Normative Statistical Solutions for Common Geotechnical Problems

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Abstract: In mathematical statistics, the most popular and the central model is the normal sample theory. Based on this theory, one can answer to such questions as what is the 95% confidence interval for this estimated mean value? Or how many samples do you need to estimate mean whose confidence interval is +10% of the estimated value? Unfortunately, we do not have such normative statistical answers for geotechnical problems, and this paper try to answer these requirements. The basic assumptions employed in this study are (1) the value to be estimated is the local average for a certain soil range (e.g., line, area or volume, which is called the local averaging range, LAR) that controls performance of a structure under consideration, and (2) samples are taken from the normal random filed whose variance and the correlation structure are known, and only the average is not known. After introducing the theory which the authors named GRASP (Geotechnical Reliability Analysis by a Simplified Procedure), several typical geotechnical normative statistical solutions are presented through examples.

Keywords: Local average; sandom field; sample size; sampling configuration; characteristic value.

1 Introduction

The objective of this paper is to propose a statistical theory that can give statistical answers to many common geotechnical problems, such as number and position of samples to obtain a characteristic value of a geotechnical parameter with specified statistical quality, and a statistical procedure to determine characteristic value of a soil parameter of the specified statistical quality from given measurements.

In the normal sample theory which is the most popular basis in the modern mathematical statistics, there are some basic assumptions. For an example, when a population mean is estimated from samples while the population variance is known, the assumptions are (1) samples from the population are independently and identically distributed following a normal distribution, (2) the variance of the normal distribution is known and (3) samples are taken randomly from the population. A statistical theory is proposed here in analogy to this theory but based on the assumptions considering the geotechnical purposes and the conditions.

First, the basic theory (GRASP, Geotechnical Reliability Analysis by a Simplified Procedure) is summarized. The full explanation of GRASP is given in Honjo and Otake (2018). Unfortunately, there is no literature yet explaining the full scope of GRASP in English, the ideas are fragmentary given in the several literature (Honjo and Otake 2012, 2013; Otake and Honjo 2013, 2014). Several typical geotechnical problems are given as examples to how he GRASP is applied to obtain the normative statistical solutions.

2 Summary of GRASP Theory

2.1 Background

The authors have demonstrated that the effect of the spatial variability of geotechnical parameters on the performance of a geotechnical structure can be evaluated with the reasonable accuracy by the local average for a certain appropriate soil range (e.g. a line, an area or a volume, which is called the local averaging range, LAR) that controls the performances of the geotechnical structure (Honjo and Otake 2013; Otake and Honjo 2013) in many typical soil mechanics problems. Based on this finding, the following facts can be concluded.

1. The effect of soil variability on performance of a geotechnical structure can be evaluated by using the variability of the LAR. In other words, one can replace a random field by a single random variable to evaluate the effects of soil variability on the performance.
2. The characteristic value of a soil parameter should be an estimate of this local average over the LAR because this is the quantity that directory controls the performance of a structure. Therefore, this is very important to recognize the local average over the LAR should be the target quantity to be estimated from the obtained samples. Otherwise, a sampling plan can be made, that ensures a certain level of statistical accuracy of this quantity, i.e., the local average over the LAR.

The second point is tackled in this paper. We have extended the traditional normal sample theory to estimate the mean of the local average over a LAR in a RF.

2.2 Basic assumptions

The following assumptions are made:

Published by Research Publishing, Singapore.
ISBN: 978-981-11-2725-0; doi:10.3850/978-981-11-2725-0_MS1-2-cd

1. A ground parameter of our concern (maybe the undrained shear strength) is assumed to follow a stationary normal random field (RF) in a stratum.
2. The variance and the autocorrelation structure, i.e. the function type and the autocorrelation distance, are assumed to be known. Furthermore, the autocorrelation function is separable in multi dimensions (Vanmarcke 1977).
3. The sampling is made in an equal interval for a specific direction. Thus, the sampling is made in a lattice form in multi dimensions in a stratum.

2.3 Estimation variance function of a local average (LA)

Let us consider a problem to estimate the mean of a LA, $Z_V(x_{est})$, based on 1-D equally spaced n samples on a sample line with length L, where x_{est} is the central coordinate of the LA and V is the size of local average range. $x_{obs}=(x_1, \dots x_i, \dots x_n)^T$ are coordinates of n sample points. The central coordinate of the sample line is indicated by x_{obs}. The LA is estimated by a weighted average of the observed values as follows:

$$\hat{Z}_V(x_{est}) = \sum_{i=1}^{n} \nu_i Z(x_i) \tag{1}$$

where ν_i is a weight whose sum should be 1.0 in order to satisfy the unbiasness condition of the estimator. The estimation variance, σ_V^2, of estimating the mean of the LA, $Z_V(x_{est})$ by $\hat{Z}_V(x_{est})$ is defined as follows:

$$\sigma_V^{\ 2} = E\left[\left\{\hat{Z}_V(x_{est}) - Z_V(x_{est})\right\}^2\right] = E\left[\left\{\sum_{i=1}^{n} \nu_i Z(x_i) - Z_V(x_{est})\right\}^2\right] \tag{2}$$

Since the estimation variance is a function of the sample line length L and the autocorrelation distance θ, it is convenient to introduce the ratio of the two, which is termed the normalized sample line length, $L_N=L/\theta$.
The estimation variance can be further expanded as follows:

$$\sigma_V^{\ 2} = E\left[\left\{Z_V(x_{est}) - \mu_Z\right\}^2\right] + \sum_{i=1}^{n}\sum_{j=1}^{n} \nu_i \nu_j E\left[\left(Z(x_i) - \mu_Z\right)\left(Z(x_j) - \mu_Z\right)\right] - 2\sum_{i=1}^{n} \nu_i E\left[\left(Z_V(x_{est}) - \mu_Z\right)\left(Z(x_i) - \mu_Z\right)\right] \tag{3}$$

The first term of Eq.(3) is actually the variance function introduced by Vanmarcke (1977):

$$\sigma_Z^{\ 2}\Gamma^2\left(V/\theta\right) = E\left[\left\{Z_V(x_{est}) - \mu_Z\right\}^2\right] \tag{4}$$

The second term is the estimation variance of the population mean. The mean estimation variance function Λ^2 is introduced to denote this term:

$$\sigma_Z^{\ 2}\Lambda^2\left(n, L_n, \nu\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \nu_i \nu_j Cov\left(x_i, x_j\right) \tag{5}$$

The third term can be described by introducing the correlation function between $Z_V(x_{est})$ and $Z\ (x_i)$, which is defined as γ_V as follows:

$$\sigma_Z^{\ 2}\gamma_V\left(Z_V(x_{est}), Z(x_i)\right) = E\left[\left(Z_V(x_{est}) - \mu_Z\right)\left(Z(x_i) - \mu_Z\right)\right] \tag{6}$$

Finally, the following equation is obtained by substituting Eqs. (4), (5) and (6) into Eq. (3):

$$\sigma_V^{\ 2} = \sigma_Z^{\ 2}\{\Gamma^2\left(V/\theta\right) + \Lambda^2\left(n, L_n, \nu\right) - 2\sum_{i=1}^{n} \nu_i \gamma_V\left(Z_V(x_{est}), Z(x_i)\right)\} \tag{7}$$

The three terms of Eq.(7) can be interpreted as follows:

1. The first term, $\Gamma^2(V/\theta)$, is the variance function, which evaluates the reduction of the LA variance over length V from σ_Z^2.
2. The second term is the mean estimation variance function, which indicates the estimation variance of the population mean, μ_Z, by the given observations. When the samples are mutually independent, this term turns out to be σ_Z^2/n.
3. The third term indicates estimation variance reduction due to the contributions of the observations made closer to the location of the LA. This is actually the interpolation effects exactly same as one experiences in Block Kriging.

2.4 General and local estimation of a LA

It is important to distinguish between the local and the general estimation problems of the mean of a LA of a geotechnical parameter in both theoretically and practically (Honjo and Otake 2013, 2018).

2.4.1 General estimation of the mean of a LA

This is to estimate the mean of a LA of a geotechnical parameter value at any arbitrary point in a stratum in geotechnical design. It is important to recognize that the relative locations of the samples and the structure under design are not considered at all in the general estimation. Since the expectation of the LA automatically is the population mean in the general estimation, the estimation variance evaluation problem becomes a problem of evaluating the estimation variance of the population mean.

In the general estimation, the estimation variance function is given as follows:

$$\sigma_V{}^2 = \sigma_Z{}^2\{\Gamma^2\left(V/\theta\right) + \Lambda^2\left(n, L_n, 1/n\right)\} = \sigma_Z{}^2\Lambda_G{}^2\left(V, n, L, \theta\right) \quad (8)$$

The third term in Eq.(7) disappears because there is no correlation between the observed values and the LA to be estimated. For the same reason, the weight for each observation is equal weight, i.e., ν_i=1/n ($i=1, \ldots, n$). The function $\Lambda_G{}^2(V,n,L,\theta)$ is termed the general estimation variance function.

2.4.2 Local estimation of the mean of a LA

This is to estimate the mean of a LA of a geotechnical parameter value at a specified location in geotechnical design. It is important to recognize that the relative locations of the samples and the structure under design are explicitly taken into account in the local estimation. This estimation problem is known as Block Kriging in geostatistics (Jounel and Huijbregts 1978).

In the local estimation, the estimation variance function is given as same form as Eq.(7) as follows:

$$\sigma_V{}^2 = \sigma_Z{}^2\{\Gamma^2\left(V/\theta\right) + \Lambda^2\left(n, L_n, \nu\right) - 2\sum_{i=1}^{n} \nu_i \gamma_V\left(Z_V(x_{est}), Z(x_i)\right)\} = \sigma_Z{}^2\Lambda_L{}^2\left(x_{est}, V, n, x_{obs}, L, \theta\right) \quad (9)$$

The weights for the observations are determined by minimizing this estimation variance, which results solving a simultaneous equation given below:

$$\begin{aligned} &\sum\nolimits_{i=1}^{n} \nu_i \rho(|x_i - x_j|) - \lambda = \gamma_V\left(Z_V(x_{est}), Z(x_j)\right) \qquad (j = 1, \cdots, n) \\ &\sum\nolimits_{i=1}^{n} \nu_i = 1.0 \end{aligned} \quad (10)$$

where λ is the Lagrange multiplier and ρ the autocorrelation function. The function $\Lambda_L{}^2(x_{est}, V, n, x_{obs}, L, \theta)$ is termed the local estimation variance function.

Based on the GRASP theory presented above, the normative solutions for some typical geotechnical problems are solved. The extension of the above theory to multi dimensions is omitted due to the space restriction, but it is relatively easy because of the separable structure of the autocorrelation functions that has been assumed.

3 Normative Solutions for Sampling Sizes and Intervals

3.1 Sand layer thickness under a stretch of river dyke embankment

3.1.1 Problem statement

It is known there is a sand layer under river dyke embankment for several kilometers. This type of layer is very susceptible for piping and liquefaction, and the key parameter is the thickness of the layer. It is known that the horizontal autocorrelation distance of the layer thickness is 200(m) and the thickness is about 5 (m) with COV of 0.3 (i.e.,SD = 1.5(m)). How much should be the interval of soil layer thickness investigations if we want to ensure the probability of our thickness estimation would not underestimate the thickness by more than 1(m) for 95% confidence.

3.1.2 The normative solution

The 5% quantile value of the standard normal distribution is $z_{0.05}$=-1.65. This lead to the estimation SD at the middle point between the two adjacent investigation points, which is expected to have the largest estimation error, should be small than 1.0/1.65 = 0.61(m). This value, 0.61 (m), is 0.61/1.5 =0.40 of the population SD, 1.5 (m), of the sand layer thickness, which requires the local estimation variance function $\Lambda_L{}^2 < (0.40)^2$. We try to find a sampling interval that makes $\Lambda_L{}^2$ smaller than $(0.40)^2$ under given conditions.

The conditions given are θ= 200(m), V = 1(m), $x_{est}=x_{obs}$=100(m), n = 20, L = $n \times \Delta L$. We calculate $\Lambda_L{}^2$ by changing ΔL from 10(m) to 100(m) to find $\Lambda_L{}^2$ that is small than $(0.40)^2$:

$$\Lambda_L{}^2\left(x_{est}, V, n, x_{obs}, L, \theta\right) = \Lambda_L{}^2\left(100, 1.0, 20, 100, 20 \times \Delta L, 200\right) \leq (0.40)^2$$

The autocorrelation distance is given as 200 (m). The local averaging range is set to 1(m), which is small compare to θ, implying that there is no effect of V on the results. V=1(m) is also considered to be physically reasonable length. x_{est}, x_{obs} and n are set for the convenience of the calculation. n = 20 is set to make sure the stationary condition is satisfied. Also, a even number is required to make the estimated local average at the middle point between the two adjacent observation points. The results of the calculation are presented below.

ΔL (m)	10	20	30	40	50	60	70	80	90	100
Λ_L	0.153	0.22	0.271	0.313	0.35	0.384	0.414	0.442	0.469	0.493

The result shows that the investigation interval of less than 60 to 70 (m) is required to satisfy the specified accuracy. This is about one third of the autocorrelation distance.

3.2 Investigation of depth of the pile bearing layer under a bridge pier foundation

3.2.1 Problem statement

An engineer wants to order PC (prestressed concrete) piles for his bridge pear pile foundation whose footing size is 8(m) by 16(m). It is roughly known from the experience that the depth to the bearing layer is about 15(m). It can be assumed that COV of the depth is 0.1 (SD=1.5(m)), and the horizontal autocorrelation distance is 30(m). He wants to get an estimate of the depth at a specified point with in the footing whose 95% confidence interval is +1(m) of the local estimate by the GRASP theory. How many investigation points and how they should be configurated to obtain an estimate of the required reliability.

In order to make the problem more specific, some examples of the configuration and number of the investigation points are presented in Fig.1. They are indicated by number of investigations for each coordinate direction, e.g. (n_1,n_2) = (2,2). We will choose among these configurations that satisfies required statistical accuracy with the minimum number of the investigation points. Presented by black dots in the figures are the investigation points, whereas white squares are the points that may have the largest estimation error within the area of the foundation, which is conveniently called the critical point.

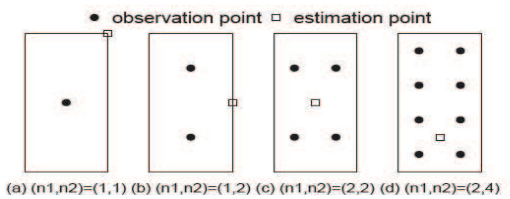

Figure 1. Some examples of the investigation points configuration and the critical estimation point.

3.2.2 The normative solution

The problem requires the estimated depth plus 1 (m) at any points within the area of the foundation should not be exceeded by 95%. The 95% quantile value, $z_{0.95}$, of the standard normal distribution is 1.64. Therefore, it is necessary to make the estimation SD to be 1 (m) / 1.64 = 0.61 (m), which is 0.61 (m) / 1.5 (m) =0.41 of the population SD. This implies the local estimation variance function at the points denoted by the white squares in Fig. 1 should be less than 0.41^2, *i.e.* $\Lambda_L^2 < (0.41)^2$.

Since the diameter of the pile is 1(m), the local averaging range is given by V_1=V_2=1(m) square area. We set the origin of the coordinate system at left bottom corner of the foundation, thus the foundation is 8(m) long in x_1-direction and 16(m) x_2-direction. The center of the investigation points is set at the center of the foundation, i.e. (x_{1obs}, x_{2obs}) = (4,8). The observation line lengths are L_1 = 8(m) and L_2 = 16(m) respectively. The estimation point should be the one which the estimation error is largest with in the foundation area. That point is given by (x_{1est}, x_{2est}) = (L_1/n_1, L_2/n_2), which is indicated by the shite square in Fig.1. This point is called the critical point.

The local estimation variance function, Λ_L^2, can be calculated, for example, as follows for (n_1,n_2) = (2,2) case (The extension of Λ_L^2 from 1-D to 2-D is not difficult under the assumption of the separability of the autocorrelation function, see Honjo and Otake (2018)):

$$\Lambda_L^2\left(x_{1est},x_{2est},V_1,V_2,n_1,n_2,x_{1obs},x_{2obs},L_1,L_2,\theta_1,\theta_2\right)=\Lambda_L^2\left(4,8,1,1,2,2,4,8,8,16,30,30\right)=(0.410)^2$$

The results of calculation for various (n_1,n_2) combinations are presented below, where the calculation has been done for both at the critical point and at the origin of the coordinate system, *i.e.* the left bottom corner of the foundation area.

From the calculated results, it is understood that $\Lambda_L^2 < (0.41)^2$ condition is satisfied at the critical point when (n_1,n_2) = (2,2) observation is done. This implies the minimum of 4 investigation points is necessary to fulfill the requirement. If one focuses on the origin (0,0), (n_1,n_2) = (2,4) observation is necessary.

(n_1,n_2)	(1,1)	(1,2)	(1,3)	(1,4)	(2,1)	(2,2)	(2,3)	(2,4)
Λ_L (at the critical)	0.798	0.574	0.543	0.527	0.695	0.410	0.357	0.327
Λ_L (at the origin)	0.798	0.664	0.610	0.581	0.737	0.580	0.512	0.474

4 Determination of a Characteristic Value of a Geotechnical Parameter

4.1 A definition of characteristic value of a geotechnical parameter

It is well known fact that there has been quite extensive discussion on how the characteristic value of a geotechnical parameter should be defined in the past 25 years. One of the triggers of this discussion was related to the development of Eurocode 7 Geotechnical design. The central definition Eurocode 7 finally adopted was "The characteristic value of a geotechnical parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state (CEN(2004), 2.4.5.1 (2))". In addition to this definition, following statement can be found in Eurocode 7.

The zone of ground governing the behaviour of a geotechnical structure at a limit state is usually much larger than a test sample or the zone of ground affected in an in situ test. Consequently the value of the governing parameter is often the mean of a range of values covering a large surface or volume of the ground. The characteristic value should be a cautious estimate of this mean value. (CEN(2004), 2.4.5.1 (7))

Actually, in the context of GRASP, this "The zone of ground governing the behaviour of a geotechnical structure at a limit state" is the local averaging range, LAR. Thus, the above definition of the characteristic value can be restated by more statistically quantified expression as flows.

The characteristic value of a geotechnical parameter shall be defined based on a local average of this parameter over a zone of ground governing the behavior of a geotechnical structure at a limit state. (This zone is termed LAR in GRASP.) Thus the characteristic value should be α% or (100- α)% quantile vale of this local average depending on the condition expected to be safe side for the structure.

We conveniently take α to be 5% in this study to solve an example.

4.2 The characteristic value of shear strength by FV test

4.1.1 Problem statement

Five sets of FV test data for an embankment construction at a site presented in Fig. 2 are given whose measurement locations are not given so as the relative position of the embankment and the investigation points. This data set is from one of six examples (i.e. EX2-5) set by ETC10 – Evaluation of Eurocode 7 - of ESSMGE (ETC10 2009).

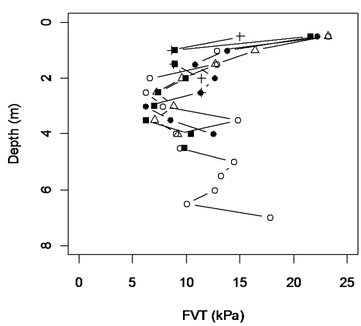

Figure 2. Undrained shear strength of the topsoil

It is known that stability of embankment is controlled by the first 4 m of the ground by the stability analysis of the embankment. Therefore the characteristic value for the first 4 m of the layer asked to be obtained.

4.1.2 The normative solution

First, the site characterization should be identified. It is observed the first measurements at 0.5(m) deep are quite different from the peat layer below 0.75(m). Thus, the ground was modeled by two layers (Table 1):

Table 1. The result of the special variation modeling of the undrained shear strength

Layer	Depth (m)	Trend component μ_{su} (kPa)	Random component σ_{su} (kPa)	Autocorrelation distance θ (m)	Number of samples
Top soil	0-0.75	21	3.44	0.25	5@1
Peat layer	0.75-4.0	$14.3\text{-}3.36x_3\text{+}0.545x_3^2$	2.36	0.25	5@5

Since the relative locations of the investigation points and the embankment is not given, the general estimation for LAR of 0.75(m) and that of 3.25(m) for each layer are obtained.

First, the characteristic value for the top soil is obtained. It is conservative assumption to postulate that the horizontal autocorrelation distances are much longer than the LAR, say the critical circle area, for the horizontal directions. Therefore, no variance reduction by local averaging takes place in the horizontal directions. It is also assumed that the investigation points are sufficiently separated to assume independence. Thus, the general estimation variance function can be calculated as follows:

$$\Lambda_G{}^2\left(V_1,V_2,V_3,n_1,n_2,n_3,L_1,L_2,L_3,\theta_1,\theta_2,\theta_3\right)=\Lambda_L{}^2(0,0,0.75,1,5,1,100,100,0.75,1,1,0.25)$$
$$=\Gamma^2\left(V_1/\theta_1\right)\Gamma^2\left(V_2/\theta_2\right)\Gamma^2\left(V_3/\theta_3\right)+\Lambda^2\left(n_1,L_{1n},1/n_1\right)\Lambda^2\left(n,L_{2n},1/n_2\right)\Lambda^2\left(n,L_{3n},1/n_3\right)$$
$$=\Gamma^2\left(0/1\right)\Gamma^2\left(0/1\right)\Gamma^2\left(0.75/0.25\right)+\Lambda^2\left(5,100,1/5\right)\Lambda^2\left(1,100,1\right)\Lambda^2\left(1,0.75,1\right)$$
$$=1.0\times1.0\times0.455+0.20\times1.0\times1.0=0.455+0.200=0.810^2$$

The first term of the above equation shows the variance reduction by taking the local average for 0.75(m), which is 0.455. The second term presents the statistical estimation error of the population mean, which in this case is 1/5=0.200. This is the error when we have 5 independent samples form the population.

Since the 5% quantile value of the standard normal distribution is -1.65, the characteristic value is given as

$$s_{uk}=\mu_{su}-1.65\times0.810\times\sigma_{su}=21.0-1.65\times0.810\times3.44=16.4\text{ (kPa)}$$

On the other hand, the characteristic value of the peat layer is obtained as a local average over depth 0.75 t0 4.00 (m). The genera estimation variance function is calculated as follows:

$$\Lambda_G{}^2\left(V_1,V_2,V_3,n_1,n_2,n_3,L_1,L_2,L_3,\theta_1,\theta_2,\theta_3\right)=\Lambda_G{}^2(0,0,2.5,1,5,5,100,100,2.5,1,1,0.25)$$
$$=\Gamma^2\left(V_1/\theta_1\right)\Gamma^2\left(V_2/\theta_2\right)\Gamma^2\left(V_3/\theta_3\right)+\Lambda^2\left(n_1,L_{1n},1/n_1\right)\Lambda^2\left(n,L_{2n},1/n_2\right)\Lambda^2\left(n,L_{3n},1/n_3\right)$$
$$=\Gamma^2\left(0/1\right)\Gamma^2\left(0/1\right)\Gamma^2\left(2.25/0.25\right)+\Lambda^2\left(5,100,1/5\right)\Lambda^2\left(1,100,1\right)\Lambda^2\left(5,2.25,1/5\right)$$
$$=1.0\times1.0\times0.180+0.20\times1.0\times025=0.180+0.050=0.479^2$$

Therefore, the characteristic value of the peat layer is given as

$$s_{uk}=\mu_{su}-1.65\times0.810\times\sigma_{su}=(14.8-3.36x_3+0.545x_3{}^2)-1.65\times0.479\times2.36=12.9-3.36x_3+0.545x_3{}^2\text{ (kPa)}$$

5 Conclusion

It is shown through this paper, statistically normative solutions for typical geotechnical problems can be obtained by applying GRASP theory. However, it is important to notify that the correlation structure of the random field need to be known in advance.

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