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Inter-Region Variability of SPT-Based Liquefaction Potential Evaluation Method

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Abstract: In China, the Standard Penetration Test (SPT) based simplified liquefaction potential evaluation method is widely used. Recent earthquakes indicate that the liquefaction potential assessment method specified in the national code for seismic design of buildings has different levels of accuracy when applied in different regions. In this paper, the inter-region variability of the model uncertainty associated with the above liquefaction potential assessment method is studied. It is found that the statistics of the model bias factor of this method in different regions are quite different, indicating that the method has different levels of accuracy when applied in different regions. The uncertainties associated with the model bias factor in different regions are also different. For a region with more case histories, the uncertainty associated with the model tends to be smaller. The model bias factor for region with no case histories has the greatest uncertainty. In the above studied method, the seismic loading is represented by critical penetration blow count N_{cr} , and the resistance of soil is represented by uncorrected standard penetration blow count N . Due to the existence of the inter-region variability, the same N/N_{cr} value implies different levels of reliability in different regions.

Keywords: Soil liquefaction; model bias factor; inter-region variability; Bayesian method.

1 Introduction

In China, the Standard Penetration Test (SPT) based method for liquefaction potential assessment as specified in the national design code (MOHURD 2016) was developed empirically based on the past case histories collected in China. In recent years, it has been observed that the method cannot reliably predict the liquefaction potential of soils in the recent earthquakes such as the 2003 Bachu earthquake (Li et al. 2012; Yuan and Sun 2011) suggested to develop a region-specific liquefaction potential assessment method to deal with this problem. Previously, Facciorusso et al. (2015), Wotherspoon et al. (2015) also observed that the cone penetration test (CPT) based liquefaction potential assessment method suggested by Robertson and Wride (RW model) (Robertson 1998, 2009; Youd and Idriss 2001) was not able to predict accurately the liquefaction potential of soils in the 2012 Emilia, and 2011 Christchurch earthquakes, respectively. Ge and Zhang (2018) also found that the RW model is less accurate in predicting the liquefaction in the 2010 Darfield, 2011 Christchurch and 2011 Tohoku earthquakes. Zhang et al. (2016) suggested that an empirical liquefaction potential assessment method developed based on a global database may have different levels of accuracy when applied to different regions, and the property of the different statistics for the same parameter between regions can be called as inter-region variability. Zhang et al. (2016) also proposed a method to characterize the inter-region variability of the model uncertainty associated with the RW model, and found the RW model with consideration of inter-region variability is better supported by existing liquefaction database than the RW model without consideration of inter-region variability. The purpose of this paper is to investigate the inter-region variability of the model uncertainty associated with the method suggested in MOHURD (2016), which may help to develop region-specific liquefaction potential assessment method. This paper is organized as follows. First, the method specified in MOHURD (2016) is briefly introduced. Then, the method for characterizing the inter-region variability of a liquefaction potential assessment model is described. Finally, the inter-region variability of the model uncertainty associated with the SPT-based method as specified in MOHURD (2016) is analyzed.

2 Liquefaction Potential Assessment Method

In MOHURD (2016), seismic loading is represented by critical penetration blow count N_{cr} , and the resistance of soil is represented by uncorrected standard penetration blow count N . Based on MOHURD (2016), saturated soil is expected to liquefy if $N < N_{cr}$, and vice versa. Let d_s and d_w denote the depth of the soil under assessment and the depth of the groundwater below ground surface, respectively. MOHURD (2016) is applicable where $d_s \leq 20\text{m}$, and N_{cr} is calculated as follows:

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$$N_{cr} = N_0 \alpha \left[\ln(0.6d_s + 1.5) - 0.1d_w \right] \sqrt{\frac{3}{\rho_c}} \quad (1)$$

$$\alpha = 0.25M - 0.89 \quad (2)$$

where ρ_c represents fine content; N_0 is the reference blow count for case of $M = 7.5$, $d_w = 2$ m, $d_s = 3$ m, and P_L (liquefaction probability) = 0.32; α is an adjustment factor and M is the Richter Scale commonly used in China. The value of N_0 is related to design seismic acceleration as shown in Table 1.

Table 1. Relationship between N_0 and design seismic acceleration (MOHURD 2016)

Design seismic acceleration (g)	0.1	0.15	0.20	0.30	0.40
N_0	7	10	12	16	19

3 Method for Inter-Region Variability Characterization

3.1 Probabilistic model

Due to the existence of model uncertainty, the seismic loading represented by N_{cr} as specified in MOHURD (2016) may not be actual. To consider such an uncertainty, let N_a denote actual critical blow count, and suppose N_{cr} can be related to N_a via a model bias factor c as follows:

$$N_a = \frac{N_{cr}}{c} \quad (3)$$

In this paper, the accuracy of the liquefaction potential assessment method is represented by model bias factor, c . Let μ_c and σ_c denote the mean and the standard deviation of c , respectively. Due to the existence of the inter-region variability, the model bias factors may vary from region to region. Figure 1 shows the probabilistic model suggested in Zhang et al. (2016) to study the inter-region variability of the model uncertainty associated with a liquefaction potential assessment model. Suppose there are case histories from k regions for model calibration. In this model, c_i denotes the model bias factor in the i th region, and μ_{ci} denotes the mean of c_i . As can be seen from this figure, different regions have different mean values of the model bias factor (i.e., μ_{ci}), but the mean values of the model bias factor in different regions follow a common distribution with a mean of μ_μ and a standard deviation of σ_μ . As c , μ_μ , σ_μ , and μ_{ci} ($i = 1, 2, \dots, k$) are non-negative, they are modeled as lognormal random variables (Huang et al. 2012). Let x_{ij} denote the j th post-earthquake observation at region i , with $x_{ij} = 1$ denoting liquefaction and $x_{ij} = 0$ denoting non-liquefaction. In this model, σ_μ measures the magnitude of the inter-region variability and will be learned based on the data from different regions. If $\sigma_\mu = 0$, there is no inter-region variability. σ_c denotes the intra-region variability and is assumed to be equal everywhere. In Figure 1, the model bias factors in different regions are connected to each other through μ_μ , σ_μ and σ_c , which will be estimated simultaneously using the data from all regions in the calibration database. As such, the estimation of model bias factor at one region is also affected by the data from other regions through their joint influence on μ_μ , σ_μ and σ_c . Such a phenomenon is called information borrowing in the literature (Rouder and Lu 2005).

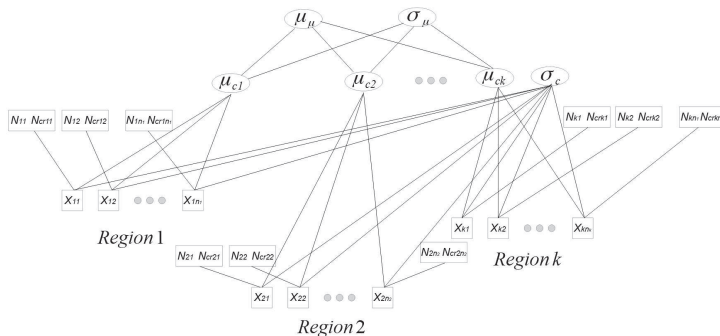


Figure 1. Relationship between region-specific observed data and the statistics of model bias factor.

3.2 Bayesian method for model calibration

In the above model, there are $(k + 3)$ parameters needing to be calibrated: μ_{ci} ($i = 1, 2, \dots, k$), μ_μ , σ_μ , and σ_c . Let $\mathbf{x}_i = \{x_{i1}, x_{i2}, \dots, x_{in_i}\}$ denote the i th observations from the i th region. Let $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k\}$ denote all the

observations from all regions. Let $f(\mu_\mu)$, $f(\sigma_\mu)$, and $f(\sigma_c)$ denote the prior probability density functions (PDFs) of μ_μ , σ_μ and σ_c , respectively. Based on Bayes' theorem, it can be shown that the posterior PDFs of $\{\mu_{ci} (i = 1, 2, \dots, k), \mu_\mu, \sigma_\mu, \sigma_c\}$ can be written as follows:

$$f(\mu_{c1}, \mu_{c2}, \dots, \mu_{ck}, \mu_\mu, \sigma_\mu, \sigma_c | \mathbf{X}) \propto f(\mu_\mu) f(\sigma_\mu) f(\sigma_c) \prod_{i=1}^k \left[\prod_{j=1}^{n_{Li}} P(x_{ij} = 1 | \mu_{ci}, \sigma_c) \right]^{w_{Li}} \prod_{j=1}^{n_{NLI}} P(x_{ij} = 0 | \mu_{ci}, \sigma_c)^{w_{NLI}} \quad (4)$$

$$P(x_{ij} = 1 | \mu_{ci}, \sigma_c) = P(N_{aij} > N_{ij} | \mu_{ci}, \sigma_c) = P(c < \frac{N_{crij}}{N_{ij}} | \mu_{ci}, \sigma_c) = \Phi \left[\frac{-\ln(\frac{N_{ij}}{N_{crij}}) - \ln(\frac{\mu_{ci}}{\sqrt{1 + \sigma_c^2 / \mu_{ci}^2}})}{\sqrt{\ln(1 + \sigma_c^2 / \mu_{ci}^2)}} \right] \quad (5)$$

$$P(x_{ij} = 0 | \mu_{ci}, \sigma_c) = 1 - P(c < \frac{N_{crij}}{N_{ij}} | \mu_{ci}, \sigma_c) = \Phi \left[\frac{\ln(\frac{N_{ij}}{N_{crij}}) + \ln(\frac{\mu_{ci}}{\sqrt{1 + \sigma_c^2 / \mu_{ci}^2}})}{\sqrt{\ln(1 + \sigma_c^2 / \mu_{ci}^2)}} \right] \quad (6)$$

where Φ is the cumulative distribution function of the standard normal random variable; n_{Li} and n_{NLI} are the numbers of liquefaction and non-liquefaction cases in the i th region; w_{Li} and w_{NLI} are weighting factors to consider the sampling bias in the liquefaction database (Zhang et al. 2016); $P(x_{ij} = 1 | \mu_{ci}, \sigma_c)$ and $P(x_{ij} = 0 | \mu_{ci}, \sigma_c)$ are the probabilities of liquefaction and non-liquefaction given μ_{ci} , σ_c , respectively. The above equation can be solved with Markov Chain Monte Carlo simulation (e.g., Gelman et al. 2013). With the above method, the PDFs of $\{\mu_{ci} (i = 1, 2, \dots, k), \mu_\mu, \sigma_\mu, \sigma_c\}$ and the statistics of c_i can be determined. One can then study the inter-region variability of the model uncertainty associated with the liquefaction potential assessment method suggested in MOHURD (2016) through the statistics of these variables.

4 Inter-Region Variability of Liquefaction Potential Assessment as Specified in MOHURD (2016)

4.1 Calibration database

To characterize the inter-region variability, the data of 174 liquefied and non-liquefied case histories that occurred in nine earthquakes in China is collected from Xie (1984) and Kong (2013). Among them, 15 cases are abandoned due to lack of in-situ SPT data. The rest 159 cases are then used for model calibration in this study, as summarized in Table 2.

Table 2. Summary of the case histories.

No.	Earthquake	M	Number of cases
1	1976 Tangshan	7.80	92
2	1975 Haicheng	7.30	12
3	1970 Tonghai	7.80	32
4	1969 Yangjiang	6.40	4
5	1969 Bohai	7.40	3
6	1967 Hejian	6.30	2
7	1966 3.8 Xingtai	7.20	7
8	1966 3.22 Xingtai	6.30	6
9	1962 Heyuan	6.40	1
Total			159

4.2. Characteristics of the inter-region variability

To apply the inter-region variability characterization model, the number of regions should be determined. In this study, the influence zones of earthquakes in Xingtai are regarded as one region, and other earthquake zones are regarded as separate regions. In such a case, there will be 8 regions in the database, and the number of random variables to be calibrated is 11. As it is believed that μ_μ , σ_μ and σ_c are positive and not greater than 3, they are assumed to be uniformly distributed between 0 and 3 in the prior distribution. The model calibration results are summarized in Table 3, where $E()$ and $Std()$ denote the mean and the standard deviation of a random variable, respectively. $Std(c_i)$ is calculated based on the method suggested in Zhang et al. (2016). From Table 3, the following phenomena are observed.

Table 3. Calibrated statistics of random variables in Figure 1.

		E()	Std()	E(c_i)	Std(c_i)	$N/N_{cr}(P_L = 0.32)$	$N/N_{cr}(P_L = 0.15)$
	μ_μ	1.596	0.376				
	σ_μ	0.912	0.737				
	σ_c	0.864	0.290				
Regions not in the database		1.596	1.231	1.596	1.532	1.268	2.008
1976 Tangshan		1.578	0.167	1.578	0.927	0.948	1.291
1975 Haicheng		1.488	0.257	1.488	0.947	1.047	1.458
1970 Tonghai		1.302	0.194	1.302	0.932	1.276	1.839
1969 Yangjiang	μ_{ci}	1.742	0.435	1.742	1.010	0.854	1.160
1969 Bohai		1.037	0.421	1.037	1.004	1.963	3.118
1967 Hejian		0.849	0.388	0.849	0.990	2.795	4.735
1966 Xingtai		1.303	0.235	1.303	0.941	1.283	1.854
1962 Heyuan		1.966	1.084	1.966	1.416	0.848	1.225

1. As mentioned previously, σ_μ measures the magnitude of inter-region variability. If $\sigma_\mu = 0$, the inter-region variability does not exist. Table 3 shows $E(\sigma_\mu) = 0.912$, indicating the existence of inter-region variability in the liquefaction potential assessment method specified in MOHURD (2016).
2. The mean values of the model bias factor as measured by $E(\mu_{ci})$ in different regions are quite different, which is a further evidence of the existence of inter-region variability. For example, the value of $E(\mu_{ci})$ in the 1962 Heyuan is 1.966, indicating on average the MOHURD (2016) model overestimates the seismic loading when applied to the Heyuan region. For comparison, the value of $E(\mu_{ci})$ of the 1967 Hejian earthquake is only 0.849, indicating that the model in MOHURD (2016) on average underestimates the seismic loading in the Hejian region.
3. The uncertainties associate with μ_{ci} ($i = 1, 2, \dots, k$) as measured by $Std(\mu_{ci})$ in different regions are quite different. For example, the values of $Std(\mu_{ci})$ in the 1976 Tangshan and the 1962 Heyuan earthquake are 0.167 and 1.084, respectively. Note that in the calibration database there are 92 case histories from the 1976 Tangshan earthquake, and only one case history from the Heyuan earthquake. It seems that if there are more case histories from a region, the uncertainty associated the model bias factor can be reduced even more. Therefore, to better characterize the model bias factor in a region, it is very useful to collect more calibration data from that region.
4. For a region not in the database, the best knowledge about $E(\mu_{ci})$ is that it equals to $E(\mu_\mu)$, i.e., $E(\mu_{ci}) = E(\mu_\mu) = 1.596$. It means its average bias factor is in between the average biases of all regions in the calibration database, which is reasonable. On the other hand, $Std(\mu_c) = 1.231$, which is greater than those of other regions, indicating the uncertainty associated with the bias factor for a region not in the database is the greatest.

As the model bias factor is uncertain, whether the soil will liquefy for a given value of N/N_{cr} is uncertain. The liquefaction probability can be calculated based on the statistics of the model bias factor with the following equation:

$$P(N_{ajj} > N_{ij} | E(c_i), Std(c_i)) = P(c < \frac{N_{crij}}{N_{ij}} | E(c_i), Std(c_i)) = \Phi \left[\frac{-\ln(\frac{N_{ij}}{N_{crij}}) - \ln(\frac{E(c_i)}{\sqrt{1 + Std(c_i)^2 / E(c_i)^2}})}{\sqrt{\ln(1 + Std(c_i)^2 / E(c_i)^2)}} \right] \quad (7)$$

Based on the above equation, Fig. 2 shows the N/N_{cr} - P_L relationships for different regions. As a result of inter-region variability, the N/N_{cr} - P_L relationships for different regions are quite scattered, indicating that the same N/N_{cr} implies different levels of reliability in different regions. For instance, when N/N_{cr} is 1, the liquefaction probability for a soil from the influence zone of the 1976 Tangshan earthquake is 0.286, while that of the 1967 Hejian earthquake is 0.739. Overall, the N/N_{cr} - P_L curve for a region not in the database is in between the N/N_{cr} - P_L curves of regions in the calibration database. The N/N_{cr} - P_L relationship is affected by both $E(c_i)$ and $Std(c_i)$. Although a region not in the calibration data has the greatest value of $Std(c_i)$, the value of $E(c_i)$ is in between those of regions with calibration data. Therefore, the N/N_{cr} - P_L curve of a region not in the calibration database is not always higher or lower than the curves of regions with calibration database.

Chen and Juang (2012) suggests that the liquefaction probability can be interpreted through information. When $0.15 \leq P_L < 0.35$, it is unlikely to liquefy. When $P_L < 0.15$, it is almost certain that the soil will not liquefy. The method specified in MOHURD (2016) is calibrated in the case of $P_L = 0.32$. To help establish the criterion

for mitigation of liquefaction hazards, the values of N/N_{cr} corresponding to $P_L = 0.32$ and $P_L = 0.15$ are also shown in Table 3. To meet the same liquefaction probability, the value of N/N_{cr} adopted in different regions are not the same. In the case of $P_L = 0.32$, in Tangshan, Yangjiang and Heyuan, the values of N/N_{cr} are smaller than 1, and the values of N/N_{cr} in other regions are all greater than 1. For a region not in the calibration database, $N/N_{cr} = 1.268$. $P_L = 0.15$ is commonly used by Cetin and Moss (Cetin et al. 2004; Moss et al. 2006), it is more conservative and the values of N/N_{cr} are much greater.

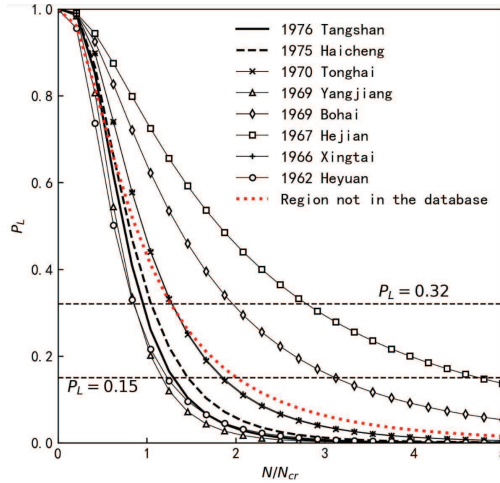


Figure 2. N/N_{cr} - P_L relationships for different regions

For ease of application, the N/N_{cr} - P_L relationships can be obtained by these equations:

$$\text{Region not in the calibration database : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) + 0.141}{0.808} \right] \quad (8)$$

$$1976 \text{ Tangshan : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) + 0.308}{0.544} \right] \quad (9)$$

$$1975 \text{ Haicheng : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) + 0.227}{0.583} \right] \quad (10)$$

$$1970 \text{ Tonghai : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) + 0.057}{0.643} \right] \quad (11)$$

$$1969 \text{ Yangjiang : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) + 0.410}{0.538} \right] \quad (12)$$

$$1969 \text{ Bohai : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) - 0.294}{0.813} \right] \quad (12)$$

$$1967 \text{ Hejian : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) - 0.594}{0.927} \right] \quad (13)$$

$$1966 \text{ Xingtai : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) + 0.054}{0.648} \right] \quad (14)$$

$$1962 \text{ Heyuan : } P_L = 1 - \Phi \left[\frac{\ln(N / N_{cr}) + 0.467}{0.646} \right] \quad (15)$$

5 Conclusions

In this paper, the inter-region variability of the model uncertainty associated with the liquefaction potential assessment method specified in MOHURD (2016) is investigated. It is found that the statistics of the model bias factor in different regions are quite different, indicating that the method given in MOHURD (2016) has different

levels of accuracy when applied in different regions. The uncertainty associated with the model bias factor in different regions are also different. For a region with more case histories, the uncertainty associated with the model uncertainty tends to be smaller. The model bias factor for a region with no case histories has the greatest uncertainty. Due to the existence of inter-region variability, the same N/N_{cr} value implies different levels of reliability. The method suggested in this paper can be used to calculate the level of liquefaction probability based on the values of N/N_{cr} in different regions.

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