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Learning about Uncertain Predictions of Rainfall-Induced Landslides from Observed Slope Performance

Ivan Depina[1,2], Emir Ahmet Oguz[3], and Vikas Thakur[3]

[1]Department or Rock and Geotechnical Engineering, SINTEF Buildings and Infrastructure, Trondheim, Norway.
[2]Faculty of Civil Engineering, Architecture and Geodesy, University of Split, Split, Croatia.
E-mail: ivan.depina@sintef.no
[3]Department of Civil and Environmental Engineering, Norwegian University of Science and Technology, Trondheim, Norway. E-mail: emir.a.oguz@ntnu.no, vikas.thakur@ntnu.no

Abstract: Accurate prediction of the occurrence of landslides triggered by rainfall can be challenging due to the uncertainties commonly associated with geotechnical, hydrological and climate parameters controlling the stability of slopes. Uncertainties may arise due to lack of knowledge (e.g., lack of geotechnical investigations) or the inherent natural variability (e.g., uncertain rainfall patterns) linked to these parameters. This work examines the potential of reducing uncertainties associated with rainfall-induced landslides by learning from observations of slope performances. The process of reducing the uncertainties is formalized by adopting the Bayesian updating framework. The performance of the implemented prediction models and the Bayesian updating framework is evaluated on the rainfall-induced landslides that occurred in 2011 in the Kvam area of central Norway. Slope failure or survival of a given rainfall event introduce additional information that can be utilized to reduce uncertainties associated with the assessment of rainfall-induced landslides in a certain area. Reducing uncertainties with site-specific observations of slope performance has a potential to increase the capacity of existing models to provide localized and more reliable predictions of rainfall-induced landslides.

Keywords: Rainfall; landslide; Bayesian; updating; slope; stability.

1 Introduction

One of the main challenges in accurate spatial and temporal predictions of the occurrence of landslides triggered by rainfall on local scales are uncertainties commonly associated with geotechnical, hydrological and meteorological parameters controlling the stability of slopes (e.g., Zieher et al. 2017). Uncertainties arise due to lack of knowledge (e.g., lack of geotechnical investigations) or the inherent natural variability (e.g., uncertain rainfall patterns) linked to these parameters (e.g., Melchiorre & Frattini 2012). The presence of uncertainties often hinders the existing advanced geotechnical, hydrological and climate models from providing reliable predictions of rainfall-induced landslides. The prediction capacity of these models can be improved by developing and implementing approaches to mitigate and reduce uncertainties. Approaches such as gathering more information on the uncertain parameters, increasing the measurement accuracy on the instruments, or improving the accuracy of the prediction models can assist in reducing uncertainties. Extensive data collection of the uncertain yet crucial parameters is not always feasible due to large and remote areas often spanning tens of square kilometres (e.g., Melchiorre & Frattini 2012).

Accordingly, this work makes an attempt to examine an alternative approach in reducing uncertainties associated with rainfall-induced landslides by learning from observations of slope performances. Observations of slope failure or an intact slope (referred as slope survival hereafter) for a given rainfall event introduce additional information that can be utilized to reduce uncertainties associated with the assessment of rainfall-induced landslides (e.g., Zhang et al. 2009). Reducing uncertainties with site-specific observations of slope performance has a potential to increase the capacity of existing models to provide localized and more reliable predictions of rainfall-induced landslides. Additional motivation for the utilization of observations of slope performance to mitigate uncertainties originates from the relatively low cost that is required to gather such information in comparison to alternative and more conventional methods for collecting data on the uncertain parameters. The process of reducing the uncertainties based on information on observed slope performance will be formalized by adopting the Bayesian updating framework. The observations of slope performance in this study will be focused on slopes surviving the rainfall event that occurred in 2011 in the Kvam area of central Norway.

2 Rainfall Infiltration Model

An analytical solution for rainfall infiltration on a slope terrain is implemented in this study based on the infiltration model developed by Iverson (2000). This infiltration model decomposes the time-dependent pressure head in a slope $\psi(t, y)$, into two components:

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$$\psi(t,y) = \psi_0(y) + \psi_1(t,y) \tag{1}$$

where t is time; y is soil depth; $\psi_0(y)$ is the long-term water pressure head corresponding to prolonged infiltration rates, while $\psi_1(t,y)$ is the short-term head response to a rainstorm. In the case of a slope parallel groundwater flow, ψ_0 is defined as a linear function of y with respect to the initial groundwater depth H_w:

$$\frac{\psi_0}{y} = \left(1 - \frac{H_w}{y}\right) cos^2\,\theta \tag{2}$$

where θ is the slope angle. The initial groundwater levels can be determined based on the long-term precipitation rates and the topographic index as shown in Melchiorre and Frattini (2012) in situations when there is a lack of field measurements of groundwater levels. The short-term head response is evaluated based on the reduced form of the Richards equation that corresponds to wet initial conditions with the saturated coefficient of permeability:

$$\frac{\partial\psi_1}{\partial t} = D_0\, cos^2\,\theta\, \frac{\partial\psi_1^2}{\partial y^2} \tag{3}$$

where D_0 is the maximum hydraulic diffusivity, corresponding to saturated conditions. Solving Eq. (3) with the boundary conditions defined in Iverson (2000) leads to the following equations:

$$\frac{\psi_1(t^*)}{y} = \frac{I_y}{k_s} R(t^*) \quad \text{when} \quad 0 \le t^* \le T^* \tag{4a}$$

$$\frac{\psi_1(t^*)}{y} = \frac{I_y}{k_s}[R(t^*) - R(t^* - T^*)] \quad \text{when} \quad t^* > T^* \tag{4b}$$

where I_y is the rainfall rate; t^* is the normalised time; T^* is the normalised duration of the precipitation, while R is the response function (Iverson, 2000). These parameters are defined as follows:

$$t^* = \frac{t}{y^2/D};\ T^* = \frac{T}{y^2/D};\ R(t^*) = \sqrt{t^*/\pi}\, exp\left(-\frac{1}{t^*}\right) - erfc\left(\frac{1}{\sqrt{t}}\right) \tag{4c}$$

where $D = 4D_0\, cos^2\,\theta$ is an effective hydraulic diffusivity; $erfc$ is the complementary error function. The infiltration model by Iverson (2000) is known to provide physically unrealistic high pressures at shallow depths. An ad-hoc solution is implemented in the infiltration model by Iverson (2000) to mitigate the problem of high pressures by limiting the predicted pressures to hydrostatic conditions defined by $\psi = y\, cos^2\,\theta$.

3 Slope Stability Model

Stability of an unsaturated slope subjected to a rainfall event is evaluated with the infinite slope model and the extended Mohr Coulomb failure criterion for unsaturated soils (e.g., Fredlund et al. 2012). Based on the extended Mohr Coulomb failure criterion, the shear strength is defined as follows:

$$\tau = c' + (\sigma - \psi)\, tan\,\varphi' \tag{5}$$

where τ is the shear strength; c' is the effective cohesion; σ is the total normal stress at the failure plane; φ' is the effective internal friction angle. The expression in Eq. (5), was derived by assuming that the rate of increase in shear strength with respect to a change in the matric suction is defined by $tan\varphi'$. The extended Mohr Coulomb failure criterion can be integrated in the expression for infinite slope stability to obtain the following expression for the minimum factor of safety, F_s, along soil depth $y' \in [0,H]$:

$$F_s(\boldsymbol{x},t) = min_{y' \in [0,H]}\{c' + tan\,\varphi'[cos^2\,\theta \int_0^{y'} \gamma(t,y)dy - \psi(t,y')]\}/\{sin\,\theta\, cos\,\theta \int_0^{y'} \gamma(t,y)dy\} \tag{6}$$

where $\boldsymbol{x}$ is a vector of model parameters in Table 1; $\gamma(t,y') = \gamma_w/[1+e][G_s + eS(t,y')]$ is the soil unit weight; $\gamma_w = 9.81\ kN/m^3$ is the water unit weight; e is the void ratio; G_s is the dry unit weight; $S(t,y') = 1.0$ if $\psi(t,y') \ge 0$ and $S(t,y') = 0.0$ otherwise is an approximation of the degree of saturation. The integral in Eq. (6) was evaluated numerically by discretizing the integration domain at 100 equally distributed segments along the soil depth.

4 Kvam Case Study

The capacity of observations of slope survival of a rainfall event will be examined on the landslides that occurred in Kvam area, central Norway, in relation to the rainfall events in June of 2011 as shown in Figure 1. The rainfall event is analysed to gain insights in the conditions controlling the occurrence of rainfall-induced landslides. These will be a valuable contribution to identifying the relative importance of the different hydrological (e.g., groundwater levels) and geotechnical (e.g., strength values) parameters in the initiation of rainfall-induced landslides. This study adopts the probabilistic framework to provide a consistent framework to model the uncertainties associated the geotechnical, hydrological and meteorological parameters. The process of reducing uncertainties in these parameters is supported by Bayesian updating. Due to lack of field investigations and direct measurements from the site, the typical values of selected input parameters are based mainly on information available in the literature. It is worth stressing that local conditions and the quaternary geology of the Kvam area is reflected in the selected values. Table 1 provides an overview of the probabilistic models adopted to model uncertainties associated with different geotechnical and hydrological parameters controlling the occurrence of rainfall-induced landslides at the Kvam area.

5 Bayesian Updating

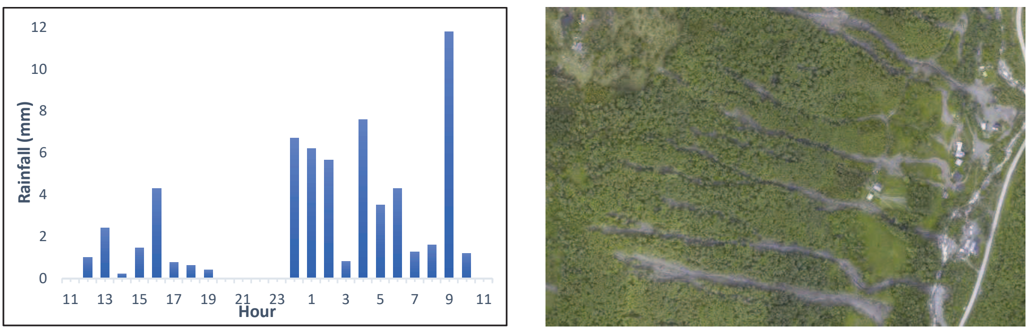
The application of Bayesian updating (e.g., Depina et al. 2017; Straub & Papaioannou 2014) is advantageous because it provides a consistent and explicit probabilistic framework to update and reduce uncertainties in the geotechnical and hydrological parameters based on additional information on the uncertain parameters. The Bayesian framework is robust and can incorporate a wide range of information including direct measurements of the uncertain geotechnical and environmental parameters (e.g., measurements of strength parameters or groundwater levels) or indirect observations (e.g., slope survival or failure after a rainfall event). Consider an n-dimensional vector of random parameters, $\boldsymbol{X} = [X_1, \ldots, X_n]^T$, that corresponds to the uncertain parameters in a slope stability model subjected to a rainfall event. $\boldsymbol{X}$ is distributed according to a joint probability density function (pdf), $f(\boldsymbol{x})$, where $\boldsymbol{x}$ is a realization of $\boldsymbol{X}$ in the corresponding outcome space, $\Omega_{\boldsymbol{X}}$. This study aims to utilize observations of a slope performance after a rainfall event to learn about the uncertain parameters controlling the occurrence of rainfall-induced landslides. The Bayesian framework can be applied in such problems, because it allows one to update a prior model of uncertainties, $f'(\boldsymbol{x})$, to a posterior probability distribution, $f''(\boldsymbol{x})$, based on observations as follows:

$$f''(\boldsymbol{x}) = L(\boldsymbol{x})f'(\boldsymbol{x})/\left[\int_{\Omega \boldsymbol{x}} L(\boldsymbol{x})f'(\boldsymbol{x})d\boldsymbol{x}\right] \tag{7}$$

where $L(\boldsymbol{x}) = Pr(Observation|\boldsymbol{X} = \boldsymbol{x})$ is the likelihood function, proportional to the probability of observing certain slope performance for a given value of model parameters. The evaluation of the likelihood function is one of the central elements in the Bayesian framework as it relates model predictions to the observation. In the case of the Bayesian updating of a slope stability model, the likelihood function relies on a link between an observed slope performance and the corresponding model prediction. Let η denote the observed performance, while $h(\boldsymbol{x})$ represents the corresponding model prediction. The observation is defined as slope survival is defined as follows:

$$Z = \{\boldsymbol{x} \in \Omega_x: h(\boldsymbol{x}) < 0, h(\boldsymbol{x}) = min\,[1.0 - F_S(\boldsymbol{x}, t = [0, T])]\} \tag{8}$$

where $F_s(\boldsymbol{x}, t)$ is the factor of safety of a slope for a given realization of the uncertain parameters at time t; T is duration of the rainfall event. Due to measurement or prediction errors, deviations occur between the observation and the model prediction, $\varepsilon = \eta - h(\boldsymbol{X})$. The distribution of deviations provides a basis for the construction of the likelihood function, such that for $\varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon)$, $L(\boldsymbol{x}) = Pr[h(\boldsymbol{x}) + \varepsilon < 0] = \Phi[-(h(\boldsymbol{x}) - \mu_\varepsilon)/\sigma_\varepsilon]$, where Φ is the standard normal cumulative density function. Bayesian updating is performed for the Kvam site based on the prior distribution of uncertainties in the parameters in Table 1. Sampling from the posterior distribution was conducted with the Markov Chain Monte Carlo algorithm developed by Au and Beck (2001). Effects of spatial variability were not modelled explicitly, and the single random variable approach was implemented.



(a) (b)

Figure 1. (a) Rainfall event in Kvam area from June 9th to 10th 2011, (b) Rainfall-induced landslides following the rainfall event in Kvam (https://www.norgeibilder.no/).

6 Slope Stability Model

Figure 2 shows the results of the Bayesian updating for the uncertain parameters based on the information on slope survival of the rainfall event in 2011 at the Kvam area for a range of slope angle values, $\theta = [20°, 25°, 30°, 35°]$. The posterior distributions were estimated empirically on 5000 samples from the posterior distribution with a non-parametric kernel density estimator (e.g., Wassermann 2006). It is important to note that the results of the updating should be interpreted within the context of the implemented slope stability model. More detailed analyses accounting for the effects of spatial variability, vegetation, slope morphology, and alternative failure modes (e.g., erosion) may provide additional valuable insights not captured with approach implemented in this study. From Figure 2 (a) it can be observed that the information on the slope survival of the rainfall event has a significant effect on the distribution of the cohesion, c. The posterior distribution has shifted to the right with increased values of the mean. The rate of increase of the mean is increasing with the value of the slope angle θ. The observation of slope survival for higher slope angles is considered as a stronger updating information than for the shallow slope angles, because the analysed rainfall is more critical for the stability of the high slope angles than for the low ones. The effects of updating on the uncertainties associated with ϕ can be examined in Figure 2 (b). Similar as the values of c, it can be observed that relatively low values of ϕ are less likely in the posterior distribution than in the prior. The posterior distribution has shifted to the right with increased values of the mean and reduced variance.

Table 1. Uncertain geotechnical and hydrogeological parameters at the Kvam site.

Parameter		Distribution	Parameters		Source
H [m]	Soil thickness	Lognormal	μ_H=-2.578·tan(θ)+2.612	σ_H=0.271	-
c [kPa]	Cohesion	Lognormal	μ_c=4.0	CoV_c=0.3	Melchiorre and Frattini (2012), Lacasse and Nadim (1997).
ϕ [°]	Friction angle	Normal	μ_ϕ=32	CoV_ϕ=0.1	Melchiorre and Frattini (2012).
Gs [-]	Dry unit weight	Normal	μ_{Gs}=2.7	CoV_{Gs}=0.02	Guan and Fredlund (1997).
e [-]	Void ratio	Normal	μ_e=0.25	CoV_e=0.1	Lacasse and Nadim (1997).
H_w [m]	Water depth	Uniform	$H_{w,\min}$=0	$H_{w,\max}$=H	-
k_s [m/s]	Saturated permeability	Lognormal	μ_{ks}=5.0·10^{-6}	CoV_{ks}=0.25	Melchiorre and Frattini (2012), Janbu (1989).
D_0 [m^2/s]	Maximum diffusivity	Lognormal	μ_{D0}=1.0·10^{-4}	CoV_{D0}=0.25	Melchiorre and Frattini (2012).
ε [-]	Slope stability model error	Normal	μ_ε=0.0	σ_ε=0.05	(Duncan & Wright, 1980)

The effects of updating on the distribution of the soil thickness, H, can be examined in Figure 2 (c). The posterior distribution overlaps closely with the prior distribution, with a slight shift to the lower values. The shift increases for the relatively high slope angle values, 30°<θ <35°. For these slope angles, the posterior distribution has slightly reduced values of the mean and a lower variance, which indicates more likely shallower slope thicknesses for these slopes. The effects of the updating on the distribution of the water depth, H_w, are presented in Figure 2 (d). From Figure 2 (d) it can be observed that slopes surviving the considered rainfall event are likely to be associated with deeper initial groundwater levels. This observation is inferred from the higher values of the mean of the posterior distribution, when compared to the prior distribution. The updating did not significantly affect the remaining uncertain parameters. The posterior distributions of the uncertain parameters controlling the occurrence of rainfall-induced landslides can be further used to update reliability estimates of landslide occurrence with respect to future rainfall events. Updating the reliability estimates enables more accurate and consistent landslide initiation probability and risk estimates thereby contributing to reduce adverse consequences on life and property. Future studies will examine the learning process that is continuously being updated by multiple sources of information on geotechnical, hydrological and meteorological conditions collected through monitoring systems (Depina 2018).

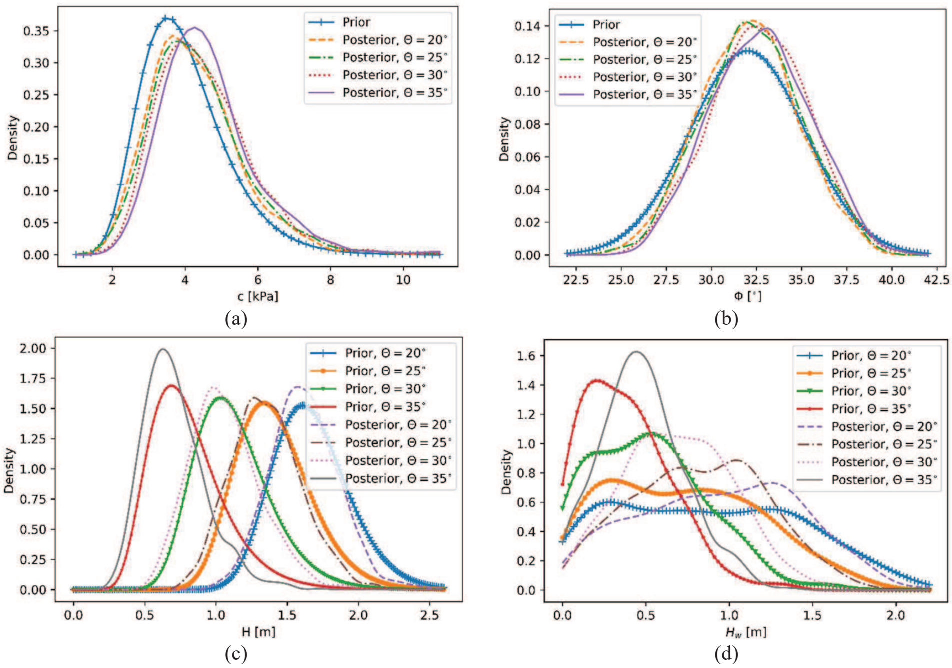


Figure 2. Prior and posterior distributions of: (a) c, (b) ϕ, (c) H and (d) H_w.

7 Conclusions

The implemented study examined the potential of utilizing information on the slope survival of a rainfall event to update uncertainties associated with geotechnical and hydrological conditions controlling the occurrence of rainfall-induced landslides. The updating was implemented within a Bayesian framework, which provides a consistent, explicit and robust framework for updating uncertainties with observations. The updating process affected significantly the probability density functions of cohesion, friction angle, slope thickness and initial groundwater depth. More detailed analyses may provide additional valuable insights not captured with the approach implemented in this study.

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