Stochastic Settlement Simulation of Soil Deposit Using a New Efficient Spatial Correlation Model

Qingxia Yue[1]

[1]School of Civil Engineering, Shandong Jianzhu University, Jinan, China.
E-mail: yueqx@sdjzu.edu.cn

Abstract: Based on the cone penetration test (CPT) data from a certain site at Shandong Province, China, the autocorrelation function for tip resistance and sleeve friction of the soil deposit is studied. Four types of soil including silty, silty sand, silty clay and clay are studied individually using four different autocorrelation models. In the models, a recently developed autocorrelation function, involving a linear, an exponential, and a cosine terms, named linear-exponential-cosine (LNCS), is used to simulate the spatial properties of the soil deposits. It is found that the autocorrelations of the tip resistance and sleeve friction are similar, and the scale of fluctuation is close, especially to the silty sand soil. Therefore, it can be assumed that the different soil parameters of the same soil deposits can use the same autocorrelation models. This can be a great help for the application of the random field in the geotechnical simulations. Further, the random field is generated using the Karhunen-Loeve expansion method based on the new autocorrelation model (LNCS). Furthermore, a stochastic settlement simulation is done based on a lab experiment; the soil settlement was tested under the uniform load. The settlement is calculated using both a random field, and a homogenous field model. The preliminary results show that the random field model can capture the settlement better than the corresponding homogenous field, as manifested by the experimental measurements.

Keywords: Autocorrelation function; random field; settlement analysis.

1 Introduction

The spatial variation of soil deposits has been widely accepted. There have been many studies on the autocorrelation of the soil properties (Vanmarcke 1977; Baecher and Christian 2003). In this context, a proper and efficient autocorrelation function is one of the key parameters describing the spatial correlation. The autocorrelation has the feature that as the separation distance increases, the correlation reduces. Thus, the most commonly used model is the simple exponential function (Liu and Chen 2010), in which the autocorrelation value decreases from one to zero as the distance increases. However, it is found that this popularly used autocorrelation model is not differentiable at the zero spatial lag, and can not accommodate negative values as the distance increases. In this regard, an improved autocorrelation model, involving a linear, an exponential and a cosine terms, named linear-exponential-cosine (LNCS), was proposed to simulate the spatial properties of the soil deposits. The new model can simulate the autocorrelation better physically, and converges faster (Yue et al. 2018). Meanwhile, it is well known that soil deposits are usually multi-layered with different types of soil. Therefore, it can be more accurate to consider the soil deposit by different soil type separately. On the other hand, there are many different parameters to capture the soil properties. It is reasonable to accept that the spatial correlation of the soil mainly due to the soil type, formation history etc. Thus, the autocorrelation of the different parameters to the same type of soil at the same site should be the same or very close. Gao (1996) studied the scale of fluctuation of different parameters of layers of soil from the same site, and found that the values are very close, while the in-situ test results bigger than the laboratory test results. This is very inspiring for the application of the random field, which means we can use the same autocorrelation parameters from the CPT data with other properties, such as elasticity, strength etc. While, Liu and Chen (2010) studied the spatial correlation structure using the CPT data, and found that the cone tip resistance has larger correlation distances than that of sleeve friction. It intrigues the great interest to study the autocorrelation function based on the same site of the same CPT soundings.

In this context, the autocorrelation of soil deposits is examined herein based on the cone penetration data (CPT) of one site at Shandong Province, China, including silty, silty sand, silty clay and clay soil. The autocorrelation of the tip resistance and sleeve friction from CPT data is analyzed with four autocorrelation models, including a newly developed linear-exponential-cosine (LNCS) model. As an example of case application, the settlement of a soil deposit is calculated and analyzed based on the random field, which is simulated using a two-dimensional Karhunen-Loéve expansion with the new autocorrelation model, and compared with the experimental results.

2 Autocorrelation Model

2.1 CPT site data analysis

Proceedings of the 7th International Symposium on Geotechnical Safety and Risk (ISGSR)
Editors: Jianye Ching, Dian-Qing Li and Jie Zhang

Published by Research Publishing, Singapore.
ISBN: 978-981-11-2725-0; doi:10.3850/978-981-11-2725-0_IS15-10-cd

CPT data have been used by many researches as a powerful tool to analyze the spatial correlation structure of soil sites (Liu et al. 2010; Uzielli et al. 2005). A CPT data provides cone tip resistance and sleeve friction information with an equal sampling interval distance. In this paper, the CPT data were gathered in Shandong province, China, and comprise measurements of cone tip resistance q_c , and sleeve friction f_s . The measurements were recorded at vertical intervals of 0.01m. The site is 300m in length, and 324 m in width; there are 51 soundings in the site, and the soil profile with soil layers is also shown in Figure 1. From the soil profile, it can be seen that the soil types mainly contain silty, clay, silty sand and silty clay. And one sample of the tip resistance q_c and sleeve friction f_s are also shown in Figure 1. Take the soil layer ② of silty soil, layer ③ of clay soil, layer ④ of silty-sand soil, and layer ⑥ of silty clay soil as the representative soil layers.

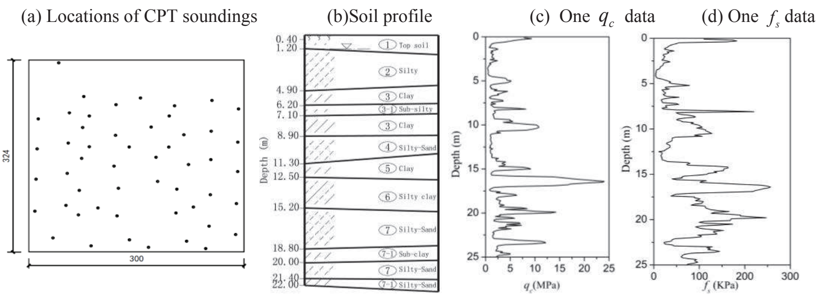


Figure 1. CPT layout and soil profile.

The mean and the coefficient of variation (COV) of the four type of soil are shown in Table 1. It can be seen that the soil data with a high variation even in one site. This also can be compared to the reference of Phoon and Kulhawy (1999), with mean value of q_c for sand 4.1MPa, and 1.59MPa for silty clay soil. The COV of q_c for the sand is between 10-81%, and for the silty clay is 5-40%. It can be seen that the uncertainties of the soil is pretty high, even to the same type of soil.

Table 1. Statistical properties of the soil.

Soil type	Mean of q_c (Mpa)	COV of q_c (%)	Mean of f_s (Kpa)	COV of f_s (%)
silty	1.56	62	25.49	55
clay	1.69	70	33.39	67
Silty sand	5.99	48	81.71	33
Silty clay	2.95	62	101.06	47

2.2 ACF calculation and autocorrelation model

In the statistical analysis of the actual soil data obtained from field or lab tests, obvious 'trends' (changes in average values) are often encountered, most typically as a function of the depth. It is commonly accepted that the trends can be viewed as segments of a large-scale fluctuation, and a large-scale fluctuation must appear as part of the statistical characterization if the trend also exist on other site inference (DeGroot and Baecher 1993; Fenton 1999). The choice of the trend to be removed is a delicate task as it affects the correlation structure and the value of the statistical parameters describing the random filed (Uzielli et al. 2005). Linear trend removal has been used in several variability studies. However, from the field data considered, no apparent linear trend with the depth exists, but it is found that the soil layer type is obvious, as shown in Figure 1.

In this case, it is advantageous to standardize the soil data by substituting each original datum point $q_c(z)$ by the equation

$$q(\mathrm{z}) = \frac{[q_c(z) - \bar{q}_c]}{\tilde{q}_c}$$

(1)

where $\bar{q}_c$ is the mean value of the layer soil, and $\tilde{q}_c$ is the standard deviation.

Next, the following procedures are done to obtain the ACF of the soil parameters. For one of the samples, assume that $x_i = q_c(z_i)$ is the value of the sample at depth $z_i = i\Delta z$, $i = 1, 2, \cdots, n$. The sample covariance function is obtained from the moment estimator

$$C(\tau_j) = \frac{1}{n}\sum_{i=1}^{n-j}(x_i - \mu_x)(x_{i+j} - \mu_x) \tag{2}$$

where $j = 0, 1, 2, \cdots, n-1$; lag $\tau_j = j\Delta z$. Here, μ_x the mean value of the data.

The sample correlation is

$$\rho(\tau_j) = \frac{C(\tau_j)}{C(0)} \tag{3}$$

Proceeding with the calculations based on the above equations, the ACF of the soil data can be obtained.

2.3 Autocorrelation model simulation of the data

Various kinds of autocorrelation models have been employed in the geotechnical literature to fit ACF (Lacasse and Nadim 1996; Phoon et al. 2003; Uzielli et al. 2005). Four kinds of autocorrelation models are considered here (Spanos et al. 2007; Yue et al. 2018), listed in Table 2. Note that the linear cosine exponential (LNCS) model is a recently proposed model, with the feature of alternating sign and differentiability at the origin of the spatial axis.

Vanmarcke (2010) has discussed the concept of the scale of fluctuation, δ , to measure the distance within which the soil property shows relatively strong correlation or persistence from point to point. This parameter can be calculated using the equation

$$\delta = 2\int_0^{\infty} R(\tau)d\tau \tag{4}$$

The scale of the fluctuation of each model is also shown in Table 2.

Table 2. Autocorrelation models.

Model	Autocorrelation function	δ
Simple exponential (SNX)	$R(\tau) = \exp(-k_{SNX}\|\tau\|)$	$2 / k_{SNX}$
Cosine exponential (CSX)	$R(\tau) = \cos(k_{CSX}\|\tau\|)\exp(-k_{CSX}\|\tau\|)$	$1 / k_{CSX}$
Linear exponential (LNX)	$R(\tau) = (1 + k_{LNX}\|\tau\|)\exp(-k_{LNX}\|\tau\|)$	$4 / k_{LNX}$
Linear cosine exponential (LNCS)	$R(\tau) = (1 + k_{LNCS}\|\tau\|)\cos(k_{LNCS}\|\tau\|)\exp(-k_{LNCS}\|\tau\|)$	$1 / k_{LNCS}$

2.4 Comparison of the ACF of different parameters

Processing the CPT data of two parameters, cone tip resistance q_c and sleeve friction f_s , that collected in the site, with the Eqs. (1)-(3), the autocorrelation can be obtained. Then, the ACF data are simulated with the autocorrelation model, for simplicity, only the linear cosine exponential (LNCS) model is adopted here for the four different soil types by least square approximation method fitting. For enhanced clarity, the average autocorrelation data with the simulated model is used in this paper, shown in Figure 2.

Figure 3 is the comparisons of the scale of fluctuation of different models for the four types of soil, it can be seen that the different autocorrelation models have different scale of fluctuation values, and the LNCS model has smaller value than other three models. Further, it can be seen that the values of model LNCS are much closer for the parameters. Furthermore, the silty sand soil is the most similar one, while the silty clay soil with the greatest variation among these four kinds of soil. Nevertheless, the value varies between 0.15-0.4. Therefore, it can be assumed that the different soil parameters of the same soil deposits can use the same autocorrelation models.

(a)Silty soil (b) Clay soil (c) Silty sand soil (d) Silty clay soil

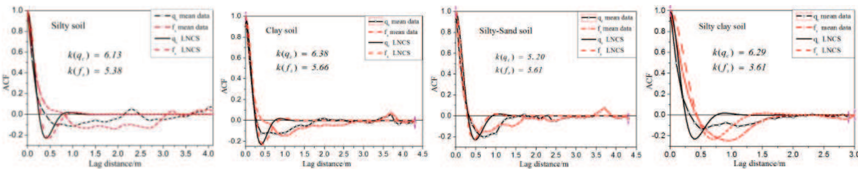


Figure 2. Autocorrelation function comparison and fitting of the mean of q_c and f_s.

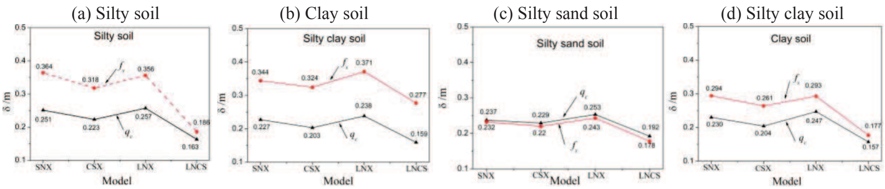


Figure 3. Scale of fluctuation for different models and soil types.

3. Random Field Simulation with Karhunen-Loeve Expansion

The autocorrelation model provides the spatial structure of the soil. For the simulation of the soil, the random field simulation supplies an approach for the application in numerical calculations of soil engineering. There are many simulation method including local average method, spectrum representation method, and the Karhunen-Loeve method, etc. The Karhunen-Loeve expansion method (Spanos and Ghanem 1989; Ghanem and Spanos 1991), often used to capture uncertainty in engineering applications is adopted here for the simulation of the random field.

3.1 Basic concept of the Karhunen-Loeve expansion

A stochastic process $X(\tau,\theta)$ indexed on a bounded domain D, and having zero mean (for convenience) and finite variance, can be represented using a finite Karhunen-Loeve (K-L) series

$$X(\tau,\theta) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\,\xi_i(\theta)\phi_i(t) \tag{5}$$

where $\xi_i(\theta)$ is a set of uncorrelated standardized random variables with zero mean and unit variance. If $X(\tau,\theta)$ is a Gaussian process, then an appropriate choice of $\xi_i(\theta)$ is a vector of uncorrelated standard Gaussian random variables; $\{\phi_i\}$ and $\{\lambda_i\}$ are the eigenfunctions and eigenvalues of the covariance function $C(\tau_1,\tau_2)$, respectively. They satisfy the homogenous Fredholm integral equation

$$\int_T C(s,t)\phi_i(t)dt = \lambda_i\phi_i(s) \tag{6}$$

In most cases, numerical methods are required to solve Eq.(6). In this paper, the integral was evaluated numerically by Simpson's quadrature scheme. The non-zero-mean stochastic process can be expressed as

$$X(\tau,\theta) = \mu + \sigma\sum_{i=1}^{M} \sqrt{\lambda_i}\,\xi_i(\theta)\phi_i(t) \tag{7}$$

The truncated Karhunen-Loeve expansion is optimal in the sense of a mean square error minimization. For a particular application, the number of terms M to be chosen depends on the desired accuracy, and on the complexity of the autocorrelation function of the random field. Ordinarily, in most engineering applications, less than 10 terms suffice.

3.2 Two dimensional field simulation using Karhunen-Loeve expansion

Note that extension of the preceding developments to two dimensional field defined for the correlation function on a rectangular domain can be achieved, as well. For a certain class of covariance functions, which are separable, the eigenvalue and the eigenfunction for $X(\mathbf{t})$ are also separable. Thus, the random field can be expressed as truncated expression in the form

$$X(\mathbf{t}) = X(t_1, t_2) = \sum_{k=1}^{N} \sqrt{\lambda_k}\,\phi_k(t_1, t_2)\xi_k$$

(8)

where $\lambda_k = \prod_{j=1}^{d} \lambda_{i_j}^{(j)}$, $\phi_k(t_1, t_2, \cdots, t_d) = \prod_{j=1}^{d} \phi_{i_j}^{(j)}(t_j)$, $i_j \in N, 1 \le i \le d$, d is the dimension.

4 A Case Study of Soil Settlement Analysis

The settlement analysis is a subject of considerable interest to practicing engineers since excessive settlement often lead to problems of serviceability. Many calculations on the settlement under foundation have been done by many researchers (Griffiths and Fenton 2009; Fenton and Griffiths 2005). While due to the wide variety of soil types, experimental data on settlement of footings founded on soil are limited. In this context, an experiment was done in the laboratory, and some numerical calculation was carried out taking the elastic modulus as random field, using the K-L expansion method mentioned above. The numerical results were compared with the experimental results.

4.1 Soil settlement experiment

The experiment was conducted in a container, The testing tank was designed as rigid box with dimensions of 1m (length), 1m (width), and 1.5m (height), shown in Figure 4. The silty clay soil was chosen for the experiment. The soil parameters are as follow: unit weight, $\gamma = 18\text{kN/m}^3$, cohesion $c = 20\text{KPa}$, friction angle $\varphi = 20^o$, and the elastic modulus $E = 7.5\text{MPa}$. The Poisson ratio is taken as 0.25. Vertical load was applied using the steel plates on a plate with size of 0.5m in square to simulate the uniformly distributed loading. Three steps of pressure were adopted for the uniform pressure as 40kN/m^2, 80kN/m^2, and 120kN/m^2. Figure 5 shows the settlement curve versus time for the three loading steps.

(a) Model box (b) loading system (a) Load step 1(40kN/m^2) (b) Load step 2(80kN/m^2) (c) Load step 3(120kN/m^2)

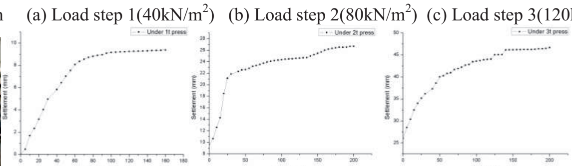


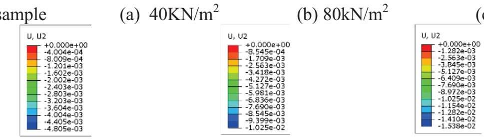
Figure 4. Test setup. **Figure 5.** The time-settlement curve of the experiment.

4.2 Finite element simulation

To study the settlement of soil using the random field model, a settlement calculation is done based on the above experimental case. A two dimensional finite element model (Figure 6a) was established with the same soil parameters as the experiment, while the elasticity of the soil was taken as the random field, and the Mohr-Coulomb model was adopted to simulate the soil. The vertical boundaries are fixed on the horizontal direction and the bottom boundary is fixed.

Using the Karhunon-Loeve expansion method, and the autocorrelation function LNCS is adopted, the soil random field can be obtained. Since the soil type is silty clay, and based on the research about the different soil parameters of the same soil is very close, and can be considered as having the same autocorrelation parameter. Thus, the K_{LNCS} in the LNCS model is taken as 6.29 from Figure 3 of the q_c in the vertical direction. Therefore, the corresponding scale of fluctuation is 1/6.29, which equals 0.16m. Note that in natural site deposits, correlation in vertical direction tends to have much shorter distances than in the horizontal direction. A ratio of about one to ten for these correlation distances is common. Here, the ratio of 10 is adopted. Thus, the horizontal scale of fluctuation is 1.6m accordingly in the simulation. Figure 6(b) is one sample of the random field. Figure 7 shows the vertical settlement of the soil with three levels of the pressure with the random field. It can be seen that the settlement at the same section is not uniform any more due to the random field effect.

(a) FE model (b) Random field sample (a) 40KN/m^2 (b) 80kN/m^2 (c) 120kN/m^2



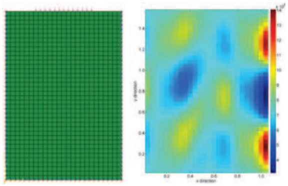


Figure 6. FE model and random field sample.

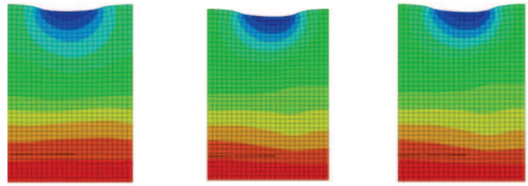


Figure 7. Settlement of the soil deposit with different pressures.

To compare the numerical results with the experimental results, a Monte Carlo simulation with 1000 times was done with the random field. Simultaneously, the mean value of soil elasticity modulus is taken as the homogenous field. Figure 8 shows the stress and settlement comparison of the test to the simulations with both random field and homogeneous (mean value) field. It can be seen that the random field simulation results are much closer to the experimental result.

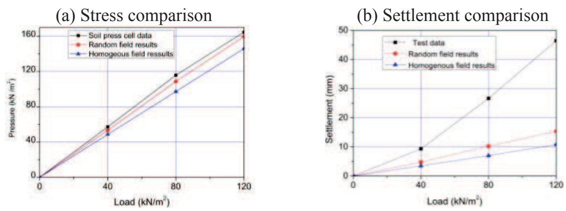


Figure 8. Comparison results between the simulation and experiment

5 Concluding Remarks

The autocorrelation model of the tip resistance and sleeve friction based on the CPT data has been analyzed, using a recently established autocorrelation model (LNCS). It has been found that the correlation parameters of various models are different, while not much variation to the values of the scale of fluctuation, especially to the silty sand soil has been noted. Further, the settlement calculations with the random field and homogenous field have been undertaken, and some comparisons have also been done. The results have shown that, in general, the random field can capture the attributes of the settlement better than the homogenous field. Additional studies are clearly warranted to further elucidate the findings of this preliminary study, and to report on this interesting problem of stochastic geotechnical analysis.

Acknowledgement

This research has been generously supported by the National Natural Science Funds of China (Grant No. 51879395 and 51678350) which is gratefully acknowledged by the authors.

References

Baecher, G.B. and Christian, J.T. (2003). *Reliability and Statistics in Geotechnical Engineering*, John Wiley & Sons.

DeGroot, D.J. and Baecher, G.B. (1993). Estimating autocovariance of in situ soil properties. *Journal of Geotechnical Engineering, ASCE*, 119(1),147-166.

Fenton, G. (1999). Random field modeling of CPT data. *Journal of Geotechnical and Geoenvironmental Engineering*, 125 (6), 486-498.

Fenton, G.A. and Griffiths, D.V. (2005).Three-dimensional probabilistic foundation settlement. *Journal of Geotechnical and Geoenvironmental Engineering*, 131(2), 232-239.

Gao, D. (1996). Safety index and its application in the geotechnical design. *Geotechnical Investigation and Surveying*, (1), 1-6. (in Chinese: 高大钊 (1996).岩土工程设计安全度指标及其应用. *工程勘察*, (1), 1-6.

Ghanem, R.G and Spanos, P.D. (1991). *Stochastic Finite Elements: A Spectral Approach*, New York, Springer-Verlag.

Griffiths, D.V. and Fenton, G.A. (2009). Probabilistic settlement analysis by stochastic and random finite-element methods. *Journal of Geotechnical and Geoenvironmental Engineering*, 135(11), 1629-1637.

Lacasse, S. and Nadim, F. (1996). Uncertainties in characterizing soil properties, In *Uncertainty in the Geologic Environment: From Theory to Practice*, 49-75, ASCE Geotechnical Special Publication No.58.

Liu, C.N. and Chen, C.H. (2010). Estimating spatial correlation structures based on CPT data. *Georisk*, 4(2), 99-108.

Phoon, K.K. and Kulhawy, F. H. (1999). Characterization of geotechnical variability. *Can. Geotech. Journal*, 36, 612-624.

Phoon, K.K., Quek, S.T., and An, P. (2003). Identification of statistically homogeneous soil layers using modified Bartlett statistics. *Journal of Geotechnical and Geoenvironmental Engineering*, 129(7), 649-659.

Spanos, P.D. and Ghanem, R. (1989). Stochastic finite element expansion for random media. *Journal of Engineering Mechanics*, 115(5), 1035-1053.

Spanos, P.D, Beer, M., and Red-Horse, J. (2007). Karhunen–Loéve expansion of stochastic processes with a modified exponential covariance kernel. *Journal of Engineering Mechanics*, 133(7), 773-779.

Uzielli, M., Vannucchi, G., and Phoon, K.K. (2005). Random field characterization of stress normalized cone penetration testing parameters. *Geotechnique,* (1), 3-20.

Vanmarcke, E. (1977). Probabilistic modeling of soil profiles. *Journal of the Geotechnical Engineering Division*, 103(11), 1227-1246.

Vanmarcke, E. (2010). *Random Fields – Analysis and Synthesis. Revised and Expanded New Edition*, World Scientific Publishing Co. Pte. Ltd, Singapore.

Yue Q., Yao J., Ang H.-S. A., and Spanos P. D. (2018). Efficient random field modeling of soil deposits Properties. *Soil Dynamics and Earthquake Engineering*, 108, 1-12.