Spatial Variability of Sandy Clay: Case Study

Rongjie He[1], Wuzhang Luo[1], and Jinhui Li[1*]

[1]Department of Civil and Environmental Engineering, Harbin Institute of Technology, Shenzhen, China.
* corresponding author, E-mail: jinhui.li@hit.edu.cn

Abstract: Soils are natural materials and demonstrate spatial variability. The autocorrelation function and scale of fluctuation of soil properties are often used to quantify the spatial variability. The assessment of the spatial variability of undrained shear strength and cone resistance have been reported by many researchers. The scale of fluctuation in horizontal direction is difficult to be evaluated due to the lack of data. Based on 138 boreholes drilled in-site in Shenzhen, China, this paper investigates the scale of fluctuation of different soil properties in horizontal direction. These data refer to five soil parameters including cohesion, friction angle, compression modulus, liquid limit and plastic limit of soils. The mean, standard deviation, coefficient of variation and spatial variability of these properties are reported. Based on random field theory, a squared exponential autocorrelation function is found to best fit the autocorrelation function. The scale of fluctuation of cohesion, friction angle, compression modulus, liquid limit and plastic limit are calculated.

Keywords: Spatial variability; cohesion; friction angle; compression modulus; liquid limit; plastic limit.

1 Introduction

Due to a combination of geologic, environmental and physical-chemical process, the soils have strong spatial variability which is often ignored in most of geotechnical engineering (Dasaka and Zhang 2012; Ching and Phoon 2013; Li et al. 2014; Lloret-Cabot et al. 2014). This variability is currently managed in practice by adopting suitably conservative parameters for design. In reality, the spatial variability has a great impact on stability of geotechnical structures. The failure mechanisms of foundations are significantly affected by the spatial variability of soil (Li et al. 2014). If the mean strength of the soil is used to calculate the bearing capacity of footing, the results are usually overestimated (Griffiths et al. 2002; Cassidy et al. 2013; Huang et al. 2013).

The spatial variability can be very different for different types of soils, directions (such as horizontal, vertical), measurement intervals and estimation methods (Jaksa 1995; Lacasse and Nadim 1996; Phoon and Kulhawy 1999; Uzielli et al. 2005). Particularly, the calculation of the scale of fluctuation in horizontal direction is difficult due to the lack of data.

Based on extensive tests that have been conducted in Shenzhen, China, the spatial variability of different soil properties are investigated in this study. The scale of fluctuation of cohesion, friction angle, compression modulus, liquid limit and plastic limit in horizontal direction are calculated.

2 Details of Site Investigation

The site is located in Bao'an district, western Shenzhen of China as shown in Figure 1(a). The longitude and latitude of the site are 113.8° E, 22.7° N, respectively. The area of the site is about 547 m x 444 m and there are 138 boreholes in this site (see Figure 1(b)). The minimum interval of the boreholes is about 9.4 m. Such dense data in the horizontal direction will benefit the spatial variability analysis. The soil samples from the boreholes were taken to the laboratory to obtain the soil properties including cohesion, friction angle, compression modulus, liquid limit and plastic limit. The cohesion and the friction angle were measured using unconsolidated undrained triaxial test.

3 Soil Properties

There are four types of soil in the site, including gravelly clay, sandy clay, completely weathered granite and strongly weathered granite (see Table 1). There are different numbers of data for different types of the soils. The sandy clay has the greatest number of data, which is enough to analyze the spatially variability accurately. Therefore, the variability of the sandy clay is investigated in the following section.

The liquid limit of the sandy clay ranges from 26.3% to 55% and the plastic limit ranges from 15.8% to 32.8%. The mean values of liquid limit and plastic limit of sandy clay are 41.86% and 24.95%, respectively. The compression modulus ranges from 3.5 MPa to 8.6 MPa and the mean value is 4.57 MPa. The mean value of cohesion and friction angle are 20.3 kPa and 22.49°, respectively. The COV of the compression modulus in this study is 0.15 which is less than the COV of the resilient modulus (i.e. 0.37) reported by Liu et al. (2016). The

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liquid limit and plastic limit have the same COV of 0.14. According to the report of Phoon et al. (1995), the COV of liquid limit ranged from 0.07 to 0.39, and the COV of plastic limit ranged from 0.06 to 0.34. The COV of friction angle is 0.14 which is in the range of 0.1 through 0.5 presented by Phoon et al. (1995). The cohesion has the largest variability and its COV is 0.3 which is in the range of 0.11 through 0.49 presented by Phoon et al. (1995).

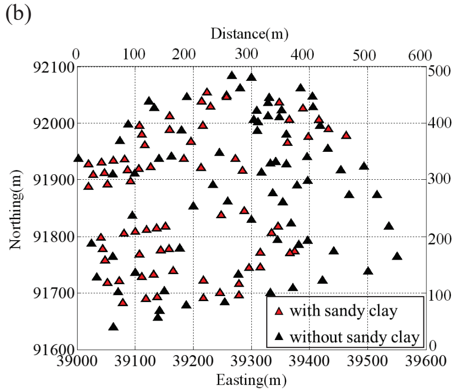

Figure 1. Details of site investigation: (a) location of the site and (b) deployment of the boreholes.

Table 1. The basic soil properties.

Soil type	Statistical items	Liquid limit (%)	Plastic limit (%)	Compression modulus (MPa)	Cohesion (kPa)	Friction angle (°)
Gravelly clay	Sample numerosity	5	5	4		
	Maximum	45.7	27.1	6.5		
	Minimum	34.8	21.2	5.2		
	Mean	41.14	24.66	5.63		
Sandy clay	Sample numerosity	94	94	93	68	68
	Maximum	55	32.8	8.6	31.6	27.9
	Minimum	26.3	15.8	3.5	6.4	16.3
	Mean	41.86	24.95	4.57	20.3	22.49
	Standard deviation	5.86	3.38	0.71	6.4	3.09
	COV	0.14	0.14	0.15	0.32	0.14
Completely weathered granite	Sample numerosity	35	35	34	26	13
	Maximum	49.2	29.2	6.1	30	27.7
	Minimum	28.4	17	4.1	7.8	19.2
	Mean	33.32	22.97	4.81	20.43	23.88
	Standard deviation	5.67	3.33	0.5	6.21	2.22
	COV	0.15	0.14	0.1	0.3	0.09
Strongly weathered granite	Sample numerosity	1	1	1		
	Maximum	33.9	20.8	5.5		
	Minimum	33.9	20.8	5.5		
	Mean	33.9	20.8	5.5		

4 Basic Concepts of Spatial Variability

Soils are natural materials undergoing a combination of geologic, environmental and physical-chemical process, which result in strong spatial variability. The properties of the soils vary from place to place. The mean value and the standard deviation are often used to describe the variability of soils. However, the spatial information among the data is masked by these parameters. Therefore, spatial variability should be used to describe the soil variability in a way that can take the spatial information into account. The spatial variability of soils is often separated into two parts: a deterministic trend and residual variability about that trend (Beacher and Christian 2003):

$$z(x) = t(x) + u(x) \tag{1}$$

where $z(x)$ is the soil property at location x, $t(x)$ is the value of the trend at x and $u(x)$ is the residual variation. The residuals are defined as a random variable with zero mean and non-zero variance.

The residual often shows spatial correlation, with neighboring location tending to have similar residuals. The scale of fluctuations, θ, can be used to represent this spatial correlation. The residuals have strong correlation within the scale of fluctuation. For two residuals of the soil properties z_1 and z_2, the correlation coefficient is defined as (Beacher and Christian, 2003):

$$\rho = \frac{Cov(z_1, z_2)}{\sqrt{Var(z_1)Var(z_2)}} \tag{2}$$

where $Cov(z_1, z_2)$ is the covariance, $Var(z_1)$ and $Var(z_2)$ is the variance. z_1 and z_2 can be different soil properties or the same soil property in different locations. In this paper, the latter one is taken into account. After calculating the correlation coefficient in different separation distances, the relationship between separation distances and correlation coefficients can be obtained. Then, the autocorrelation function can be fitted and the scale of fluctuations can be calculated according to Table 2.

Table 2. Typical autocorrelation functions.

Model	Autocorrelation	Variance reduction function	Scale of fluctuation, θ
White noise	$R_x(\delta)=1$, if $\delta=0$ $R_x(\delta)=0$, otherwise	$\gamma(X)=1$, if $X=0$ $\gamma(X)=0$, otherwise	0
Linear	$R_x(\delta)=1-\lvert\delta\rvert/\delta_0$, if $\delta<=\delta_0$ $R_x(\delta)=0$, otherwise	$\gamma(X)=1-X/3\delta_0$, if $X<=\delta_0$ $\gamma(X)=(\delta_0/X)(1-\delta_0/3X)$, otherwise	δ_0
Single Exponential	$R_x(\delta)=\exp(-\lvert\delta\rvert/\delta_0)$	$\gamma(X)=2(\delta_0/X)^2(X/\delta_0-1+\exp^2(-X/\delta_0))$	$2\delta_0$
Squared exponential	$R_x(\delta)=\exp^2(-\lvert\delta\rvert/\delta_0)$	$\gamma(X)=(\delta_0/X)^2(\Phi(-X/\delta_0)\pi^{0.5}X/\delta_0+\exp^2(-X/\delta_0)-1)$ in which Φ is error function	$\pi^{0.5}\delta_0$

5 Horizontal Spatial Variability

There are 67 boreholes (marked in red in Figure 1(b)) that contains sandy clay layer. The spatial variability of cohesion, friction angle, compression modulus, liquid limit and plastic limit in horizontal direction is investigated in the sandy soil. Note that the horizontal autocorrelation is assumed to be independent of compass direction in the following study.

5.1 Cohesion

In order to investigate the spatial variability of the cohesion of the sandy clay, the trend of the cohesion should be removed from the observing data first. For example, the cohesions of the sandy clay at depth range of 4 m to 7 m are shown in Figure 2(a). The vertical axis of the dot points gives the cohesions of the soil at different locations. The trend surface has been fitting using linear regression method as follows:

$$t = 0.0009x + 0.0027y - 269.1 \tag{3}$$

where x, y and t represent easting coordinate, northing coordinate and trend values, respectively. The trend are then removed from each observing data and the residuals can be attained (as shown in Figure 2(b)).

As the borehole locations are spaced in an irregular pattern in this site, the data are grouped into subsets according to their distances. Thirty-six subsets are established for data pairs separated by different distances. In order to ensure the accuracy of the results, the data pairs at different depths (e.g. from 1.8m to 24.6 m) have been employed to calculate the correlation coefficient at the same separation distances. The trend functions at different depths are removed separately. The correlation coefficients for each subset with different separation distances are calculated using Eq. (2) and the results are shown in Figure 3. The results were then fitted by white noise model, linear model, single exponential model and squared exponential model. Sum of squared errors were used to present the fitting degree and calculated by using Eq.(4).

$$S_{error} = \sum (X_{predicted} - X_{known})^2 \tag{4}$$

where S_{error} is sum of squared error, $X_{predicted}$ is the predicted value and X_{known} is the known value.

The S_{error} of the white noise model, linear model and single exponential model are 4.89, 4.82 and 4.81, respectively. A squared exponential autocorrelation function with S_{error} of 4.74 is found to best fit the autocorrelation structure. The scale of fluctuation in horizontal direction was calculated as 54.8 m (as shown in Figure 3), which is in the range of 46.0 m through 60.0 m presented by Phoon et al. (1995).

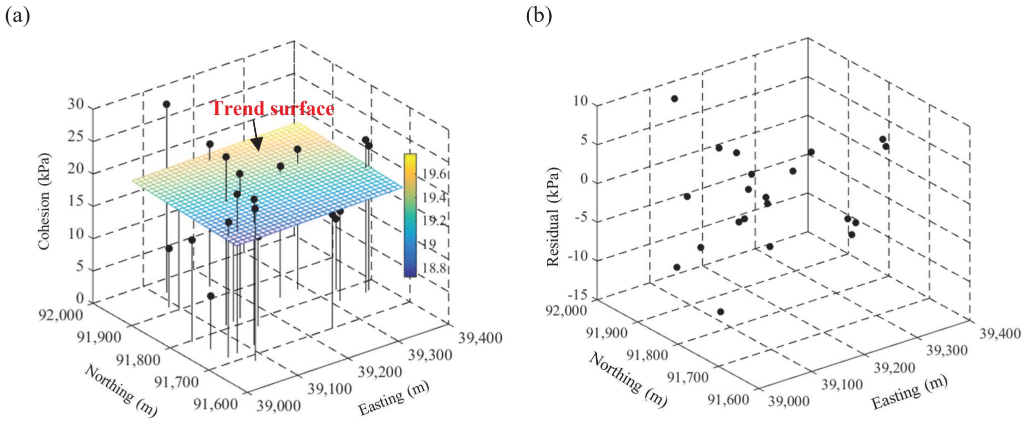

Figure 2. Cohesions and the residual of cohesions of the sandy clay at depth range of 4 m to 7 m: (a) Cohesions and (b) the residual of cohesions.

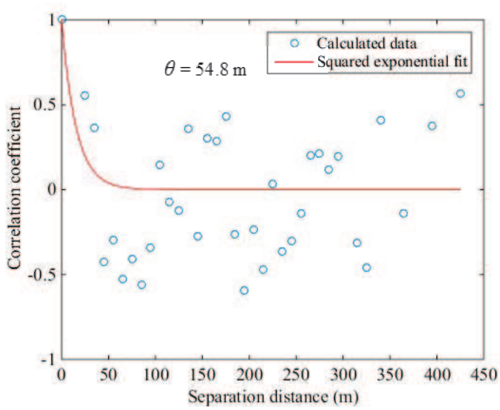

Figure 3. Autocorrelation structure and scale of fluctuation of cohesion in horizontal direction.

5.2 Friction angle

Similarly, the correlation coefficients with different separation distances for friction angle can be calculated and the results are shown in Figure 4(a). A squared exponential autocorrelation function is found to best fit the autocorrelation structure. The scale of fluctuation of the friction angle in horizontal direction is 87.2 m which is larger than that of cohesion.

5.3 Compression modulus

The compression modulus, E_s, is a parameter indicates the compressibility of a soil, which is defined as,

$$E_s = \frac{1}{m_v} \tag{5}$$

where m_v is the modulus of volume compressibility. The variation of correlation coefficients with different separation distances for compression modulus are shown in Figure 4(b). Again, a squared exponential autocorrelation function is found to best fit the autocorrelation structure. The scale of fluctuation in horizontal direction is 50.6 m which is smaller than that of cohesion and friction angle.

5.4 Liquid limit

The variation of correlation coefficients with different separation distances for liquid limit are shown in Figure 4(c). The scale of fluctuation in horizontal direction is 28.3 m, which is much smaller than that of the cohesion.

5.5 Plastic limit

The correlation coefficients for plastic limit are shown in Figure 4(d). The scale of fluctuation in horizontal direction is 31.8 m which is close to that of liquid limit.

(a) (b)

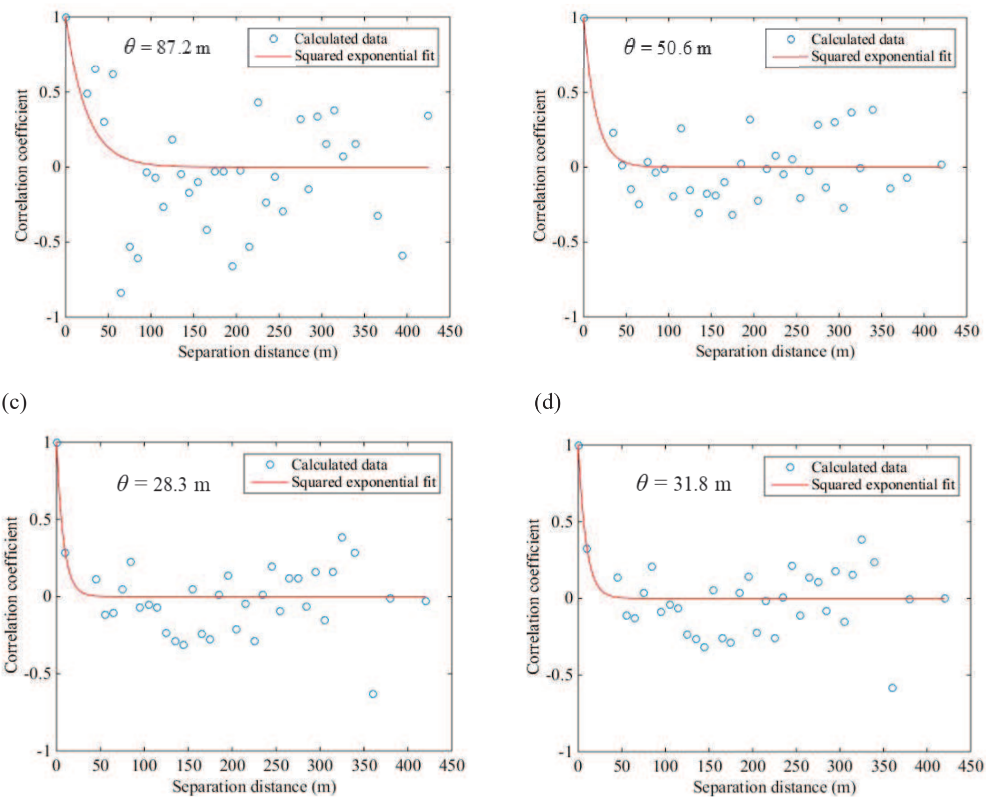

Figure 4. Autocorrelation structure and scale of fluctuation in horizontal direction for: (a) friction angle, (b) compression modulus, (c) liquid limit and (d) plastic limit.

Based on the scale of fluctuation and autocorrelation structure, random fields can be generated and it can be furthered used to assess the safety of geotechnical structures.

6 Conclusions

The spatial variability of cohesion, friction angle, compression modulus, liquid limit and plastic limit is investigated in Shenzhen based on 138 boreholes data. The soil profile at this site consists of four types of soils, including gravelly clay, sandy clay, completely weathered granite and strong weathered granite. The scale of fluctuation in horizontal direction is investigated in the layer of sandy soil. A squared exponential autocorrelation function was found to be the best fit for the calculated autocorrelation coefficient at various separation distances. As for cohesion, friction angle, compression modulus, liquid limit and plastic limit, the scale of fluctuation in horizontal direction are 54.8 m, 87.2 m, 50.6 m, 28.3 m and 31.8 m, respectively. The results can be used to generate random fields and guide the practical engineering design in site.

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References

Beacher, G.B. and Christian, J.T. (2003). *Reliability and Statistics in Geotechnical Engineering*, John Wiley & Sons Ltd. Chichester, England.

Cassidy, M. J., Uzielli, M., and Tian, Y. H. (2013). Probabilistic combined loading failure envelopes of a strip footing on spatially variable soil. *Computers and Geotechnics*, 49, 191-205.

Ching, J. and Phoon, K.K. (2013). Probability distribution for mobilized shear strengths of spatially variable soils under uniform stress states. *Georisk*, 7(3), 209-224.

Dasaka, S.M. and Zhang, L.M. (2012). Spatial variability of in situ weathered soil. *Geotechnique*, 62(5), 375-384.

Griffiths, D.V., Fenton, G.A., and Manoharan, N. (2002). Bearing capacity of rough rigid strip footing on cohesive soil: Probabilistic study. *Journal of Geotechnical and Geoenvironmental Engineering*, 128(9), 743-755.

Huang, J., Lyamin, A.V., Griffiths, D.V., Krabbenhoft, K., and Sloan, S.W. (2013). Quantitative risk assessment of landslide by limit analysis and random fields. *Computers and Geotechnics*, 53, 60-67.

Jaksa, M.B. (1995). *The Influence of Spatial Variability on the Geotechnical Design Properties of a Stiff, Overconsolidated Clay*, Ph.D. thesis, University of Adelaide, Australia.

Lacasse, S. and Nadim, F.(1996). Uncertainties in characterising soil properties. *In C.D. Shackleford, P.P. Nelson and M.J.S.Roth (eds), Uncertainty in the Geologic Environment: From theory to Practice*, Geotechnical Special Publication, No. 58, 269-286.

Li, J.H., Huang, J., Cassidy, M.J., and Kelly, R. (2014). Spatial variability of the soil at the Ballina National Field Test Facility. *Australia Geomechanics*, 49(4), 41-47.

Lloret-Cabot, M., Fenton, G.A., and Hicks, M.A. (2014). On the estimation of scale of fluctuation in geostatistics. *Georisk*, 8(2), 129–140.

Li, J., Tian, Y., and Cassidy, M.J. (2014). Failure mechanism and bearing capacity of footings buried at various depths in spatially random soil. *Journal of Geotechnical and Geoenvironmental Engineering*, 141(2), 04014099.

Liu, S., Zou, H. Cai, G., Bheemasetti, B. V., Puppala, A. J., and Lin J. (2016). Multivariate correlation among resilient modulus and cone penetration test parameters of cohesive subgrade soils. *Engineering Geology*, 209, 128–142

Phoon, K.K., Kulhawy, F.H, and Grigoriu, M.D. (1995). *Reliability-Based Design of Foundations for Transmission Line Structures*, Electric Power Research Institute, Palo Alto, Report TR-105000.

Phoon, K.K. and Kulhawy, F.H. (1999). Characterization of geotechnical variability. *Canadian Geotechnical Journal*, 36, 612-624.

Uzielli, M., Vannucchi, G., and Phoon, K.K. (2005). Random field characterization of stress-normalised cone penetration testing parameters. *Géotechnique*, 55(1), 3-20.