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Efficient Sample Manipulation for Direct Monte Carlo Simulation in Slope System Reliability Analysis

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Abstract: Probabilistic slope stability analysis based on limit equilibrium methods is often formulated as a series system reliability problem because there are a large number of (denoted by N_s) potential slip surfaces for a given slope. Direct Monte Carlo simulation (DMCS) provides a straightforward way to incorporate these slip surfaces into slope system reliability analysis, which can be easily implemented by repetitively selecting a sample as input to perform slope stability analysis considering N_s slip surfaces. However, DMCS has often been criticized by its inefficiency since it often needs a large number of (denoted by N) random samples to guarantee the accuracy of reliability estimates, especially for estimation of small failure probabilities. Therefore, the total computational costs for DMCS-based slope reliability analysis are of $O(NN_s)$ in terms of the computational costs for slope stability analysis with a single slip surface. One important reason for the expensive computational costs of DMCS is the ratio of failure samples to all the random samples simulated in DMCS is as low as the failure probability. To remove this hurdle, an efficient method called Adaptive MCS (AMCS) is proposed for slope reliability analysis. The proposed method aims to efficiently identify all failure samples from all the random samples generated by DMCS. The computational efficiency of slope system reliability analysis is significantly improved by iteratively selecting a trial slip surface for identification of failure samples. For illustration, both DMCS and AMCS are adopted to estimate system failure probability of a two-layer soil slope example in spatially variable soils.

Keywords: Monte Carlo simulation; slope stability; failure samples; computational efficiency.

1 Introduction

Probabilistic slope stability analysis based on limit equilibrium methods is often formulated as a series system reliability problem because there are a large number of (denoted by N_s) potential slip surfaces (e.g., Ji and Low 2012; Li et al. 2014; Zhang and Huang 2016). Direct Monte Carlo simulation (DMCS) provides a straightforward way to incorporate these slip surfaces into slope system reliability analysis (e.g., El-Ramly et al. 2002; Jiang et al. 2015). Using DMCS, the slope system failure probability P_f is estimated by

$$P_f = \frac{1}{N}\sum_{i=1}^{N} I\left(\min_{j=1}^{N_s} FS_j\left(\mathbf{x}_i\right) < 1\right) = \frac{N_f}{N} \tag{1}$$

where $\mathbf{x}$ denotes the uncertain parameters; N is the total number of random samples of $\mathbf{x}$; $I(\cdot)$ is the indicator function; $FS_j(\mathbf{x}_i)$ denotes the safety factor of the j-th slip surface given $\mathbf{x}_i$; N_f is the number of failure samples. DMCS needs to identify the N_f failure samples from N random samples for calculating P_f. It is often criticized due to its inefficiency because N has to be large enough to guarantee the accuracy of estimation of P_f, especially for small failure probabilities (Baecher and Christian 2003; Ching et al. 2009; Ji and Low 2012).

The total computational costs for DMCS-based slope reliability analysis are of $O(NN_s)$ in terms of the computational costs for slope stability analysis with a given slip surface. There are two types of operations to implement DMCS: (a) repetitively performing a complete slope stability analysis with N_s slip surfaces (referred to as CSSA in this study) for N times. (b) repetitively performing slope reliability analysis with N random samples using a prescribed slip surface (referred to as SRAP in this study) for N_s times. The former operation is generally adopted, but it is inefficient because finding failure samples needs to perform CSSA for each sample. A single run of CSSA is able to find only one failure sample at maximum. On the other hand, a single run of SRAP might find more than one failure samples.

Therefore, a new method called Adaptive MCS (AMCS) is proposed in this study for slope reliability analysis. The proposed method aims to efficiently identify all the failure samples from the random samples generated by DMCS. Both CSSA and SRAP are performed in a cooperative way. For illustration, both DMCS and AMCS are adopted to estimate system failure probability of a two-layer soil slope example in spatially variable soils.

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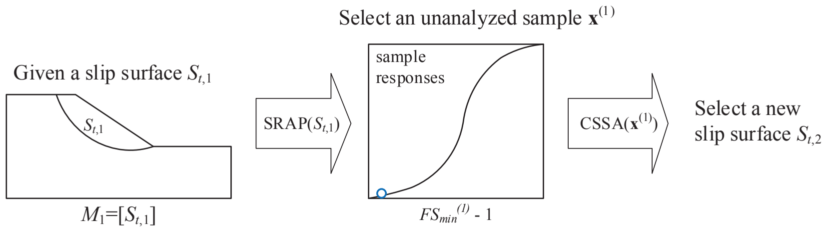
2 Adaptive MCS for Slope System Reliability Analysis

The proposed AMCS iteratively constructs a slope subsystem M_k with k trial slip surfaces, $M_k = [S_{t,1}, S_{t,2}, \ldots, S_{t,k}]$, the failure probability $P_f^{(k)}$ of which converges to the system failure probability P_f and is written as:

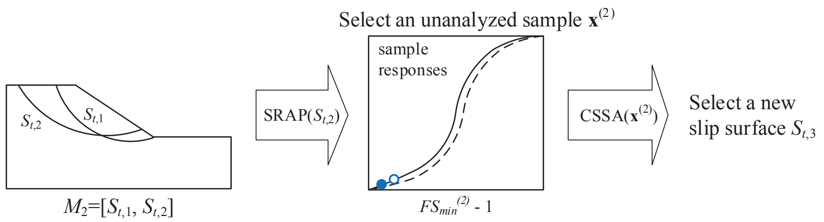
$$P_f^{(k)} = \frac{1}{N}\sum_{i=1}^{N} I\left[FS_{\min}^{(k)}\left(\mathbf{x}_i\right) < 1\right] = \frac{N_f^{(k)}}{N} \rightarrow P_f \tag{2}$$

where $FS_{\min}^{(k)}\left(\mathbf{x}_i\right)$ denotes the minimum slope safety factor for sample $\mathbf{x}_i$ at k-th iteration; $N_f^{(k)}$ is the number of failure samples at k-th iteration.

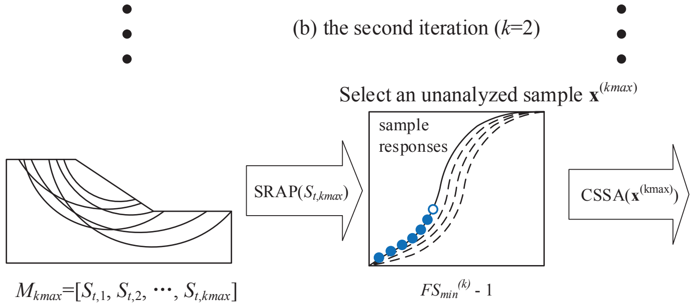
Figure 1 illustrates the implementation procedure of AMCS in slope system reliability analysis. AMCS starts with generating N random samples of uncertain parameters $\mathbf{x}$. At the first iteration (k = 1), an initial trial slip surface $S_{t,1}$ is arbitrarily prescribed and it constitutes a subsystem $M_1 = [S_{t,1}]$. $S_{t,1}$ is used to perform SRAP with N random samples, and $FS_{\min}^{(1)}\left(\mathbf{x}_i\right)$ is obtained as $FS_{\min}^{(1)}\left(\mathbf{x}_i\right) = FS_{t,1}\left(\mathbf{x}_i\right)$ for k = 1. A random sample corresponding to the minimum $FS_{\min}^{(1)}$ is then selected as $\mathbf{x}^{(1)}$ and used to perform CSSA. The minimum slope safety factor corresponding to $\mathbf{x}^{(1)}$ is obtained and its corresponding slip surface is selected as the second trial slip surface $S_{t,2}$. The number of failure samples is counted as $N_f^{(1)}$.



(a) the first iteration (k=1)



(b) the second iteration (k=2)



(c) the k_{max}-th iteration

Figure 1. Illustrative scheme of Adaptive MCS for slope reliability analysis.

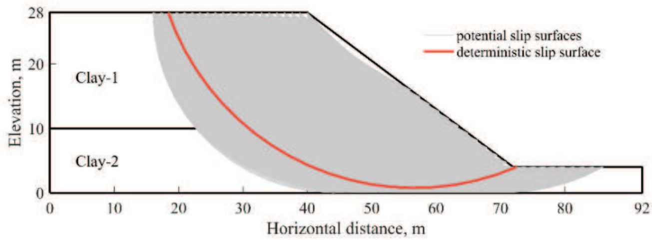
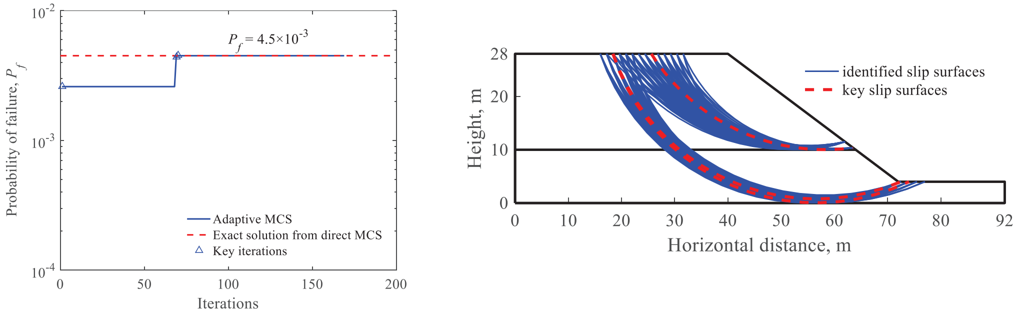


Figure 2. Geometry and potential slip surfaces of a two-layer slope example.



(a) Probability of failure (b) Identified slip surfaces

Figure 3. Failure probability and slip surfaces obtained from AMCS.

For the second iteration (k = 2), $S_{t,2}$ is added to the subsystem M_2 = [$S_{t,1}$, $S_{t,2}$]. It is also used to perform SRAP with N samples. The sample responses are updated as $FS_{\min}^{(2)}(\mathbf{x}_i) = \min\left\{FS_{\min}^{(1)}(\mathbf{x}_i), FS_{t,2}(\mathbf{x}_i)\right\}$. The sample corresponding to the smallest $FS_{\min}^{(2)}$ is selected as $\mathbf{x}^{(2)}$ and is used to perform CSSA. The minimum safety factor for sample $\mathbf{x}^{(2)}$ is calculated, and its corresponding slip surface is selected as $S_{t,3}$. The number of failure samples is counted and denoted as $N_f^{(2)}$. The iterative process described above is repeated k_{max} times when $N_f^{(k)}$ converges to $N_f^{(k_{\max})}$ (i.e., relative differences in $N_f^{(k_{\max})}$ less than 0.001 for additional k_t iterations where k_t is a threshold value of iterations). Then, $N_f^{(k_{\max})}$ is used to calculate the slope failure probability using Eq. (2). When k_{max} is equal to N_s, AMCS is reduced to DMCS. Thus, AMCS can be seen as a variant of DMCS. In AMCS, the estimation of failure probability is monotonically increasing as more failure samples are obtained. The value of k_t reflects a trade-off between computational accuracy and efficiency. In practice, k_t often accounts for only a small percentage of N (e.g., 0.5%) to achieve convergence.

The computational cost of slope reliability analysis can be measured by the equivalent number of CSSA, N_T, which is equal to N for the DMCS with N random samples, i.e., $N_{T,\,DMCS} = N$. For each iteration, AMCS performs both CSSA with N_s slip surfaces for one sample (i.e., the computational cost is equivalent to 1 time of CSSA) and SRAP with one slip surfaces for N sample (i.e., the computational cost is equivalent to N_s / N times of CSSA). The computational cost of AMCS with k_{max} iterations $N_{T,AMCS}$ is expressed as:

$$N_{T,AMCS} = k_{\max}\left(N / N_s + 1\right) \tag{3}$$

As indicated by Eq. (3), $N_{T,AMCS}$ can be much less than $N_{T,\,DMCS}$ if a large number of slip surfaces are considered.

3 Illustrative Example

As shown in Figure 2, a two-layer soil slope is used to illustrate the proposed method. The slope has a height of 24 m and a slope ratio of 4:3. It is comprised of two clay layers with equal unit weight of 19 kN/m^3. The undrained shear strengths of the two clay layers were modeled using random variables (Ching et al. 2009; Zhang and Huang 2016) and random fields (Li et al. 2016; Jiang et al. 2017). The proposed method is adopted to evaluate the slope failure probability for both cases, i.e., random variable case and random field case. For the sake of conciseness, details on uncertain soil parameters and their statistics in the two cases are referred to Ching et al. (2009) and Li et al. (2016), respectively. A total of 34458 potential circular slip surfaces are considered in this example, and the safety factor of slope stability is computed using simplified Bishop method.

3.1 Slope reliability analysis results

For random variable case, a total of 20000 random samples are generated using DMCS. Both AMCS and DMCS are adopted to evaluate the failure probability of this example. Figure 3(a) shows the iterative estimation of slope failure probability using AMCS. The slope failure probability converges to the estimation from the DMCS at 70-th iteration and terminates at 169-th iteration, thus identifies 169 slip surfaces by the proposed method, as shown in Figure 3(b). The slope failure probability is estimated as $P_f = 4.50\times10^{-3}$. Table 1 shows the probabilistic analysis results using different methods. The result (i.e., $P_f = 4.50\times10^{-3}$) from AMCS is favorably comparable with those (i.e., $P_f = 4.40\times10^{-3}$ and 3.80×10^{-3}, respectively) obtained using DMCS and Importance Sampling (IS) reported by Ching et al. (2009).

Table 1. Slope reliability analysis results in random variable case.

Method	P_f	N_T	COV	Unit COV[b]	Source
Adaptive MCS	4.50×10^{-3}	267	0.127[a]	2.08	This study
Direct MCS	4.50×10^{-3}	2×10^{4}	0.105	14.87	This study
Direct MCS	4.40×10^{-3}	1×10^{4}	0.150	15.04	(Ching et al. 2009)
Importance Sampling	3.80×10^{-3}	1×10^{3}	0.066	2.09	(Ching et al. 2009)

Note: (a) Estimated from 30 independent runs; (b) Unit COV = COV $\sqrt{N_T}$

Table 2. Slope reliability analysis results in random field case (θ_H = 24m, θ_V = 2.4m).

Method	P_f	N_T	COV	Unit COV[b]	Source
Adaptive MCS	1.70×10^{-4}	624[a]	0.223[a]	5.57	This study
Direct MCS	1.90×10^{-4}	1×10^{5}	0.243	76.84	This study
Direct MCS	1.80×10^{-4}	6×10^{4}	0.304	74.53	(Li et al. 2016)
Subset Simulation	1.90×10^{-4}	2.3×10^{3}	0.370	17.75	(Li et al. 2016)

Note: (a) Estimated from 30 independent runs; (b) Unit COV = COV $\sqrt{N_T}$

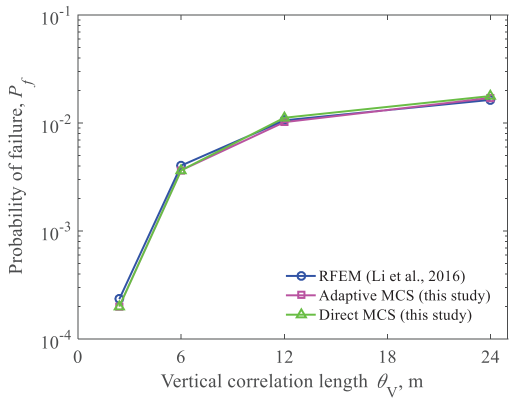


Figure 4. Effects of vertical correlation length on slope failure probability (horizontal correlation length θ_H = 24 m).

The computational costs of different methods are also shown in Table 1 and denoted by N_T. The computational cost of the proposed method is calculated using Eq. (3) as $N_{T,AMCS} = 169 \times (2\times10^4 / 34458 + 1) = 267$. It indicates that the computational cost for the proposed method is only about 1/75 that of DMCS, even though the proposed method uses the same number of random samples as DMCS. Besides, the unit COV is also calculated to compare the performance of different reliability methods. A smaller unit COV value indicates higher computational efficiency. The unit COV values of AMCS and IS are 2.08 and 2.09, respectively. This indicates a higher computational efficiency of AMCS and IS in comparison with DMCS.

For random field case, the slope is discretized into 1 m×1 m square and triangular elements (Li et al., 2016). The soil parameters are modeled using lognormal random fields. Table 2 shows slope reliability analysis results obtained as the horizontal and vertical correlation length equal to θ_H = 24 m and θ_V = 2.4 m, respectively. The slope failure probability is obtained as 1.70×10^{-4}, which is in good agreement with those from DMCS and Subset Simulation from Li et al. (2016). The proposed method terminates after 160 iterations, resulting in a computational cost of $N_{T,AMCS}$ = 160 × (10/ 34458 + 1) = 624. The unit COV of the proposed method is equal to 5.57, which is 1/12 and 1/3 of the respective unit COVs of DMCS and Subset Simulation. The proposed approach considerably improves the computational efficiency of slope system reliability analysis.

Figure 4 shows that the slope system failure probability increases from 1.70×10^{-4} to 1.71×10^{-2} as vertical correlation length increases from 2.4 m to 24 m and horizontal correlation length θ_H = 24 m. The results agree well with those reported by Li et al. (2016). The proposed method provides reasonable estimates of slope failure probability with consideration of spatial variability in a cost-effective manner.

3.2 Discussion on the differences between AMCS and representative slip surface methods

Representative slip surface (RSS) methods have been proposed in previous studies (e.g., Li et al. 2014; Zhang and Huang 2016; Jiang et al. 2017). Although both the proposed method and RSS methods use a small portion of potential slip surfaces, instead of using all of them, they are conceptually different. Three main differences are summarized as below:

1. The proposed method incrementally adds a new slip surface for approximation, while RSS methods selects RSSs from a given set of potential slip surfaces. RSS methods need to prescribe all slip surfaces. In contrast, the proposed method can be equally applicable to cases considering an unlimited number of slip surfaces, for example, non-circular slip surfaces, which may be unknown before slope stability analysis.
2. RSS methods needs to calculate correlation coefficients between slip surfaces. The number of correlation coefficients between slip surfaces is of $O(N_s^2)$, which increases significantly as the number of slip surfaces N_s increases. It might not be a trivial task to compute correlation coefficients between slip surfaces, particularly as the spatial variability is considered and is explicitly modelled using random fields. On the other hand, AMCS does not need the information of correlation among responses of different slip surfaces.
3. Determining the threshold value of correlation coefficient of responses of different slip surfaces for selecting RSSs is not trivial. This is because the number of RSSs is uncertain in different cases, especially in spatially variable soils where failure mechanisms vary. A trial-and-error process is often necessary.

4 Conclusion

This study presents a new MCS-based method called Adaptive MCS for slope reliability analysis using limit equilibrium methods. AMCS adopts an iterative procedure to progressively approximate estimation of slope failure probability. It is shown that AMCS is able to provide reasonably accurate estimation of slope failure probability in comparison with brute-force DMCS and advanced MCS methods (e.g., Importance sampling and Subset Simulation), but its computational cost is largely reduced compared with brute-force DMCS. Although both AMCS and RSS methods uses a small portion of potential slip surfaces, instead of using all of them, AMCS is conceptually different from RSS methods, avoiding the need of information on the correlation of responses of different slip surfaces, and it is also applicable to slope stability problems considering non-circular slip surfaces, which is undergoing exploration.

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