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# Rock Engineering Insights from Reliability Analysis to Improve Eurocode 7 Design Approach

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**Abstract:** Insights obtained from reliability analysis and reliability-based design (RBD) are presented, in the context of three rock engineering ultimate limit states, namely 2D and 3D rock slope stability and underground excavation stability. The aim is to show how reliability-based design (RBD) via the first-order reliability method (FORM) can overcome some of the limitations and ambiguities of the Eurocode 7 (EC7) when applied to rock engineering. The reliability analysis and RBD are discussed with respect to parametric correlations and sensitivity information revealed by the design point in FORM. The differences and similarities between the design point in RBD and that in EC7 are discussed. The probabilities of failure based on RBD are compared with Monte Carlo simulations. The ability of RBD to provide interesting information at its design point and to automatically reflect parametric uncertainties, correlations, and case-specific sensitivities are demonstrated. It will be shown that reliability analysis and RBD via FORM can improve EC7 design approach when (i) partial factors for rock engineering parameters are not yet covered in EC7; (ii) the sensitivities of parameters vary from case to case; (iii) realistic considerations warrant correlation among parameters; (iv) a resistance or action parameter possesses stabilizing-destabilizing duality; (v) different target reliability index values are aimed at to reflect different consequence of failure.

**Keywords:** First-order reliability method; reliability-based design; Monte-Carlo simulation; rock slope; underground excavation; Eurocode 7.

## 1. Introduction

The design approach based on overall factor of safety has long been used by geotechnical engineers. More recent alternatives are the characteristic/ nominal values and partial factors used in the limit state design approach of Eurocode 7 (EC7) and the load and the resistance factor design (LRFD) approach in North America. Yet another approach, namely design based on a target reliability index of the first-order reliability method (FORM), can play at least a useful complementary role to EC7 and LRFD, as elaborated in Low and Phoon (2015), Low et al. (2017), and Low (2017). A special case of FORM is the earlier Hasofer-Lind index for correlated normal random variables. The classical mathematically intricate u-space approach for the FORM and the Hasofer-Lind method is described in Ang and Tang (1984), Haldar and Mahadevan (1999), Baecher and Christian (2003), for example. In addition, Low and Tang (2007) presented practical procedures for FORM-based reliability analysis and reliability-based design (RBD), which obtain the same results as the classical u-space approach but are much more transparent than the latter. Response surface method can be used to bridge standalone numerical software with FORM analysis, e.g. Chan and Low (2012a).

This paper discusses reliability analyses and reliability-based designs based on FORM, for a 2-dimensional rock slope containing a discontinuity plane and a tension crack, a 3-dimensional tetrahedral wedge in rock slope, and roof stability of an underground excavation in rocks. Extension from FORM to the more accurate second-order reliability method (SORM) is also discussed.

Focus is on interesting insights and sensitivity information revealed in the design point of FORM, and on the complementary role that FORM can play towards partial factor design approaches like EC7 and LRFD.

The useful insights and information from FORM-based reliability analysis and design may not be obtainable in other probabilistic approaches like the first-order second-moment (FOSM) method and the Monte Carlo simulation method.

## 2. SORM and FORM Analyses of a Two-Dimensional Rock Slope

Fig. 1 shows a two-dimensional rock slope with five correlated random variables: the shear strength parameters  $\phi$  (degrees) and  $c$  (tonne/m<sup>2</sup>) of the discontinuity plane (inclined at  $\psi$ ), the depth  $z$  (m) of the tension crack, the ratio  $zw/z$  which affects the water pressure in the tension crack and in the discontinuity plane, and the coefficient  $\alpha$  of horizontal earthquake acceleration. While  $\phi$ ,  $c$  and  $z$  are assumed to be normally distributed,  $zw/z$  and  $\alpha$  obey the highly asymmetric truncated exponentials. The limit equilibrium stability formulations are as in Hoek (2007, Chapter 7). The statistical inputs and probability distributions in Fig. 1 follow those in Hoek (2007,

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Chapter 8) which investigates the probability of failure using Monte Carlo simulations.  $\tan\phi$  can be used (next section), if desired, instead of  $\phi$ .

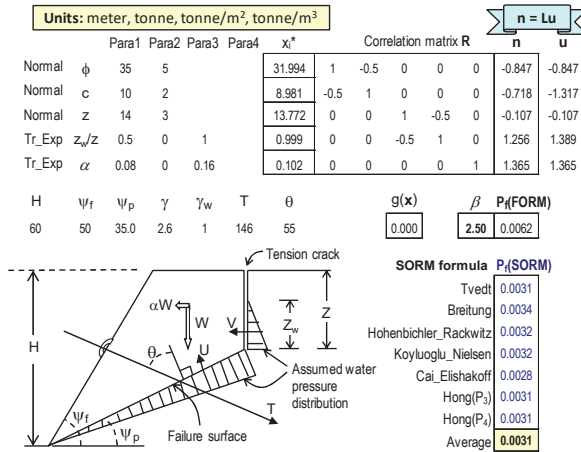


Figure 1. FORM-based RBD for a target reliability index  $\beta = 2.5$ , and comparison with SORM.

If the reinforcing force T is zero in Fig. 1, the FORM reliability index  $\beta$  is found to be 1.89, and the probability of failure is about 3%.

To raise the reliability index from 1.89 to a target of  $\beta = 2.50$ , a reinforcing force of  $T = 146$  tons per m length of slope is required. The probability of failure based on FORM  $\beta$ ,  $P_f(\text{FORM}) \approx \Phi(-\beta)$ , where  $\Phi$  is the standard normal cumulative distribution, is exact only if the random variables are normally distributed and the performance function  $g(x)$  is linear. These two conditions are not satisfied for the case in hand, hence the  $P_f(\text{FORM}) = 0.62\%$  shown in Fig. 1 is approximate.

SORM analysis can be done using FORM results, to estimate the curvatures of the limit state surface (LSS) at the design point. In Fig. 1, SORM is implemented via the Chan and Low (2012b) approach, obtaining an average  $P_f(\text{SORM})$  of about 0.31%. For comparison, five Monte Carlo simulations each with 500,000 trials yielded  $P_f$  values within the range 0.33% to 0.35%.

There is no unique SORM  $P_f$  value. It depends on the method used for estimating the curvatures at the design point, and on the formula used to compute  $P_f$  based on FORM  $\beta$  and the curvatures at the FORM design point. Nevertheless, the seven SORM formulas give consistent  $P_f$  values for the case in Fig. 1, and are more accurate than FORM  $P_f$ . If desired, one can extend the FORM analysis into the SORM analysis. Should the curvatures of the LSS turn out to be negligible, all the SORM formulas will approach  $P_f(\text{FORM}) = \Phi(-\beta)$ , with the result that the computed SORM probability of failure will be the same as FORM probability of failure. Note that the Breitung formula yields a SORM  $P_f$  of 0.34%, which is the same as the average of five Monte Carlo  $P_f$  values.

Mathematicians may emphasize that the FORM  $P_f$  of 0.62% for the case in hand is twice as high as the SORM  $P_f$  of 0.31% or the Monte Carlo  $P_f$  of about 0.34%. However, designers may adopt the pragmatic engineering stance, and regard both 0.62% and 0.31% as sufficiently small. If so, an RBD aiming at a target FORM  $\beta$  of 2.5 is adequate if the aim is not for a precise  $P_f$  but for a sufficiently small  $P_f$  (say  $<1\%$ ). Conducting SORM based on the results of FORM is then not necessary.

The following may be noted for the RBD example in Fig. 1:

1. The design point (five values under the column labelled  $x_i^*$ ) is akin to the design point in EC7. However, the FORM design point is the most probable point of failure at the contact point of an expanding equivalent dispersion hyperellipsoid with the LSS (Fig. 2(a), but in 5D hyperspace), and reflects context-sensitivity and parametric correlations in a way the design point of EC7 cannot, because the design point in EC7 is obtained by applying code-specified partial factors to conservative characteristic values, Fig. 2(b).
2. It is difficult for EC7 to anticipate and provide partial factors for all relevant parameters in various rock engineering problems. For example, for the case in hand, while partial factors for c and  $\phi$  may be specified in the code, those for tension crack depth z, ratio  $z_w/z$  and horizontal earthquake acceleration  $\alpha$  may not.

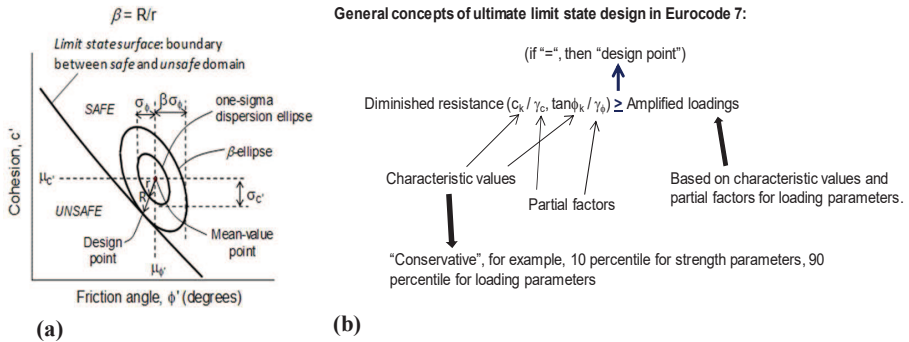


Figure 2. (a) FORM design point; this perspective is also valid for non-normal distributions, when viewed as “equivalent ellipsoids”; (b) EC 7 design point.

3. It is difficult for EC7 to consider context-sensitive parametric correlations in recommending partial factors. For the case in Fig. 1, the tension crack depth  $z$  and the extent to which it is filled with water ( $zw/z$ ) are negatively correlated. For illustrative purpose, a negative correlation coefficient of  $-0.5$  is assumed between  $z$  and  $zw/z$ . This means that shallower crack depths tend to be water-filled more readily (i.e.,  $zw/z$  ratio will be higher when  $z$  is small) than deeper crack depths, consistent with the scenario suggested in Hoek (2007) that the water which would fill the tension crack in this Hong Kong slope would come from direct surface run-off during heavy rains.
4. Some random variables, e.g.  $z$  for the depth of the tension crack, may possess stabilizing-destabilizing duality. Increasing  $z$  increases the weight of the sliding block but decreases the inclination angle  $\psi$  of the failure surface, besides its interacting effect with the water pressure via  $zw/z$ . FORM-based RBD automatically resolves this duality issue, which is difficult for EC7 to deal with. Other examples with stabilizing-destabilizing duality are presented in Low et al. (2017).
5. FORM can be used in two ways to provide insights pertinent to EC7:
  - a. Estimate statistical inputs, then conduct FORM reliability analysis on a design that derives from EC7, to estimate the reliability index and the probability of failure, and to compare the design points of FORM and EC7. If desired, partial factors can be back-calculated from the design point of FORM, for comparison with those specified in EC7, as done in Low et al. (2017), and Low 2017 (for LRFD).
  - b. Estimate statistical inputs, then obtain a design based on FORM target  $\beta$  (e.g. 2.5), for comparison with the design from EC7 which is based on applying specified partial factors to conservative characteristic values.

The above and other insights/deliberations will be discussed further in the context of the two other rock engineering problems in the sections below.

### 3. Reliability Analysis of 3D Tetrahedral Wedge Mechanisms in Rock Slopes

The stability analysis of 3D tetrahedral wedges formed by two intersection joints in rock slopes (Fig. 3) requires resolution of forces in three-dimensional space. The problem has been extensively treated, for example in Hoek and Bray (1977). The methods used include stereographic projection technique, engineering graphics, and vector analysis.

Low (2007) presented compact closed form equations for analyzing the stability of tetrahedral wedges. In Fig. 3, the uncertainties of discontinuity orientations ( $\beta_1, \delta_1, \beta_2, \delta_2$ ), shear strength of joints ( $\tan\phi$  and  $c/\gamma h$ ), and water pressure in joints (dimensionless parameter  $G_w$ ) are modelled by the versatile beta general distributions which can assume non-symmetrical bounded probability density function.

The reliability analysis here assumes the means and standard deviations of  $\tan\phi$ ,  $c/\gamma h$  and  $G_w$  on joint plane 1 are identical to those on joint plane 2. These assumptions are for simplicity, not compulsory. Reliability analysis yielded  $\beta = 1.92$  against sliding on both planes,  $\beta = 1.39$  against sliding on plane 1, and  $\beta > 5$  for other modes. This means that although the governing failure mode at mean values is sliding on both planes, the reliability index  $\beta$  against sliding on plane 1 is—in the presence of uncertainty in discontinuity orientations ( $\beta_1, \delta_1, \beta_2, \delta_2$ )—more critical than that against sliding on both planes. This information would not be revealed in a deterministic analysis, or in a reliability analysis that considers only one failure mode.

The values under the column labelled  $n$  denote the distances between the equivalent normal mean values and the design point values of the seven random variables, in units of their respective equivalent normal standard deviations. The design point values under the column labelled  $x^*$  and the dimensionless design point indices

under the column labelled n indicate that this ultimate limit state is reached, not surprisingly, by increasing the value of the water pressure parameter  $G_w$ , and decreasing the shear strength values of  $\tan\phi$  and  $c/\gamma h$ , and that stability is more sensitive to these three random variables than to  $\beta_1$ ,  $\delta_1$ ,  $\beta_2$  and  $\delta_2$ . If desired, partial factors (not required in FORM analysis and RBD) for all the seven random variables can be back-calculated from the results of FORM analysis, to provide guidance for future versions of EC7. Note however that the FORM reliability index and design point automatically reflects context-sensitivity (whether uncertainty is high or low, and whether some parameters are correlated), resulting in case-specific back-calculated partial factors for different situations, which is logical. For example, if the value of the correlation coefficient between  $\tan\phi$  and  $c/\gamma h$  is -0.5 (instead of 0.0) in the correlation matrix of Fig. 3, a reliability index of  $\beta = 2.21$  is obtained; the sensitivity indices (under the column labelled n) also differ to some extent from those shown in Fig. 3, including higher sensitivity for the dip direction parameters.

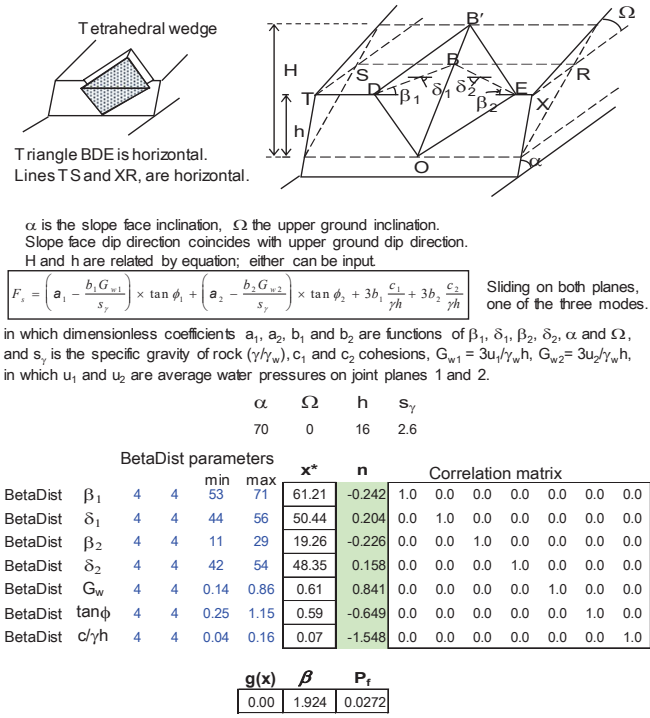


Figure 3. Reliability analysis of 3-D rock wedge with uncertain discontinuity orientations, and with negatively correlated  $\tan\phi$  and  $c/\gamma h$ .

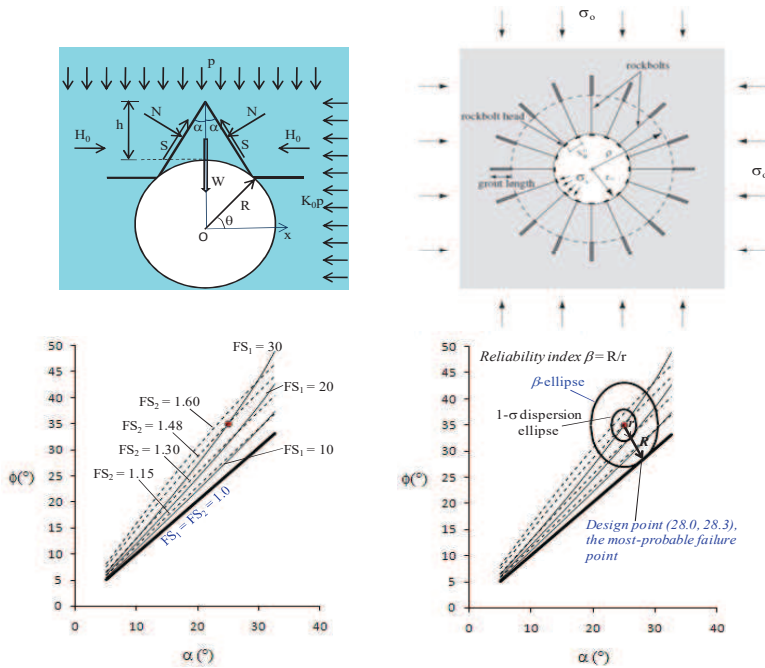
The four random variables of discontinuity orientations are  $\beta_1, \delta_1, \beta_2$  and  $\delta_2$ , where  $\beta_1$  and  $\beta_2$  are related to dip directions of joints 1 and 2, and  $\delta_1$ , and  $\delta_2$ , are dip angles. It may be difficult for EC7 to recommend partial factors for dip directions because stability is affected not by actions of  $\beta_1$  and  $\beta_2$  separately, but by their interactions in forming the tetrahedral wedge. Even if EC7 decides to let partial factors of dip directions be 1.0 due to difficulty in deciding whether it is unfavorable to rotate the dip directions to the left or to the right, one still needs to decide on conservative characteristic values of the dip directions, not an easy task for designers because difficult to see which perturbation (to the left or to the right of the mean value?) of the dip direction is more destabilizing.

Much insights can be revealed to resolve the above ambiguities and to suggest sensible partial factors for discontinuity orientations and other relevant parameters if FORM analysis is conducted in the two ways recommended in point (5) of the previous section.

4. Reliability-Based Design and Analysis of Tunnels in Rocks

Low and Einstein (2013) discussed the ambiguous nature of the factor of safety of a tunnel with a roof wedge, Fig. 4, where two different definitions of the  $F_s$  are shown to be reconcilable via the first-order reliability method (FORM). RBD via FORM was then applied to a circular tunnel supported with elastic rockbolts in elasto-plastic

ground with the Coulomb failure criterion (Fig. 4, top right). The 3D effects of excavation were approximated using the established  $\beta_\sigma$  method. The spacing and length of rock bolts were designed so as to achieve a target reliability index. The similarities and differences between the ratios of FORM design-point values to mean values, on the one hand, and the partial factors of limit state design, on the other hand, are discussed. Like the previous two cases,  $c$  and  $\phi$  are among the random variables modelled, with negative correlation of  $-0.5$ . The design point values from FORM indicate that the wedge stability is much more sensitive to  $\phi$  than to  $c$ , different from those in the previous 2D and 3D rock slope examples.



A tale of two factors of safety, and reconciliation via FORM

Figure 4. FORM analysis and RBD of tunnels in rocks.

Unlike design point based on partial factors, the design point in FORM is obtained as a by-product of target reliability index (and associated Pf), and reflects input uncertainties, sensitivities, and correlations from case to case in a way that design point based on rigid partial factors cannot. However, more statistical input information is required in RBD than in EC7. In its current version, EC7 covers little on the characteristic values and partial factors of rock engineering parameters like orientations of discontinuities, in situ stresses, properties of joints and rock material, or the less common  $\beta_\sigma$  parameter which approximates 3D excavation in 2D analysis. RBD via FORM is a more flexible approach in dealing with case-specific uncertainties of myriad input variables and can potentially complement EC7 (and LRFD) by providing insights (which can be subtle and unexpected) and guidance on characteristic values and partial factors, if FORM is conducted in the two ways recommended in point (5) of Section 2.

### 5. Summary and Conclusions

Reliability analyses and reliability-based designs were conducted for three rock engineering ultimate limit states, namely 2D and 3D rock slope stability and underground excavation stability. The aim is to show how RBD via the first-order reliability method can overcome some of the limitations and ambiguities of Eurocode 7 when applied to rock engineering. Among the merits of FORM is the information contained in its design point (the most probable point of failure) where an expanding dispersion ellipsoid (or equivalent ellipsoid if nonnormal distributions are involved) just grazes the limit state surface. The differences and similarities between the design point in RBD and that in EC7 are discussed. The probabilities of failure based on RBD-via-FORM are compared with second-order reliability method and with Monte Carlo simulations. The ability of RBD-via-FORM to provide interesting and useful information at its design point and to automatically reflect parametric uncertainties,



correlations, and case-specific sensitivities are emphasized. It may be concluded that reliability analysis and RBD-via-FORM can provide insights and guidance to the evolving EC7 design approach when (i) Partial factors for rock engineering parameters are not yet covered in EC7; (ii) The sensitivities of parameters vary from case to case; (iii) Physical considerations justify modelling of parametric correlations; (iv) A resistance/action parameter possesses stabilizing-destabilizing duality; (v) different target reliability index values are aimed at to reflect different consequence of failure.

It is suggested that FORM can be used in the following two ways to provide insights and guidance to the evolving EC7 and to detect potential pitfalls/inconsistencies (e.g. stabilizing/destabilizing duality) in the latter:

1. Estimate statistical inputs, and conduct FORM analysis on a design that derives from EC7, to estimate the reliability index and the probability of failure, and to compare the design points of FORM and EC7. If desired, partial factors can be back-calculated from the design point of FORM, for comparison with those specified in EC7. One should note that RBD, like EC7, aims at a sufficiently safe design, not a design with a precise  $P_f$ .
2. Estimate statistical inputs, then obtain a design based on a target  $\beta$  (e.g. 2.5) of FORM, for comparison with the design from EC7 which is based on applying specified partial factors to conservative characteristic values.

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