# Efficient Probabilistic Back Analysis of Slopes Accounting for Spatial Variation of Soil Properties

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**Abstract:** The probability distributions of uncertain soil properties can be updated with multiple sources of information including in-site measurements and field observations via probabilistic back analysis. The updated probability distribution can be further used for more realistic slope stability assessment. However, few attempts have been made to conduct probabilistic back analyses accounting for the inherent spatial variation of soil properties. This paper proposes an efficient probabilistic back analysis approach by integrating random field modeling, Bayesian updating and subset simulation. A real slope is investigated to illustrate the effectiveness of the proposed approach, in which the field observations on slope failure and location of the slip surface are incorporated in Bayesian updating.

Keywords: Slope; spatial variability; probabilistic back analysis; Bayesian updating.

## 1 Introduction

A precise determination of material properties is an important prerequisite for slope stability analysis. Back analysis integrating with in-situ and/or laboratory testing, monitoring and field observation is one of important paths to understand the material properties and reduce slope failure risk (e.g., Duncan et al. 1999). However, it is not realistic to conduct in-situ testing and monitoring everywhere, some uncertainty remains due to the spatial variability of soil properties between measurement locations. The spatial variability not only can increase the uncertainty in predicting the material properties, but also can significantly influence the slope stability (e.g., Phoon and Kulhawy 1999; Jiang et al. 2016; Li et al. 2019). Although many sophisticated back analysis methods have been proposed for the estimation of soil properties, uncertainties due to the spatial variation are not properly accounted for in the back analyses (e.g., Gilbert et al. 1998; Zhang et al. 2010a, b; Wang et al. 2013; Ering and Sivakumar Babu 2016). Therefore, it is essential to perform probabilistic back analysis incorporating the spatial variability of soil properties based on multiple sources of site-specific information (e.g., test data, monitoring data and field observations).

Many researchers have realized the importance of the inherent spatial variation of soil properties and taken it into account in the probabilistic back analyses and even in the reliability updating of geotechnical systems (e.g., Miranda et al. 2009; Papaioannou and Straub 2012; Ering and Sivakumar Babu 2017; Huang et al. 2018; Yang et al. 2018). Although the posterior distributions of soil properties can be evaluated analytically or numerically by sampling approaches in the back analyses, it is computationally demanding and not easy to implement. This is because the inference of the posterior distribution requires solving a high-dimensional integral. In addition, the current commonly-used analytical and numerical approaches are not effective to tackle the back analysis problems incorporating the spatial variation. For instance, the analytical solution of posterior distribution can be achieved only for particular conjugate priors (Ang and Tang 2007). The maximum likelihood method may induce a biased estimate of posterior distribution when the output response is a nonlinear function of uncertain input parameters (e.g., Zhang et al. 2010a; Wang et al. 2013). The Markov chain Monte Carlo simulation is inefficient for high-dimensional Bayesian inference problems due to the limitations including slow convergence, the choice of the proposal probability density function (PDF) and determination of the burn-in period (e.g., Ching and Wang 2016). The BUS approach (Bayesian Updating with Structural reliability methods) originally proposed by Straub and Papaioannou (2015) has been shown to be effective in sampling high dimensional posterior distribution (e.g., Betz et al. 2018; Jiang et al. 2018), but it is tedious and time-consuming because the evaluating of likelihood multiplier is coupled with subset simulation run (DiazDelaO et al. 2017). To this end, this study proposes an efficient probabilistic back analysis approach of slopes accounting for the spatial variability of soil properties and develops a stopping criterion for subset simulation (SuS) to solve the Bayesian updating problems. The proposed approach does not require evaluating the likelihood multiplier with the aid of the developed stopping criterion.

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## 2 Probabilistic Back Analysis of Slopes

### 2.1 Construction of likelihood function

The BUS approach is extended herein for probabilistic back analysis of spatially varying soil properties. One critical step for Bayesian updating is to construct likelihood function, $L(\boldsymbol{x})$, which is proportional to the probability of the observation event given $\boldsymbol{X} = \boldsymbol{x}$ (e.g., Ang and Tang 2007). Where $\boldsymbol{X}$ is the vector of uncertain input parameters with a size of $n$, $\boldsymbol{X} = \left(X_1, X_2, \cdots, X_n\right)^{\mathrm{T}}$ ; $\boldsymbol{x}$ is the realization of $\boldsymbol{X}$. In geotechnical practice, the observed slope failure or landslide information can be well utilized to back analyze the uncertain input parameters and update the understanding on the slope performance (e.g., Zhang et al. 2010b; Wang et al. 2013; Ering and Sivakumar Babu 2016, 2017). In theory, a slope failure or landslide implies that the factor of safety of the slope at the moment of failure is equal to unity. In practice, there might be uncertainties in defining slope failure. It is assumed that the slope failure is well defined such that the uncertainties associated with slope failure definition are minimized (e.g., Zhang et al. 2010b; Wang et al. 2013). In this way, the likelihood function that indicates the chance to observe slope failure can be established as

$$L(\boldsymbol{x}) = \phi\left[\frac{FS(\boldsymbol{x}) + \mu_\zeta - 1.0}{\sigma_\zeta}\right] \tag{1}$$

where $\phi(.)$ is the cumulative distribution function of a standard normal variable; $\zeta$ is a model correction factor for characterizing the uncertainty of slope stability model, which is frequently assumed to follow the normal distribution with a mean of $\mu_\zeta$ and a standard deviation of $\sigma_\zeta$ (e.g., Christian et al. 1994; Zhang et al. 2010b; Wang et al. 2013); $FS(\boldsymbol{x})$ is the factor of safety calculated using limit equilibrium or finite element methods.

### 2.2 Inference of posterior distribution

The BUS approach has been utilized to learn the probability distribution of spatially varying soil properties (e.g., Straub and Papaioannou 2015; Jiang et al. 2018) through transforming a high-dimensional updating problem into an equivalent structural reliability problem. The SuS is then employed to solve the structural reliability problem (Au and Beck 2001). Within the BUS approach with SuS, the likelihood function $L(\boldsymbol{x})$ is utilized to define an observation domain $\Omega_X$ in an augmented outcome space $\boldsymbol{x}_+ = [\boldsymbol{x}; u]$:

$$\Omega_X = \{u < cL(\boldsymbol{x})\} \tag{2}$$

where $u$ is the outcome of a standard uniform random variable $U$ in [0, 1]; $c$ is a likelihood multiplier satisfying the following inequality for all $\boldsymbol{x}$:

$$cL(\boldsymbol{x}) \le 1.0 \tag{3}$$

The probability of the event $Z = \{\boldsymbol{x}_+ \in \Omega_X\}$, $P(Z)$, which is referred to as "acceptance probability" can be evaluated as a product of larger conditional probabilities of a set of nested intermediate events:

$$P(Z) = P\left[H(\boldsymbol{x}_+) < 0\right] = P(Z_1)\prod_{i=2}^{m} P\left(Z_i \middle| Z_{i-1}\right) \tag{4}$$

where $P(.)$ denotes the probability of an event; $H(\boldsymbol{x}_+)$ is a driving variable, $H(\boldsymbol{x}_+) = u - cL(\boldsymbol{x})$ ; $Z_1 \supset Z_2 \supset \ldots \supset Z_{m-1} \supset Z_m$ are intermediate events defined as $Z_i = \{H(\boldsymbol{x}_+) < g_i\}$, in which $g_i$, $i = 1, 2, \ldots, m$, are threshold values satisfying $g_1 > g_2 > \ldots > g_{m-1} > 0 \ge g_m$; $P(Z_1)$ is the probability corresponding to the first level of SuS; $P(Z_i|Z_{i-1})$ is the conditional probability of $Z_i$ given $Z_{i-1}$; $m$ is the number of levels of SuS required to reach the observation domain. Therefore, sampling the posterior distribution becomes equivalent to sampling the failure domain of the structural reliability problem for determining $P(Z)$. The thresholds $g_i$, $i = 1, 2, \ldots, m$, are chosen adaptively such that the intermediate conditional probabilities take a target value $p_0$.

According to Eq. (3), the largest admissible value $c_{\max} = 1\big/\max\limits_{X} L(\boldsymbol{x})$ of the multiplier $c$ can be o although the value of $c$ is unknown before computation (Straub and Papaioannou 2015). Note that bias will be induced in the distribution of the samples if a value larger than $c_{\max}$ is used. Conversely, adopting a value smaller than $c_{\max}$ can obtain correct samples following the posterior distribution but less efficient. Moreover, the computational cost of the BUS approach decreases linearly with the logarithm of $P(Z)$, which in turn is proportional to the value of the multiplier $c$ as deduced from Eqs. (2) and (4). Thus, it is beneficial to choose $c$ as large as possible such that the inequality in Eq. (3) holds (Betz et al. 2018; Jiang et al. 2018). Although Straub and Papaioannou (2015), Betz et al. (2018) and Jiang et al. (2018) developed adaptive approaches to evaluate the value of $c$ as the reciprocal of the maximum of the likelihood function, it is found that they are tedious and time-consuming since the evaluation of $c$ is coupled with the SuS run. To remove the limitations, the observation domain $\Omega_X$ is rewritten as

$$\Omega_X = \left\{ \ln\left[\frac{L(\boldsymbol{x})}{u}\right] > -\ln c \right\} \tag{5}$$

Correspondingly, the driving variable $H(\boldsymbol{x}_+)$ is $\ln\left[\frac{L(\boldsymbol{x})}{u}\right]$, and the acceptance probability $P(Z)$ is evaluated as

$$P(Z) = P\left[H(\boldsymbol{x}_+) > b\right] = P(Z_1)\prod_{i=2}^{m} P\left(Z_i \middle| Z_{i-1}\right) \tag{6}$$

where $b$= -ln$c$ is the threshold value. The intermediate events are defined as $Z_i = \{H(\boldsymbol{x}_+) > b_i\}$, in which $b_i$, $i$ = 1, 2, …, $m$, are threshold values satisfying $b_1 < b_2 < \ldots < b_m$. With such an adjustment, the driving variable $H(\boldsymbol{x}_+)$ no longer depends on the multiplier $c$ and the choice of $c$ is deliberately decoupled with the SuS run. The multiplier $c$ only affects determination of the threshold value $b_{\min} = -\ln c_{\max}$ beyond which the samples can be accepted as the posterior samples. The samples obtained from the highest level of SuS will invariably follow the posterior distribution as long as the multiplier $c$ is sufficiently small to satisfy the inequality in Eq. (3). This implies that the distribution of the samples conditional on $Z$ will settle at the posterior PDF of spatially varying soil properties as long as $b$ is larger than $b_{\min}$.

### 2.3 Determination of stopping criterion for SuS

Similar to $c_{\max}$, the value of $b_{\min}$ is generally unknown but does not affect the SuS run for evaluating $P(Z)$ in the improved BUS formulation. The SuS can be carried out with increasing levels until one determines that the threshold value $b_m$ of the highest level has passed $b_{\min}$. One crucial problem is how to judge whether $b$ has passed $b_{\min}$. It turns out to be a more well-defined task if the complementary cumulative distribution function (CCDF) of $H(\boldsymbol{x}_+)$, i.e. $P\left[H(\boldsymbol{x}_+) > b\right]$ versus $b$, is found to have distinctly different characteristics for $b < b_{\min}$ and $b > b_{\min}$ (DiazDelaO et al., 2017). Following DiazDelaO et al. (2017), the probability $P\left[H(\boldsymbol{x}_+) > b\right]$ can be expressed as

$$P\left[H(\boldsymbol{x}_+) > b\right] = P_D e^{-b} \qquad (b > b_{\min}) \tag{7}$$

where $P_D$ is model evidence. As seen from Eq. (7), the CCDF of $H(\boldsymbol{x}_+)$ will be converted into an exponential decay function once $b$ has passed $b_{\min}$. The corresponding curve of $\ln P\left[H(\boldsymbol{x}_+) > b\right]$ versus $b$ will be changed to a decreasing line with a slope of -1:1 once $b > b_{\min}$. Whether $b$ has passed $b_{\min}$ can be determined according to the variation of $\ln P\left[H(\boldsymbol{x}_+) > b\right]$ with $b$. Obviously, this stopping criterion for SuS is subjective and qualitative. It is of necessity to develop a stopping criterion for SuS that has good computational operability.

As mentioned earlier, to obtain the samples following the posterior PDF, $b_m > b_{\min}$ as well as the inequality in Eq. (3) must be guaranteed. In other words, the following inequalities should be met at the same time:

$$e^{-b_i} L(\boldsymbol{x}) < e^{-b_{\min}} L(\boldsymbol{x}) \le 1.0 \tag{8}$$

It is not possible to determine an analogous right-hand side of inequality in Eq. (8) because the $b_{\min}$ is unknown in advance. To this end, an inadmissible set $B_i = \left\{e^{-b_i} L(\boldsymbol{x}) > 1.0\right\}$ is defined, and the prior probabilities of the inadmissible sets $B_i$ are given by (DiazDelaO et al. 2017)

$$a_i = P(B_i) = P\left[L(\boldsymbol{x}) > e^{b_i}\right] \tag{9}$$

Thereafter, the values of $a_i$, $i$ = 1, 2, …, $m$, are estimated, respectively, to judge whether the SuS run shall be terminated. As confirmed in DiazDelaO et al. (2017), as the level of SuS increases, the inadmissible set is monotonously decreasing and gradually approaches a null set, and the corresponding probability $a_i$ is a monotonously decreasing sequence of values converging to zero. Note that direct computation of $a_i$ is challenging since it involves a multiple integral. In this study, the value of $a_i$ is evaluated by means of performing an inner SuS run with $b_i$ obtained from the outer SuS run as input. The outer SuS run will be terminated once $a_i$ is found to be below $10^{-8}$. Then the failure samples are extracted from the highest level of outer SuS to infer the posterior distributions of spatially varying soil properties. It should be mentioned here that the value of $a_i$ can be set to be zero if it is less than $10^{-8}$, which leads to significant savings in computational costs, without significant loss of accuracy in the estimate of $a_i$. Additionally, the inner SuS run for evaluating $a_i$ is independent of the outer SuS run for determining $P(Z)$ and inferring the posterior distributions of soil properties.

## 3 Application to Congress Street Cut in Chicago

In this section, the proposed approach is applied to back analyze the uncertain shear strength parameters of a real slope named Congress Street cut with four soil layers. The reliability of this cut considering the spatial variability of soil properties has been investigated by Jiang et al. (2016) and Li et al. (2019). A portion of the Congress Street "superhighway", just east of Halsted Street, in Chicago, was built in an open cut. As reported in Ireland (1954), Congress Street cut was located mainly in saturated clays. The cut on south side with a length of about 200 feet failed in an undrained manner in 1952 during the construction of Congress Street in Chicago. The effect of pore water pressures on the slope stability was negligible.

**Table 1.** Prior knowledge of undrained shear strengths.

| Soil layer | Variable | Distribution | Mean | Standard deviation | Lower bound | Upper bound |
|---|---|---|---|---|---|---|
| Clay 1 | $s_{u1}$ (kPa) | Truncated normal | 136 | 50 | 0 | 272 |
| Clay 2 | $s_{u2}$ (kPa) | Truncated normal | 80 | 15 | 0 | 160 |
| Clay 3 | $s_{u3}$ (kPa) | Truncated normal | 102 | 24 | 0 | 204 |

Following Chowdhury and Xu (1995), the cut has a height of 14.1 m and two slope angles of 36.3° and 36°, respectively. Below the upper layer of sand, there are these clay layers denoted, from the top down, as clays 1, 2 and 3, respectively, each of which has its corresponding undrained shear strength (see Figure 1). The sand layer has negligible influence on the slope stability because of zero cohesion and low normal stress (Chowdhury and Xu 1995). Therefore, the cohesion and friction angle of sand are treated as deterministic quantities, they equal to 0 and 30°, respectively. The undrained shear strengths, $s_{u1}$, $s_{u2}$ and $s_{u3}$, of three clay layers are modeled as truncated normal random fields. A two-dimensional exponential autocorrelation function is adopted to characterize the spatial variation of the undrained shear strength in each clay layer. The horizontal and vertical scales of fluctuation of 40 m and 4.0 m are selected based on the typical variation ranges summarized in Phoon and Kulhawy (1999). Table 1 summarizes the prior knowledge of the undrained shear strengths. The random fields in three clay layers are discretized into 434, 773 and 522 elements with a side length of 0.5 m, respectively. The total unit weights of four soil layers are 18.5 kN/m$^3$. Based on the means of uncertain input parameters, the factor of safety calculated using ordinary method of slices is equal to 2.27, which is identical to the value (i.e., 2.1396) as reported in Chowdhury and Xu (1995). In addition, the critical slip surface is also located which passes through four different soil layers as shown in Figure 1.

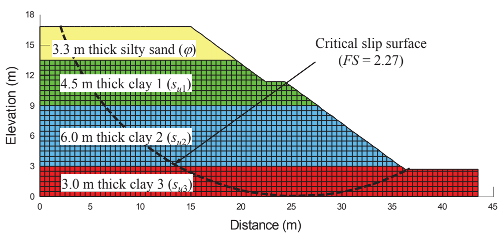



**Figure 1.** Geometry, random field mesh and stability analysis result of Congress Street cut in Chicago.

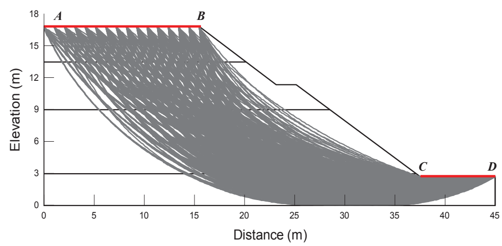
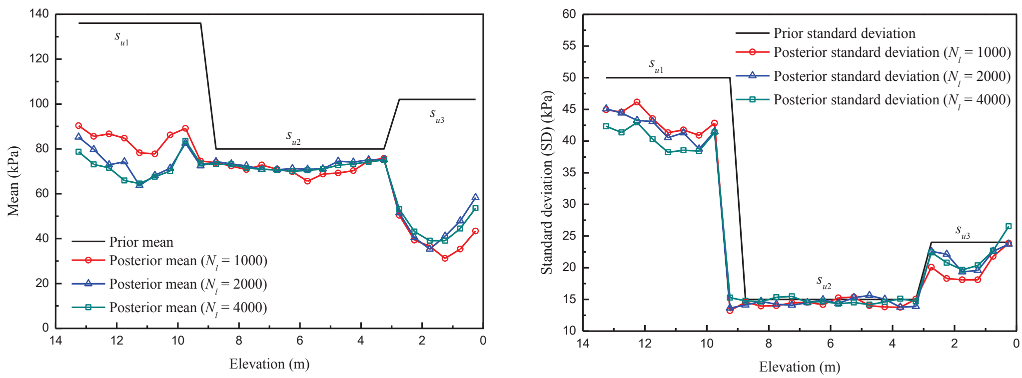



**Figure 2.** Slope with 1164 randomly generated potential slip surfaces.

Figure 1 in Ireland (1954) showed the approximate geometrical size of the cut and the location of slip surface at the moment of cut failure, from which two important field observations can be obtained: (1) The observed slope failure (i.e., $FS \leq 1.0$) information can be used to construct the likelihood function using Eq. (1); (2) The observed entry and exit regions of the slip surface can be utilized to generate the potential slip surfaces for slope stability analysis. As shown in Figure 2, a total of 1164 potential slip surfaces that cover the failure domain of the cut are generated randomly based on the entry and exit regions (*AB* and *CD*).

Following Christian et al. (1994) and Zhang et al. (2010b), the model correction factor $\zeta$ that is assumed to be normally distributed with mean $\mu_\zeta$ = 0.05 and standard deviation $\mu_\zeta$ = 0.07 is employed to construct the likelihood function. With the likelihood function and prior knowledge of undrained shear strengths, the proposed approach is adopted to infer the posterior distributions of $s_{u1}$, $s_{u2}$ and $s_{u3}$. To yield satisfactory computational results, 10 independent runs of the BUS approach with SuS are carried out. The averages of the posterior statistics of input parameters obtained from these 10 runs are taken as the final results. The conditional probability $p_0$ = 0.1 is chosen, and the number of samples at each level ($N_l$) is determined through parameter sensitivity analysis. Figure 3(a) and (b) compare the posterior means and standard deviations of $s_{u1}$, $s_{u2}$ and $s_{u3}$

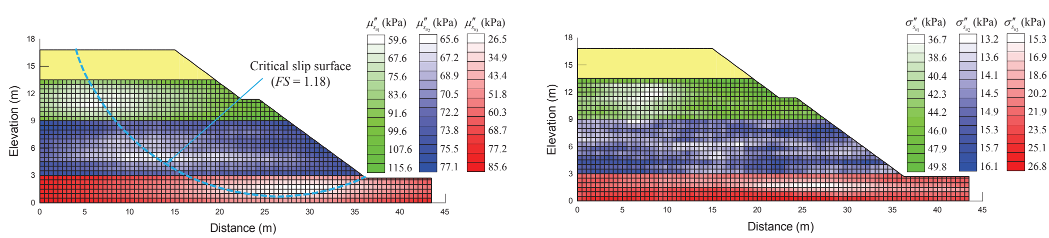
along the vertical direction ($x$ = 15.25) associated with $N_l$ = 1000, 2000 and 4000, respectively. The prior means and standard deviations are also plotted in Figure 3 for comparison. It can be observed that the $N_l$ also has an important effect on the posterior means and standard deviations of $s_{u1}$, $s_{u2}$ and $s_{u3}$. Nevertheless, the posterior means and standard deviations of $s_{u1}$, $s_{u2}$ and $s_{u3}$ gradually converge as $N_l$ increases. To balance the computational accuracy and efficiency, $N_l$ = 2000 is chosen. After incorporating the field observations, the random fields of $s_{u1}$, $s_{u2}$ and $s_{u3}$ are no longer stationary since the posterior means and standard deviations vary distinctly along different spatial locations. As seen from Figure 3, the means and standard deviations of $s_{u1}$ and $s_{u3}$ are modified more noticeably than those of $s_{u2}$. It indicates the undrained shear strengths underlying the first and third clay layers affect the slope stability more significantly in comparison to that underlying the second clay layer.




(a) Means (b) Standard deviations

**Figure 3.** Comparison of prior and posterior statics of undrained shear strengths along the vertical direction ($x$ = 15.25 m).

Figure 4(a) and (b) depict the estimated posterior means ($\mu''$) and standard deviations ($\sigma''$) of $s_{u1}$, $s_{u2}$ and $s_{u3}$ within the slope profile. The dark and light shaded regions indicate areas of high and small values, respectively. After incorporating the observed slope failure information, the obtained random fields of $s_{u1}$, $s_{u2}$ and $s_{u3}$ are no longer stationary. Based on the posterior means of $s_{u1}$, $s_{u2}$ and $s_{u3}$, the *FS* calculated using ordinary method of slices is 1.18, which matches with the field observation of slope failure within an allowable model error range. In addition, the statistics of the undrained shear strengths at the locations around the slip surface are updated the most and the updating weakens gradually for the locations away from the slip surface (see Figure 4).




(a) Posterior means (b) Posterior standard deviations

**Figure 4.** Posterior means and standard deviations of undrained shear strengths within the slope profile.

## 4 Conclusions

This paper proposes an efficient probabilistic back analysis approach of slopes for learning the probability distributions of spatially varying soil properties. The effectiveness of the proposed approach has been demonstrated by a real slope example. The proposed approach can well infer the posterior distributions of spatially variable soil properties. Unlike the original BUS approach, the proposed approach does not require evaluating the likelihood multiplier in advance. A stopping criterion for subset simulation is also developed. With the aid of the developed stopping criterion, the proposed approach can obtain failure samples that invariably follow the target posterior distributions more conveniently. The effectiveness of the stopping criterion can be validated through the complementary cumulative distribution function (CCDF) of driving variable. The field observations and inherent spatial variation has an important influence on learning the probability distributions of soil properties. Once the field observations as well as inherent spatial variation are incorporated into Bayesian updating, the obtained random fields of soil properties are no longer stationary. To promote the application of the proposed approach in engineering practice, a user-friendly code that can be easily integrated to standalone numerical codes needs to be developed for reliability-based design of slopes.

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