Predicting Time-Dependent Tunnel Convergences Using the Bayesian Approach

Xianda Feng[1] and Rafael Jimenez[2]

[1]School of Civil Engineering and Architecture, University of Jinan, No. 336, West Road of Nan Xinzhuang, Jinan 250022, Shandong Province, P.R. China. E-mail: feng.xianda@hotmail.com
[2]ETSI Caminos, C. y P., Technical University of Madrid, C/ Profesor Aranguren s/n, 28040 Madrid, Spain. E-mail: rjimenez@caminos.upm.es

Abstract: The prediction of tunnel convergences is of significant importance to the design and safe construction of tunnels. This study proposes a Bayesian approach to improve time-dependent convergence predictions, updating them with new information provided by successively convergence measurements. The proposed approach allows to consider various sources of uncertainties such as model uncertainty, model parameters uncertainty and measurement uncertainty. One real tunnel project, i.e., the GCS drift of the Underground Research Laboratory (URL) of the French National Radioactive Waste Management Agency (Andra), is used to demonstrate the applicability and performance of the proposed approach. Results show that the prediction accuracy is improved, and that its uncertainty is reduced, after the measured convergences are employed to update prior predictions.

Keywords: Tunnel time-dependent convergences; Bayesian approach; uncertainty.

1 Introduction

Tunnel excavation produces stress redistributions which result in deformations of the surrounding rock masses and of the tunnel support, leading to the closure (convergence) or instability of the excavated area (Kontogianni et al. 2006). The prediction of tunnel convergences proves to be vital to the design and safe construction of tunnels (Kavvadas 2005). Several approaches, such as analytical solutions, empirical models or numerical methods, have been employed to analyze or predict time-dependent tunnel convergences. In this study, we focus on the empirical models (Kontogianni et al. 2006; Sulem et al. 1987) because of their simplicity in practical use.

However, the convergence measurements are usually unavailable (or scarce) in the primary design stage, hence leading to difficulty in using such empirical convergence models because it is difficult to accurately estimate the required parameters based on limited information (González del Álamo and Jiménez 2012). In any case, it is reasonable to expect that previous information from adjacent tunnels or sections can provide useful *a priori* estimates that can be later improved when more data are available. To illustrate this, this study proposes a Bayesian approach to update the prior predictions of tunnel convergence computed with the empirical model proposed by Sulem et al. (1987) when the new convergence data become available. The convergence data from the GCS drift of the Underground Research Laboratory (URL) of the French National Radioactive Waste Management Agency (Andra) are employed to demonstrate the application and results of the proposed approach.

2 Empirical Models of Tunnel Convergences

Sulem et al. (1987) demonstrated that the total convergence C(x, t) can be represented by the combination of (i) the face advance effect and (ii) of rheological effects. It can be written as

$$C(x,t) = C_{\infty x}\left[1-\left(\frac{X}{X+x}\right)^2\right]\left[1+m\left(1-\left(\frac{T}{T+t}\right)^{0.3}\right)\right] \tag{1}$$

where X is a parameter related to "the distance of influence of the face"; x is the "distance of the measuring section to the tunnel face"; $C_{\infty x}$ is the "instantaneous closure" that would be obtained, without the influence of other time-dependent effects, for an infinite rate of face advance; T is the "characteristic parameter of the time-dependent properties of the ground"; t is the time passed since the first measurement at this section was conducted.

3 Bayesian Updating to Predict Time-Dependent Tunnel Convergences

3.1 Prior prediction of tunnel convergences

Let vector $\boldsymbol{\alpha}$ = [X, T, $C_{\infty x}$, m] represent the model parameters in Eq. (1), which are all assumed as random variables. In addition, let U_i (i = 0, 1, 2 , …, n) represent the (real) value of convergence at $x = x_i$ and $t = t_i$; and let $C(x_i,t_i;\ \boldsymbol{\alpha})$ denote (theoretical) convergences predicted by the model at $x = x_i$ and $t = t_i$. A model bias factor

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(*BF*) is introduced to characterize model uncertainty (Hsiao et al. 2008; Kung et al. 2007; Li et al. 2016); then we have

$$U_i = BF \cdot C(x_i, t_i; \boldsymbol{\alpha}) = h(x_i, t_i; \boldsymbol{\Theta}) \tag{2}$$

where $\boldsymbol{\Theta}$ is the complete vector of (uncertain) model parameters given by $\boldsymbol{\Theta} = [\boldsymbol{\alpha}, BF]$.

If we use $f_0(\boldsymbol{\Theta})$ to represent the prior probability density function (PDF) on $\boldsymbol{\Theta}$, then the (prior) mean predicted convergences can be computed as (Li et al. 2016):

$$\mathrm{E}(U_{i0}) = \int_{-\infty}^{+\infty} h(x_i, t_i; \boldsymbol{\Theta}) f_0(\boldsymbol{\Theta})\, d\boldsymbol{\Theta} \quad (i = 1, 2, ..., n) \tag{3}$$

The variables in $\boldsymbol{\Theta}$ are assumed to be independent and $f_0(\boldsymbol{\Theta})$ is assumed to follow a multivariate normal distribution.

3.2 Updating convergence prediction with measured data

Once such an *a-priori* probabilistic model is available, the monitored convergences can be employed to successively update the predictive model of tunnel convergences. In this study, the model is updated after each new convergence measurement. That is, if a set of convergence measurements ($i = 1, \ldots, n$), u_i (at $x = x_i$ and $t = t_i$), become available, the updating process can be successively repeated so that the posterior PDF of the j-th updating (for $1 \leqslant j \leqslant n$) can be expressed as (Li et al. 2016):

$$f_j(\boldsymbol{\Theta} \mid u_1, \ldots, u_j) = \frac{f_{j-1}(\boldsymbol{\Theta} \mid u_1, \ldots, u_{j-1}) \cdot \phi_\varepsilon[u_j - h(x_j, t_j; \boldsymbol{\Theta})]}{\int_{-\infty}^{+\infty} f_{j-1}(\boldsymbol{\Theta} \mid u_1, \ldots, u_{j-1}) \cdot \phi_\varepsilon[u_j - h(x_j, t_j; \boldsymbol{\Theta})] d\boldsymbol{\Theta}} \tag{4}$$

where $\phi_\varepsilon[u_j - h(x_j, t_j; \boldsymbol{\Theta})]$ is the likelihood of the u_j and $\phi_\varepsilon(\cdot)$ is the PDF of the convergence measurement error, ε.

The means of the j-th updated future convergence predictions can be expressed as (Kloek and van Dijk 1978; Li et al. 2016):

$$\mathrm{E}(U_{lj}) = \int_{-\infty}^{+\infty} h(x_l, t_l; \boldsymbol{\Theta}) \cdot f_j(\boldsymbol{\Theta} \mid u_1, \ldots, u_j) d\boldsymbol{\Theta} \quad (j < l \le n) \tag{5}$$

4 One Illustrative Example

To illustrate the application of the proposed approach, we consider a real tunnel project, i.e., the GCS drift of the Underground Research Laboratory (URL) of the French National Radioactive Waste Management Agency (Andra). The GCS drift has a length of 63.3 m and an overburden depth between 420 m and 550 m. The cross-section is circular with a diameter of 5.2 m, and with six measuring points being located along its perimeter (See Fig.1). Only the horizontal convergences measured are used in this study.

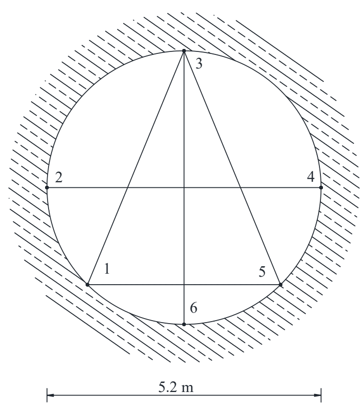

Figure 1. Cross-section of the GCS drift, modified from Guayacán-Carrillo et al. (2016).

Here, we focus on Section D at PM 40.7, which is one of the six monitoring sections (at PM: 16.9 - 30.1 - 40.7 - 49.2 -52.9 - 57.7) in this drift whose convergences were investigated by Guayacán-Carrillo et al. (2016). In this case, the values of the four parameters in $\boldsymbol{\alpha}$ fitted using data from a previously excavated section (Section B at PM 16.9; see Table 3 of Guayacán-Carrillo et al. (2016)) are employed as the mean values of the prior distributions of parameters for Section D. The parameters in $\boldsymbol{\alpha}$ are assumed to be normally distributed with COV

= 0.10. BF is assumed to follow a normal distribution with mean μ_{BF} = 1 and a COV$_{BF}$ = 0.15. Kontogianni et al. (2006) and Kontogianni and Stiros (2002) indicate that the standard error of good quality geodetic convergence measurements is 3 – 4 mm, therefore the measurement error, ε, is assumed to follow a normal distribution with mean μ_{ε} = 0 and standard deviation σ_{ε}= 3 mm in this study.

The (means of) prior convergence predictions can then be computed and the results are shown in Fig.2. Although both Section B and Section D are close to each other, and although they excavated in the Callovo-Oxfordian claystone, Fig.2 shows that differences between the prior convergence predictions and the measured convergences are substantial. The updated predictions (updating the model using the measured convergences from Section D, one by one and as they become available) are also shown in Fig.2, showing that predictions improve as the updating proceeds. In other words, the model improves as it incorporates information from the newly available measured convergences.

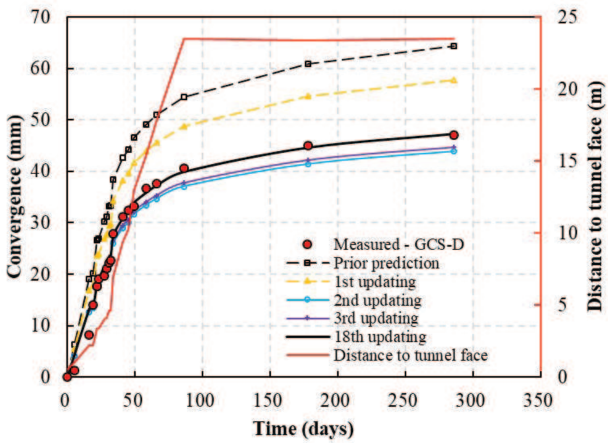

Figure 2. Face advance sequence and convergence information.

Figure 3 shows that the Bayesian approach improves the quality of average predictions and that it reduces their uncertainty, with ±σ error bounds that become narrower as more measured convergences are used to update the model.

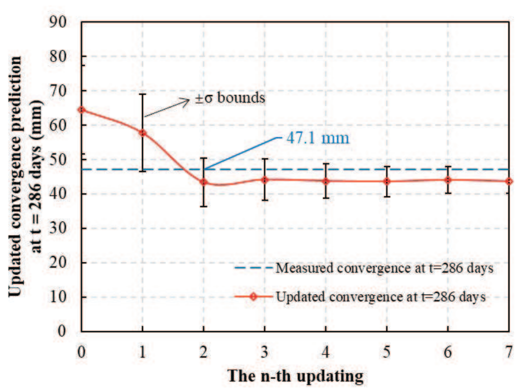

Figure 3. Prior and updated mean "last" convergence values (for t = 286 days), with their corresponding ±σ bounds.

5 Conclusions

This paper proposes a Bayesian updating approach to predict tunnel time-dependent convergences. Prior predictions of tunnel time-dependent convergences, which are often not good approximation to the measured convergences, are successively updated using the measured convergences that become available. The real project of the GCS drift is used to demonstrate the effect of this approach. Results based on show the prediction accuracy is improved and the predictive uncertainty is reduced.

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