The paper was published in the proceedings of the 7th International Symposium on Geotechnical Safety and Risk (ISGSR 2019) and was edited by Jianye Ching, Dian-Qing Li and Jie Zhang. The conference was held in Taipei, Taiwan 11-13 December 2019.

Reliability Analysis of Rock Slope Stability Using Rock Mass Properties Estimated from Bayesian Method and Sparse Site-Specific Measurements

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Abstract: Stability analysis of rock slopes is a complex problem because of uncertainties involved in the rock mass properties. It involves investigation into the uncertain influence of rock parameters on reliability of rock slope. Cohesion (c) and friction angle (ϕ), and their correlation affect results of reliability analysis of rock slope stability. This makes c and ϕ as well as their correlation important parameters required for reliability analysis of rock slope stability. However, the characterization of joint probability distribution of c and ϕ through which their exact correlation can be estimated requires a large amount of rock property data, which are often not available for most rock engineering projects. To overcome this problem, a Bayesian approach is presented to characterize the correlation between c and ϕ through their joint probability distribution using available limited data pairs of c and ϕ from a rock project, and an expanded reliability-based design (RBD) approach is presented to use the equivalent sample pairs of c and ϕ from the Bayesian approach to perform reliability analysis of a rock slope. Using a design example, insight is given into the propagation of the correlation between c and ϕ through their joint probability into the reliability analysis, and their influence on the calculated reliability of the rock slope.

Keywords: Reliability analysis; rock slope; uncertainties; Bayesian approach; equivalent samples.

1 Introduction

Reliability analysis of rock slopes requires parameters of shear strength of discontinuities (i.e., cohesion (c) and friction angle (ϕ)) (e.g., Li et al. 2011). These parameters (i.e., c and ϕ) are often treated as random variables, and there is site-specific correlation between them. Evaluation of the reliability of rock slopes requires that the joint probability density function (PDF) of correlated rock parameters is known, to quantify the correlation between them. However, in most geotechnical engineering projects, the joint PDF is often unknown because of limited data from field and laboratory tests (e.g., Wang and Akeju 2016). Because of limited data, c and ϕ are often modelled in reliability analysis using only their means and standard deviations without considering the correlation between them or it is simply assumed (e.g., Jimenez-Rodriguez et al. 2006). Wang and Aladejare (2016) noted that if the correlation between random variables is not properly modelled in reliability analysis, the failure probability obtained from there reliability analysis might differ by orders of magnitude. Hence, it is of practical interest to develop approaches to quantify site-specific correlation between c and ϕ, and to assess their influence on reliability analysis of rock slopes. This study addresses these challenges through reliability analysis of rock slope stability using rock mass properties estimated from Bayesian equivalent samples. A Bayesian approach is used to derive joint PDF of c and ϕ, which is incorporated into Markov chain Monte Carlo (MCMC) simulation to generate many sample pairs of c and ϕ. The large number of sample pairs generated using MCMC represents the site-specific joint distribution of c and ϕ from which joint probability distributions as well as the correlation between c and ϕ can be evaluated. The sample pairs of c and ϕ are used as input in expanded reliability-based design (RBD) of rock slopes, that formulates the design process as an expanded reliability problem in which Monte Carlo simulations (MCS) are used in the design. For illustration purpose, reliability analysis of rock slope to assess the influence of correlation during the determination of the maximum rock slope height for safe excavation is presented.

2 Bayesian Approach for Characterization of Joint Distribution of c and ϕ

Bayesian framework is used to obtain the joint PDF of c and ϕ for quantifying the correlation between them (e.g., Ang and Tang 2007). The Bayesian framework formulates the characterization of joint PDF of c and ϕ as an inverse analysis problem, and it is useful even when only limited site observation data pairs of c and ϕ are available. Previous studies have modelled c and ϕ as normal random variables (e.g., Low 2007), therefore, in this study, c and ϕ are modelled as normal random variables with means μ_c and μ_ϕ respectively; and standard

Proceedings of the 7th International Symposium on Geotechnical Safety and Risk (ISGSR)
Editors: Jianye Ching, Dian-Qing Li and Jie Zhang

Published by Research Publishing, Singapore.
ISBN: 978-981-11-2725-0; doi:10.3850/978-981-11-2725-0_IS16-1-cd

deviations σ_c and σ_ϕ, respectively. The correlation between c and ϕ is quantified by coefficient of correlation, ρ, which is also treated as a random variable in the Bayesian approach. A bivariate normal distribution is used to model the site-specific joint probability distribution of c and ϕ from available site observation data. Distribution parameters of a bivariate normal distribution include μ_c, μ_ϕ, σ_c, σ_ϕ and ρ, which are needed to completely depict the joint distribution of correlated c and ϕ at a site. Both site-specific observation data and knowledge or information available prior to collection of site-specific observation data (i.e., prior knowledge) are used to estimate the distribution parameters μ_c, μ_ϕ, σ_c, σ_ϕ and ρ. Using the theorem of total probability, the joint PDF of c and ϕ for a given set of prior knowledge and site observation data is expressed as:

$$P(c,\phi|\text{Data,Prior}) = \int_{\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho} P(\text{c},\phi|\ \mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho) \times P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho|\text{Data,Prior})d_{\mu_c}d_{\mu_\phi}d_{\sigma_c}d_{\sigma_\phi}d_\rho \quad (1)$$

where $P(\text{c},\phi|\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)$ is the joint conditional PDF of c and ϕ for a given set of μ_c, μ_ϕ, σ_c, σ_φ and ρ. The joint conditional PDF of c and ϕ is expressed as:

$$P(c,\phi\,|\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho) = \frac{1}{2\pi\sigma_c\sigma_\phi\sqrt{1-\rho^2}} \times exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{c-\mu_c}{\sigma_c}\right)^2 - 2\rho\left(\frac{c-\mu_c}{\sigma_c}\right)\left(\frac{\phi-\mu_\phi}{\sigma_\phi}\right) + \left(\frac{\phi-\mu_\phi}{\sigma_\phi}\right)^2\right]\right\} \quad (2)$$

$P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho|\text{Data,Prior})$ reflects the integrated knowledge on μ_c, μ_ϕ, σ_c, σ_ϕ and ρ based on prior knowledge and site observation data pairs of c and ϕ. In Bayesian framework, $P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho|\text{Data,Prior})$ is simplified as $P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho|\text{Data})$, and expressed as (e.g., Wang and Akeju 2016):

$$P(\mu_c,\mu_\varphi,\sigma_c,\sigma_\varphi,\rho|\text{Data}) = KP(\text{Data}|\mu_c,\mu_\varphi,\sigma_c,\sigma_\varphi,\rho)P(\mu_c,\mu_\varphi,\sigma_c,\sigma_\varphi,\rho) \quad (3)$$

where K is a normalizing constant; $\text{Data} = \{(c_j,\phi_j),\ j = 1,2,3,\ldots,n_s\}$ is a set of site-specific data pairs of c and ϕ; $P(\text{Data}|\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)$ is the likelihood function and $P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)$ is the prior distribution of μ_c, μ_ϕ, σ_c, σ_ϕ and ρ The likelihood function $P(\text{Data}|\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)$ and prior distribution $P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)$ are expressed in Eqs. (4) and (5), respectively as:

$$P(\text{Data}|\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho) = \prod_{j=1}^{n_s}\frac{1}{2\pi\sigma_c\sigma_\phi\sqrt{1-\rho^2}} \times exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{c_j-\mu_c}{\sigma_c}\right)^2 - 2\rho\left(\frac{c_j-\mu_c}{\sigma_c}\right)\left(\frac{\phi_j-\mu_\phi}{\sigma_\phi}\right) + \left(\frac{\phi_j-\mu_\phi}{\sigma_\phi}\right)^2\right]\right\} \quad (4)$$

$$P\left(\mu_c,\sigma_c,\mu_\phi,\sigma_\phi,\rho\right) = \begin{cases} \begin{bmatrix} (\mu_{c_{\max}} - \mu_{c_{\min}}) \times (\sigma_{c_{\max}} - \sigma_{c_{\min}}) \times \\ (\mu_{\phi_{\max}} - \mu_{\phi_{\min}}) \times (\sigma_{\phi_{\max}} - \sigma_{\phi_{\min}}) \times \\ (\rho_{\max} - \rho_{\min}) \end{bmatrix}^{-1} & for\, \mu_c \in [\mu_{c_{\min}},\mu_{c_{\max}}],\ \sigma_c \in [\sigma_{c_{\min}},\sigma_{c_{\max}}], \mu_\varphi \in [\mu_{\phi_{\min}},\mu_{\phi_{\max}}],\ \sigma_\phi \in [\sigma_{\phi_{\min}},\sigma_{\phi_{\max}}]\, and\ \rho \in [\rho_{\min},\rho_{\max}] \\ 0 & others \end{cases} \quad (5)$$

The prior distribution in Eq. (5) is defined in this study by the typical ranges of the joint distribution parameters, and they are readily available in geotechnical literatures (e.g. Aladejare and Wang 2018). Using the updated knowledge of the joint distribution parameters given by Eq. (3), the joint PDF of c and ϕ given site observation data pairs of c and ϕ and a set of prior knowledge given in Eq. (1) can be expressed as Eq (6):

$$P(c,\phi|\text{Data,Prior}) = K\int_{\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho} P(\text{c},\phi|\ \mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho) \times P(\text{Data}|\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)P(\mu_c,\mu_\phi,\sigma_c,\sigma_\phi,\rho)d_{\mu_c}d_{\mu_\phi}d_{\sigma_c}d_{\sigma_\phi}d_\rho \quad (6)$$

MCMC simulation is used to generate a sequence of sample pairs of c and ϕ from the joint PDF presented in Eq. (6). Details of the MCMC simulation representation of sample pairs of c and ϕ can be found in Wang and Akeju (2016).

3 Reliability Analysis of Rock Slope

Reliability analysis of rock slope is performed by formulating the determination of a safe slope height (H) as an expanded reliability-based design (RBD) problem. Monte Carlo simulations (MCS) is used to perform the design analysis, with design objective to find the maximum value of H that satisfies the factor of safety (FS) requirements and achieves the target probability of failure P_T or target reliability index β_T. In this study, failure refers to a condition where the ratio of the resisting force to driving force of a slope is less than one (i.e., FS < 1.0). In the expanded RBD, the input parameter H is treated as an independent discrete random variable with a uniformly distributed probability mass function P(H). The failure probabilities (i.e., conditional probability P(Failure | H)) is calculated for various values of H and compared with P_T. Feasible designs are identified when the failure probability corresponding to various values of H, P(Failure | H) $\leq P_T$. Using Bayes' theorem, P(Failure | H) is expressed as (e.g., Ang and Tang 2007):

$$\mathrm{P(Failure|H)} = \frac{\mathrm{P(H|Failure)P(Failure)}}{\mathrm{P(H)}} \tag{7}$$

where P(H | Failure) is the conditional probability of H provided that failure occurs; P(H)=1/n_H; P(Failure) is the probability of occurrence of failure over simulation samples. n_H is the number of possible discrete values for H, which is determined by setting the range of possible values of H and increment within the range.

3.1 Monte Carlo simulation

Monte Carlo simulation is used to incorporate large number of samples of H, c and ϕ in the reliability analysis of a rock slope. Samples of H are simulated using uniform probability mass function within a prescribed range, while samples of c and ϕ are simulated using the Bayesian approach presented in the earlier section. The reliability analysis uses each set of random samples as input to calculate FS and perform check of rock slope requirement for safety, to judge whether failure occurs or not. This process of using set of random samples to calculate FS and check if there is failure or not is repeated until all sets of random samples have been used as inputs. From the results of the analysis, the total number of MCS samples (n_t), number of MCS samples where failure occurs (n_f) and number of MCS samples where failure occurs for a specific value of H (n_v) are easily estimated. These statistics are used to calculate P(H | Failure) and P(Failure) as follows:

$$\mathrm{P(H|Failure)} = \frac{n_v}{n_f} \tag{8}$$

$$\mathrm{P(Failure)} = \frac{n_f}{n_t} \tag{9}$$

Feasible designs of the rock slope are identified when the failure probability corresponding to specific value of H, P(Failure | H)$\leq P_T$. The maximum slope height (H_{max}) for safe slope is the maximum value of H for which P(Failure | H) $\leq P_T$. H_{max} is the height which satisfy the FS requirement, and it is the vertical extent to which excavation of rock slope benches can be performed without safety problems. To improve the accuracy of the MCS results which increases as the number of MCS samples increases, 5,000,000 MCS samples is used in this study for illustration.

3.2 Deterministic limit equilibrium model

A two-dimensional limit equilibrium model with single failure model, in which the rock slope is assumed with a 1-m thick slice through the slope, which is shown in Figure 1 is used in this study to perform reliability analysis. Note that the rock geometry shown in Figure 1 has been widely used in the literature (e.g., Jimenez-Rodriguez et al. 2006; Li et al. 2011). All forces acting on the slope are resolved into components that are parallel and normal to the sliding surface. The vector sum of the block weight acting on the plane is termed the driving force, while the product of normal forces and the tangent of friction angle, plus the cohesion force, is the resisting force. The FS is calculated as the ratio of the sum of resisting forces to the sum of driving forces (Hoek and Bray 1981).

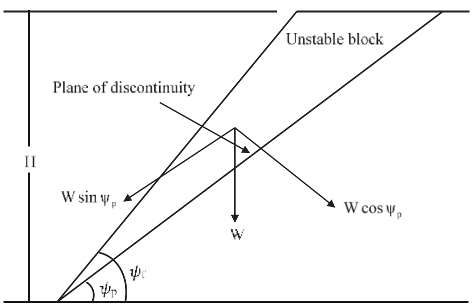


Figure 1. Illustration of a rock slope with dry tension crack (modified after Hoek 2007).

In this study, a rock slope is taken to be safe when the FS is greater than one (FS > 1) and fails or unsafe when the FS is less than one (FS < 1). These assumptions are consistent with previous studies on reliability of rock slopes (e.g., Jimenez-Rodriguez et al. 2006; Aladejare and Wang 2018). This study considers a case of rock slope with dry tension crack in the analysis of rock slope using the deterministic limit equilibrium model (e.g., Hoek 2007), in which FS is calculated as:

$$\mathrm{FS} = \frac{cA + W\cos\psi_p \tan\phi}{W\sin\psi_p} \tag{10}$$

where c is the cohesion of joint surface; ϕ is the friction angle of the joint surface; ψ_p is the angle of failure surface; ψ_f is the slope angle, measured from horizontal; $A = \frac{1}{4}H/\sin\psi_p$ is the base area of the wedge; $W = 0.5\gamma_r H^2(\cot\psi_p - \cot\psi_f)$ is the weight of rock wedge resting on the failure surface (MN); and γ_r is the unit weight of rock.

4 Design Example

This study adopts limit equilibrium model presented in Section 3.2 to perform expanded RBD of a rock slope with dry tension crack. Different slope heights, H, are considered, with H ranging from a minimum of 10 m to a maximum of 100 m with an increment of 5 m (i.e., $n_H = 19$). Adopting the slope parameters from Hoek (2007), ψ_p=35°, ψ_f =50° and $\gamma_r = 0.026\ \mathrm{MN/m^3}$, and they are all assumed to be fixed values. Data pairs of c and ϕ obtained for fractured rock surface at Forsmark, Sweden (Lanaro and Fredriksson 2005) are used in the Bayesian approach to simulate joint distribution of c and ϕ. Table 1 presents the 27 data pairs of c and ϕ from the laboratory tests performed in this site. Such number of data pairs are not often available in most rock engineering projects, and the Bayesian approach only requires a limited number of site-specific data pairs. Therefore, only 10 randomly

Table 1. Laboratory results performed on samples of fractured rock at Forsmark (from Lanaro and Fredriksson, 2005).

Sample No	c (MPa)	ϕ (°)
1	0.22	32.5
2	0.20	37.9
3	0.55	33.7
4	1.01	32.3
5	0.00	36.2
6	0.37	33.0
7	0.72	32.1
8	0.95	30.5
9	0.26	32.8
10	0.94	27.3
11	0.54	31.9
12	0.84	38.0
13	0.58	33.9
14	0.27	36.7
15	0.64	35.4
16	0.53	35.1
17	0.40	36.1
18	0.75	36.9
19	0.92	33.6
20	0.40	32.8
21	0.39	35.4
22	0.76	34.5
23	0.78	32.6
24	0.33	30.5
25	0.58	30.4
26	0.53	39.1
27	1.11	37.6
Mean	**0.58**	**34.0**
Standard deviation	**0.28**	**2.8**
Correlation	**-0.18**	

selected data pairs out of the 27 data pairs are used as input data. The 10 randomly selected data pairs of c and ϕ include: [0.58 MPa and 33.9°], [0.84 MPa and 38.0°], [0.54 MPa and 31.9°], [0.58 MPa and 30.4°], [0.37 MPa and 33.0°], [1.11 MPa and 37.6°], [0.33 MPa and 30.5°], [0.78 MPa and 32.6°], [0.20 MPa and 37.9°] and [0.40 MPa and 32.8°]. The 10 selected data pairs of c and ϕ are combined with typical ranges of distribution parameters of c and ϕ ([0.1 MPa, 1.3 MPa] for μ_c , [0 MPa, 0.4 MPa] for σ_c , [16°, 40°] for μ_ϕ , [0°, 4°] for σ_ϕ and [-1, 0.1] for ρ) (e.g., Aladejare and Wang 2018) in the Bayesian approach to characterize the joint distribution of c and ϕ. The joint distribution of c and ϕ obtained is used as inputs in expanded RBD of rock slope, to perform reliability analysis and assess the influence of correlation on reliability analysis of rock slope.

4.1 Joint distribution of c and ϕ

Figure 2 presents the scatter plot of the joint distribution of c and ϕ, and includes their histograms. The correlation coefficient between Bayesian equivalent sample pairs of c and ϕ is – 0.19, and it is close to – 0.18 estimated from the 27 data pairs of c and ϕ obtained at the site. The small difference in both estimations of correlation shows that the Bayesian approach satisfactorily characterizes the joint distribution of c and ϕ at the site. In real-life engineering practice, limited data is typically available, and such a direct comparison is not as obvious as shown in Table 1.

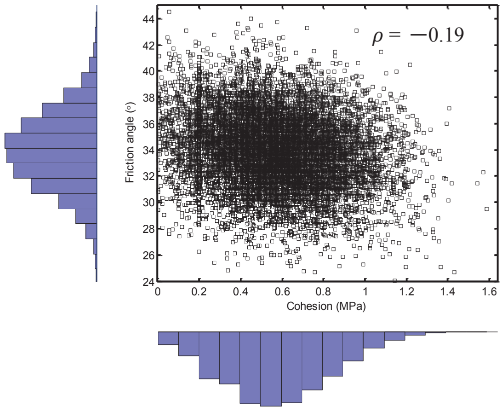


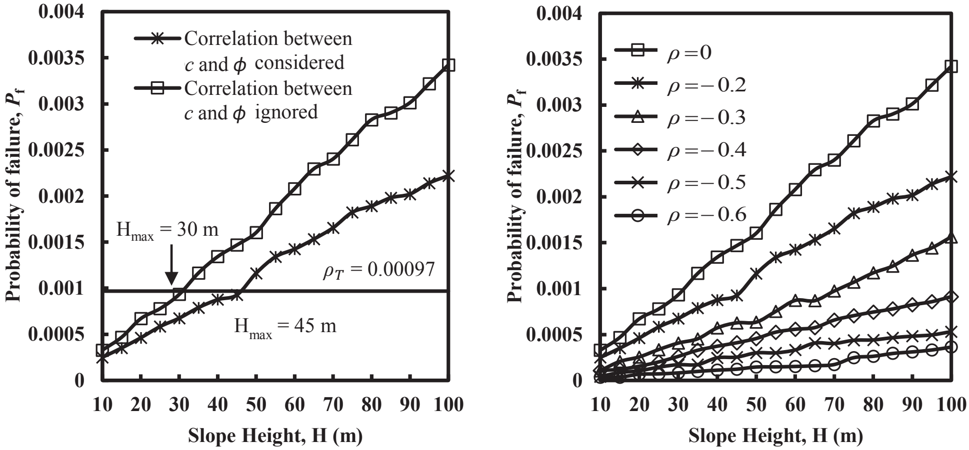
Figure 2. Scatter plot of the Bayesian equivalent sample pairs of c and ϕ with their marginal histograms.

The mean and standard deviation of c obtained from Bayesian approach are 0.57 MPa and 0.29 MPa, respectively, while those of the laboratory measurements of c are 0.58 MPa and 0.28 MPa, respectively. The relative differences between the mean and standard deviation from the two approaches are 1.7 % and 3.6 %, respectively. Also, the mean and standard deviation of ϕ obtained from Bayesian approach are 33.9° and 2.9°, respectively, while those of the laboratory measurements of ϕ are 34.0° and 2.8°, respectively. The relative differences between the mean and standard deviation from the two approaches are 0.3 % and 3.6 %, respectively. The small relative differences show that the estimates of the marginal distributions of c and ϕ are consistent with the laboratory measurements.

4.2 Reliability analysis of rock slope

Reliability analysis are performed to design the maximum height of a rock slope with dry tension crack. Two sets of analysis are done simultaneously, with one set of 5,000,000 sample pairs of c and ϕ which has = -0.19 (obtained in Section 4.1) and another set of 5,000,000 sample pairs of c and ϕ when the correlation is ignored or not considered (i.e., ρ = 0), which are also generated through Bayesian approach developed in Section 2. The difference between the two sets is that correlation is considered in one and ignored in the other. Eq. (10) is used to calculate FS for reliability analysis of the rock slope. For each set of 5,000,000 calculations of FS, the number of samples (n_v) where failure occurs at a specific value of H are counted. Then, conditional probability P(Failure | H) for failure are estimated for each set. The target failure probability, P_T = 0.00097 (i.e, reliability index, β_T =3.1) is used as the reliability constraint. Feasible designs are the maximum value of H (i.e., $\mathrm{H_{max}}$) that fall below the P_T shown in Figure 3(a). The feasible designs for the rock slope is $\mathrm{H_{max}}$ = 45 m when correlation between c and ϕ is considered, while $\mathrm{H_{max}}$ = 30 m when correlation ignored. When the correlation is ignored, the reliability analysis produced $\mathrm{H_{max}}$ which is smaller and leads to under excavation of safe slope benches at the rock site. This is consistent with the observation of Wang and Aladejare (2016) that when the correlation between rock parameters is ignored or not properly modelled in reliability analysis, the failure probability obtained from the reliability analysis might differ by orders of magnitude. Hence, the feasible design of the maximum slope height $\mathrm{H_{max}}$= 45 m, obtained when correlation between c and ϕ is considered, can be taken as the final design for the rock slope.

To further explore the effect of correlation on reliability analysis of rock slope, MCS is used to further simulate 4 additional sets of 5,000,000 sample pairs of c and ϕ, with ρ = -0.3, -0.4, -0.5, and -0.6, respectively for each of the sets. That leads to 6 sets of 5,000,000 sample pairs of c and ϕ when added with the ones previously generated. Figures 3(b) presents the results of the reliability analysis for the rock slope using the 6 sets. It shows that the feasible slope heights increase as the correlation between c and ϕ becomes stronger. When correlation is ignored in reliability analysis, the productivity of rock slopes may be reduced, as smaller slope heights means fewer benches can be worked at rock site. When a strong correlation exists between rock parameters but is ignored during reliability analysis, it could have more effect than those present in this study.



(a) Conditional probability of failure for slope height of the rock slope

(b) Effect of correlation between c and ϕ on rock slope stability.

Figure 3. Reliability-Based design of rock slope height.

5 Conclusions

In this study, a reliability analysis of rock slope is performed by using Bayesian equivalent samples obtained from Bayesian approach in expanded reliability-based Design (RBD) approach. The Bayesian approach uses the available limited data pairs of cohesion and friction angle together with prior knowledge to characterize their joint distribution and correlation. The equivalent sample pairs of cohesion and friction angle generated from their joint distribution are used in the expanded RBD to perform reliability analysis of rock slope stability. The probabilities of failure for rock slope associated with different correlation between cohesion and friction angle or rock can differ considerably. The probability of failure decreases with increasing correlation strength, leading to increasing maximum slope height that is safe for excavation at rock slope site. Ignoring or simply assuming correlation between rock parameters in reliability analysis might cause the failure probability to differ by orders of magnitude.

References

Aladejare, A. E. and Wang, Y. (2018). Influence of rock property correlation on reliability analysis of rock slope stability: From property characterization to reliability analysis. *Geoscience Frontiers*, 9(6), 1639-1648.

Ang, A. H-S. and Tang, W. H. (2007). *Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering*, Wiley, New York.

Jimenez-Rodriguez, R., Sitar, N., and Chacon, J. (2006). System reliability approach to rock slope stability. *International Journal of Rock Mechanics and Mining Sciences*, 43(6), 847-859.

Hoek, E. (2007). *Practical Rock Engineering*, Institution of Mining and Metallurgy, London.

Hoek, E. and Bray, J. (1981). *Rock Slope Engineering*, third ed, Institution of Mining and Metallurgy, London.

Lanaro, F. and Fredriksson, A. (2005). *Rock Mechanics Model: Summary of the Primary Data: Preliminary Site Description Forsmark Area-Version 1.2.*, Swedish National Testing and Research Institute, Sweden. http://www.skb.se.

Li, D. Q., Chen, Y., Lu, W., and Zhou, C. B. (2011). Stochastic response surface method for reliability analysis of rock slopes involving correlated non-normal variables. *Computers and Geotechnics*, 38 (1), 58-68.

Low, B. K. (2007). Reliability analysis of rock slopes involving correlated non-normals. *International Journal of Rock Mechanics and Mining Sciences*, 44(6), 922-935.

Wang, Y. and Akeju, O. V. (2016). Quantifying the cross-correlation between effective cohesion and friction angle of soil from limited site-specific data. *Soils and Foundations*, 56(6), 1057–1072.

Wang, Y. and Aladejare, A. E. (2016). Bayesian characterization of correlation between uniaxial compressive strength and Young's modulus of rock. *International Journal of Rock Mechanics and Mining Sciences*, 85, 10-19.