partial Y_i}{\partial\xi_i}\frac{\partial\xi_i}{\partial\sigma_i}=\left(1-\frac{\ln X_i-\lambda_i}{\xi_i^2}\right)\cdot\frac{\sigma_i}{\xi_i(\mu_i^2+\sigma_i^2)} \tag{10}$$

Substituting Eqs. (5), (7) or (10) into Eq. (4), the standard deviation of reliability index (σ_β) can calculated. Note that $\partial g(\boldsymbol{x}^*)/\partial x_i$ and $\sigma_{\mathrm{g}}(\boldsymbol{x}^*)$ are calculated in the FORM algorithm, and all the other parameters in Eqs. (4), (5), (7) or (10) are simply to be calculated, so the calculation of σ_β can be seen as a by-product of the FORM algorithm. Therefore, the new RGD using the SRI algorithm is computational efficient because it does not need the 7-pint PEM which is composed of $7n_{\mathrm{c}}$ FORM iteration processes.

3 Implementation of the New RGD Using Spreadsheet

3.1 Design example of a shallow foundation

Tan et al. (2019) carried out the RGD implemented with SRI-based FORM using the scientific programming language Matlab, which requires users to be familiar with this language. Considering the popularities of the spreadsheet of Excel, we implement the new RGD using Excel. To compare the designs of the RGD using Excel and Matlab, the same illustrative example, the design of a shallow spread foundation under the ultimate limit state (ULS) (Juang and Wang 2013), is used. The target reliability index (β^{T}) for the ULS of the shallow foundation is 3.8.

The shallow foundation is built directly above the ground water table (Fig. 1), the embedded depth of this foundation is D = 0.8 m. The vertical permanent load is G = 900 kN and the live load is Q = 458.7 kN. The effective shear strength parameters are c' = 0 and φ' = 36.4°. Similar to Juang and Wang (2013), the effective friction angle φ' and load Q are taken as independent random variables or noise factors. These two variables are lognormally distributed. The means of φ' and Q are 36.4° and 458.7 kN, and the COVs of φ' and Q are 0.08 and 0.15, respectively. The COV of φ' is seemed as a random variable, whose standard deviation is 0.08.

The performance function for the ULS of the foundation under vertical load is as follows:

$$g(\boldsymbol{X})=R_{\mathrm{U}}-(G+W)-Q \tag{11}$$

where W is the gravity of the shallow foundation ($W=\gamma_{\mathrm{c}}BL$, γ_{c}=24 kN/m^3 is the unit weight of concrete, B and L are the two side lengths of the shallow spread foundation, respectively); R_{U} is the bearing capacity of the foundation and it is calculated as follows (Juang and Wang 2013; Wang 2011):

$$R_{\mathrm{U}}=\left(\frac{1}{2}\gamma' BN_{\gamma}s_{\gamma}+\gamma DN_{q}s_{q}\right)BL \tag{12}$$

where γ and γ' are unit weight and buoyant unit weight of foundation soil, respectively (γ=22 kN/m^3, γ'=12.2 kN/m^3); N_γ and N_q are bearing capacity factors and they are functions of the effective friction angle φ'; s_γ and s_q are shape factors and they are functions of φ' and foundation size.

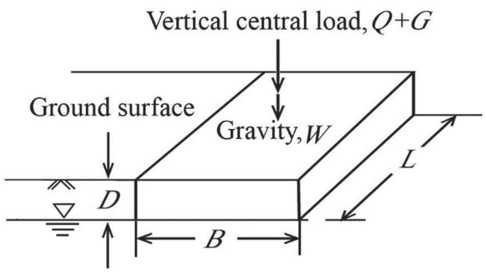

Figure 1. Sketch of a shallow foundation.

As shown in Fig. 1, the goal of the RGD for this foundation is to select the robust optimum design parameters (***d***), which are foundation width *B* and length *L*. A robust optimum design means that the safety requirement (β or P_f) is satisfied, the robustness index (β_β or P_c) is maximized, and the construction cost of the foundation is minimized. The detail computation formula for the cost of the shallow foundation is referred to Gong et al. (2014) and Wang and Kulhawy (2008). Similar to Juang and Wang (2013), both *B* and *L* are in the range of 1.0 m to 4.0 m, and $1\leq L/B\leq 10$ is assumed for the design of a rectangular foundation.

3.2 RGD of shallow foundation using the spreadsheet

According to the analyses of Section 2.2 and Section 3.1, we set up a spreadsheet model for performing the new RGD for the shallow foundation. In Fig. 2, the top and the left parts is for the calculation of reliability index (β), which is similar to Low (2014) and Khoshnevisan et al. (2015). The boxed content in the lower right part of the spreadsheet is for the calculation of the feasibility robustness index (β_β). The four squared ranges with gray background are cells which need input values, and other boxed cells represent they contain formulas.

The calculation of reliability index using the FORM in a spreadsheet is an optimization problem. By using *Solver* to automatically minimize the reliability index β in D16, and changing the ***n*** column in L5:L6, subject to $g(\boldsymbol{X}) = 0$ in D15, the reliability index can be obtained. This process can be performed for all values to be selected for *B* and *L*. Note that the computation of β does not depend on the values of the lower right part of the spreadsheet shown in Fig. 2, additional iterative process is unneeded for the computation of β_β. Therefore, the RGD based on the sensitivity of FORM using spreadsheet is very easy to use.

3.3 Design results

By changing the candidate values of *B* and *L* in cells P5:P6 and using the *Solver* function, different values of reliability index (β), failure probability (P_f), and feasibility robustness index (β_β and P_c) can be obtained. Then, comparing the reliability index with the target reliability index of β^T=3.8, those designs whose reliability indices are greater than or equal to β^T are considered as acceptable designs. And then, for all the acceptable designs, the feasibility robustness indices and construction costs are taken into account to select an optimal design.

Figure 3(a) shows the relationship between the feasibility robustness index β_β and foundation width *B* for all acceptable designs, and Fig. 3(b) shows the corresponding $P_c \sim B$ curves for different foundation lengths. The results of Tan et al. (2019) obtained using Matlab were also plotted in Fig. 3 for comparison.

It can be clearly found that both the feasibility robustness indices β_β and P_c which were obtained using Excel are the same as their corresponding values which were obtained using Matlab. This proves the spreadsheet model for the RGD shown in Fig. 2 is correct. As shown in Fig. 2, only a few cells should be added to a spreadsheet model using FORM, so the spreadsheet model for the RGD is easy to set up and user-friendly.

The relationship between the feasibility robustness index (β_β) and the construction cost (*C*) are shown in Fig. 4 using hollow symbols for all acceptable designs. As can be seen from Fig. 4, the feasibility robustness index β_β increases with the increase of construction cost and foundation size. A high value of β_β represents a high level of design robustness, but the construction cost is also high. All the designs in Fig. 4 are acceptable because all the reliability indices of these designs are greater than the target reliability index β^T. The hollow square means the knee point which indicates the best compromise between design robustness and construction cost. Based on Fig. 4, the client can select a proper design based on his or her preference. For example, if the client prefers high level of design robustness, the design of B=4.0 m and L=4.0 m can be selected; if the user prefers low construction cost, the design of B=1.8 m and L=2.0 m can be selected; and if the user does not have special requirement on design robustness and construction cost, a design corresponding to the knee point of the $\beta_\beta \sim C$ curve, (B=3.0 m and L=2.8 m), is suggested to be selected.

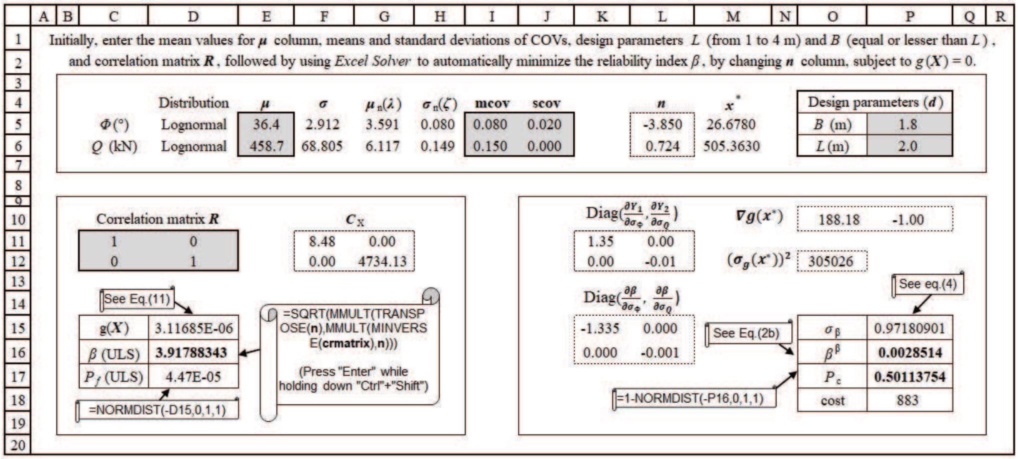

Figure 2. Layout of a spreadsheet for the new RGD.

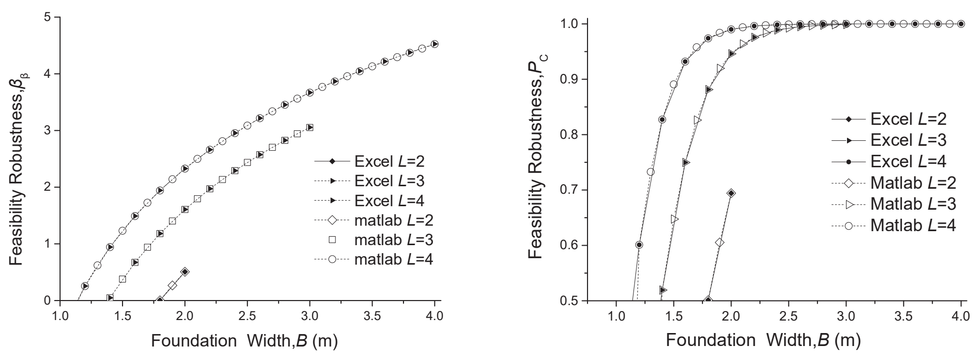

Figure 3. Feasibility robustness index. (a) β_β; (b) P_c for acceptable designs.

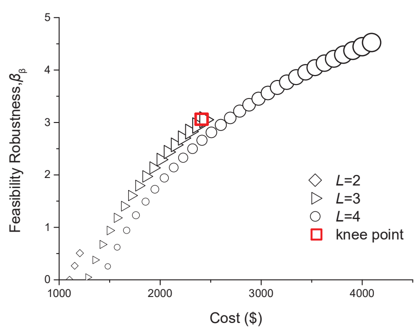

Figure 4. Relationship between the Cost and feasibility robustness (β_β) for acceptable designs.

4 Conclusion

There are many uncertainties in geotechnical engineering. The uncertainties of statistics of soil parameters will results in the over dangerous or over conservative designs of geotechnical structures. To ensure a design is robust against the variation of soil parameters, the robust geotechnical design (RGD) method is proposed and used in geotechnical engineering. In this paper, a modified RGD based on the sensitivity analysis of FORM using spreadsheet is developed. Because the sensitivity of reliability index (SRI) with respect to the distribution parameter of basic variables is a by-product of the first order reliability method (FORM), the feasibility robustness index can be easily computed using the FORM. Take the design of a shallow foundation for example, a spreadsheet model for the new RGD is set up. The compassion between the designs with those using Matlab demonstrates the correctness and the simplicity of this spreadsheet model. Although only a shallow foundation was designed in this paper, the spreadsheet model set up in this paper can be used for the design of other geotechnical structures after minor modification.

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