Developments in LRFD Calibration for Internal Limit States for Mechanically Stabilized Earth Walls in North America

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Abstract: The paper describes advances in reliability-based load and resistance factor design (LRFD) calibration of simple linear limit state functions used for internal limit states design of mechanically stabilized earth (MSE) walls (i.e., tensile rupture and pullout of the reinforcing elements). The method has been applied to MSE walls reinforced with steel reinforcing elements and modern polymeric soil reinforcement materials. The general approach considers the accuracy (or bias) of the underlying deterministic models that are used by engineers to compute nominal values of load and resistance terms in the limit state equations for the two limit states mentioned above. The calibration method has the flexibility to include the concept of level of understanding used in Canadian practice by linking it quantitatively to project-dependent uncertainty in the choice of nominal values at the time of design.

Keywords: MSE walls; internal stability; LRFD; calibration.

1 Introduction

Load and resistance factor design (LRFD) is adopted in North American design codes for geotechnical foundation structures and retaining walls. In the US the AASHTO (2017) "LRFD Bridge Design Specifications" is used, while in Canada the corresponding document is the Canadian Highway Bridge Design Code (CHBDC). (CSA 2019). Both documents undergo periodic revisions. A notable difference between the codes is the notion of "level of understanding" for project-specific conditions that is adopted in the Canadian code to select a suitable resistance factor (Fenton et al. 2016).

This paper describes a methodology to compute resistance factors for simple linear limit states and demonstrates the approach using the example of internal stability design of mechanically stabilized earth (MSE) walls (i.e., tensile rupture and pullout of the reinforcing elements). The general approach considers the accuracy (or bias) of the underlying deterministic models that are used by engineers to compute nominal values of load and resistance terms for the two limit states mentioned, *plus* the concept of level of understanding used in Canadian practice. A convenient closed-form solution proposed by Bathurst et al. (2017) is used to demonstrate the general approach. The paper will be of interest to code developers in the US and Canada when future editions of the two national codes are prepared, and for regulators in other countries where the LRFD approach for MSE walls is being contemplated.

2 Internal Limit States and General Approach

The two internal limit states that are the most important for internal stability design of reinforced mechanically stabilized earth (MSE) walls are reinforcement rupture and pullout. These limit states can be referenced to Figure 1 for the case of steel reinforced MSE walls. For the case of a single load term the general factored limit state function of interest is:

$$g = \frac{\varphi R_n}{\gamma_Q Q_n} - 1 \tag{1}$$

Here, R_n and Q_n are the nominal resistance and nominal load computed at time of design using closed-form solutions that are found in North American design codes, and parameters φ and γ_Q are resistance and load factor, respectively. The probability of failure for limit states of this form [$P_f = P(g < 0)$] is best computed using nominal values that have been transformed to true (measured) values for resistance (R_m) and load (Q_m) using bias values. Bias (λ) is the ratio of measured to predicted value; hence, bias values for resistance and load terms are:

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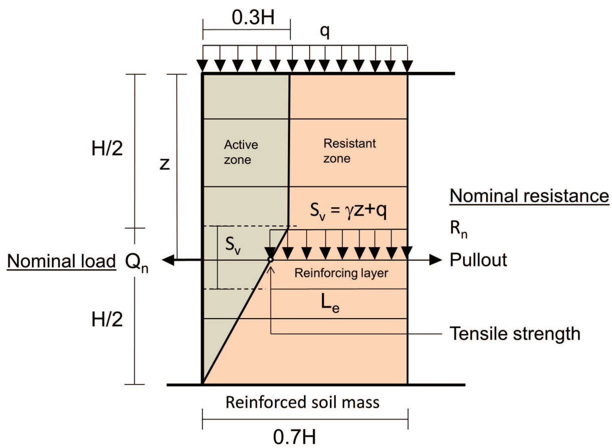

Figure 1. MSE wall components and internal stability limit states for steel reinforced MSE walls.

$$\lambda_R = R_m / R_n \tag{2a}$$

$$\lambda_Q = Q_m / Q_n \tag{2b}$$

Fortunately, measured R_m and Q_m data are available from databases collected by the writers and co-workers. Recent examples are described in papers by Allen and Bathurst (2015, 2018), Allen et al. (2019), Bathurst et al. (2019), Miyata and Bathurst (2019) and Miyata et al. (2018a, b).

For a prescribed load factor (γ_Q), Monte Carlo simulation can be used to find a value of resistance factor that satisfies a target probability of failure, or equivalently, reliability index (β). An alternative approach to find φ for simple limit state functions described by Eq. (1), is the closed-form solution proposed by Bathurst et al. (2017):

$$\varphi = \gamma_Q \frac{(\mu_{\lambda R}/\mu_{\lambda Q})\sqrt{(1+COV_{Qn}^2)(1+COV_{\lambda Q}^2)/(1+COV_{Rn}^2)(1+COV_{\lambda R}^2)}}{\exp\left\{\beta\sqrt{\ln\left[\frac{(1+COV_{Qn}^2)(1+COV_{\lambda Q}^2)(1+COV_{Rn}^2)(1+COV_{\lambda R}^2)(1+\rho_R COV_{Rn}COV_{\lambda R})^2(1+\rho_Q COV_{Qn}COV_{\lambda Q})^2}{(1+\rho_n COV_{Rn}COV_{Qn})^2}\right]}\right\}} \tag{3}$$

Here, μ_{Rn} and COV_{Rn}, μ_{Qn} and COV_{Qn}, $\mu_{\lambda R}$ and $COV_{\lambda R}$, and $\mu_{\lambda Q}$ and $COV_{\lambda Q}$ are mean and coefficient of variation (COV) values for R_n, Q_n, λ_R and λ_Q, respectively. Parameters ρ_R and ρ_Q are *bias dependencies* which are the Pearson correlation coefficients between R_n and λ_R and between Q_n and λ_Q, respectively. When ρ_R and ρ_Q are non-zero, the *on average* accuracies ($\mu_{\lambda R}$ and $\mu_{\lambda Q}$) of the underlying load and resistance models that appear in the limit state design equation (i.e., Eq. 1) will vary with the magnitude of the nominal values (R_n and Q_n). Parameter ρ_n is the correlation coefficient between R_n and Q_n and is non-zero when equations for R_n and Q_n share one or more input parameters that are random variables. Hereafter, ρ_n is referred to as *nominal correlation.*

If there is no variability in the accuracy of the load and resistance models that appear in a limit state design equation and there is no correlation between nominal values R_n and Q_n, then bias values disappear and Eq. (3) simplifies to:

$$\varphi = \gamma_Q \frac{\sqrt{(1+COV_{Qn}^2)/(1+COV_{Rn}^2)}}{\exp\left\{\beta\sqrt{\ln\left[(1+COV_{Qn}^2)(1+COV_{Rn}^2)\right]}\right\}} \tag{4}$$

Treating nominal resistance and load values (R_n and Q_n) as random variables is not unreasonable since reinforcement load models for internal stability design of MSE walls are functions of soil friction angle and unit weight which have statistical uncertainty. The same is true for the calculation of ultimate pullout capacity for most pullout limit state equations that appear in design codes.

Nevertheless, in earlier closed-form formulations that appear in the related literature to compute resistance factor φ, only the bias of the load and resistance models was considered (e.g., Withiam et al. 2001; Baecher and

Christian 2003; Allen et al. 2005; Bathurst et al. 2008) and bias dependencies were ignored. For these conditions, Eq. (3) devolves to:

$$\varphi = \gamma_Q \frac{(\mu_{\lambda R}/\mu_{\lambda Q})\sqrt{(1+COV_{\lambda Q}^2)/(1+COV_{\lambda R}^2)}}{\exp\left\{\beta\sqrt{\ln\left[(1+COV_{\lambda Q}^2)(1+COV_{\lambda R}^2)\right]}\right\}} \tag{5}$$

In fact, uncertainty in both nominal values computed at time of design, *and* model bias are expected. Furthermore, correlations between nominal and bias values are possible. For all of these reasons, Eq. (3) is attractive. Another advantage of Eq. (3) is that the COV of nominal load and resistance terms can be linked to *level of understanding* of project-specific conditions as discussed later in the paper. It can be noted that Eq. (3) requires that all nominal and bias variables are lognormally distributed. This assumption has proven to be a reasonable practical assumption for the internal rupture and pullout limit states that are the focus of this paper.

Solutions for resistance factor φ can be found using conventional Monte Carlo (MC) simulation techniques which can also account for correlations between nominal and bias variables. However, Eq. (3) has the advantage that it gives the same outcomes but is easily implemented in a spreadsheet which is convenient for sensitivity analyses. Furthermore, the influence of changes in parameter variables is better understood by examining their location in Eq. (3), and φ outcomes present smoothly when plotted against other parameters.

3 Example Bias Statistics

Example load bias values expressed as cumulative distribution function (CDF) plots are shown in Figure 2 for a relatively poor load model (LM1) and a relatively good load model (LM2). Load model (LM1) can be found in US and Canadian design codes (AASHTO 2017; CSA 2019) and is called the Simplified Method. Load Model (LM2) is a modified load model that can be used for both steel and polymeric reinforced MSE walls and is called the Simplified Stiffness Method (Allen et al. 2015). The data for both models can be seen to deviate, at least visually, from a straight line in standard normal variable – log of bias value space. However, it is the data in the top of the distribution that control the probability of failure because these data correspond to the largest under-estimates of true (measured) load values. Hence, estimating the mean and COV of bias values by fitting over the upper part of the distributions is recommended. In this plot, fitting the CDF approximation to all data for each model does provide a reasonable statistical characterization of the data in the upper tail. However, for other datasets the difference in the distribution of the tail compared to the rest of the data may not be as subtle, requiring a fit to tail to better estimate probability of failure.

A relatively good load model has mean bias close to 1 (or just less than 1) and a small spread in bias values. A relatively poor model is the reverse. In the examples here, the poor load model predicts maximum tensile loads under operational conditions that are 2.3 times the measured values *on average*, which is judged to be excessively conservative for design. Another feature of a relatively good load model is that model accuracy (i.e., bias) does not vary with the magnitude of the predicted value. This is a desirable feature of any model regardless whether design is carried out in conventional allowable (working) stress (factor of safety approach) or probabilistic design frameworks. Using this criterion, the Pearson's correlation coefficient ($\rho_{\lambda Q}$) for load bias and computed tensile load was -0.41 for the poor model (LM1), and zero at a level of significance of 5% for the good model (LM2). Similar CDF plots for two different pullout models are shown in Figure 3. The approximation to both CDF plots using the mean and COV of bias values computed from the entire database of 318 values can be seen to reasonably capture the measured data including the lower end of both distributions, at least visually. It is the distribution of resistance bias values at the lower tail that strongly influences probability of failure and thus the magnitude of the calculated resistance factor. Pullout model PM2 is a better model than model PM1 based on the mean bias value which is close to but slightly greater than 1, and the smaller spread in bias values. Bias dependency with computed pullout capacity is not present with pullout model PM2, but is significant for the poorer pullout model PM1 ($\rho_{\lambda R}$ = -0.46).

4 Selection of Load Factor

In practice, the load factor γ_Q in the factored limit state design equation (Eq. 3) is prescribed in LRFD codes used by the designer. For MSE walls this load factor does not conform to a consistent target level of load exceedance as recommended by Allen et al. (2005). In structural engineering LRFD calibration for bridges, a 2% load exceedance criterion has been used in practice. The cumulative frequency plot in Figure 4 shows that this criterion corresponds to a load bias value of λ_Q = 1.75 which is equivalent to a load factor γ_Q = 1.75. In the AASHTO (2017) code the specified (dead) load factor is 1.35 and in the CSA (2019) code the value is 1.25. Clearly, the actual load exceedance values are different between codes and both are greater than the target value that has been recommended for structural engineering LRFD calibration. The important lesson here is that

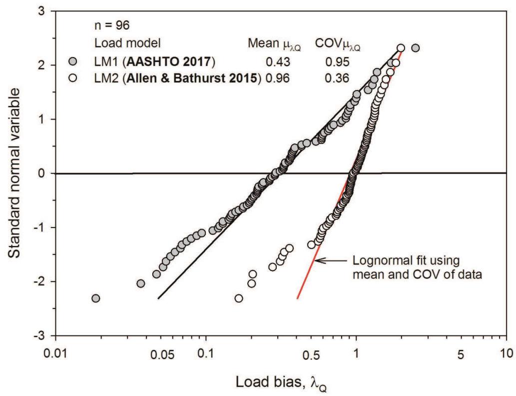

Figure 2. Example CDF plots for maximum tensile load in geogrid reinforced MSE walls (data from Bathurst et al. 2019).

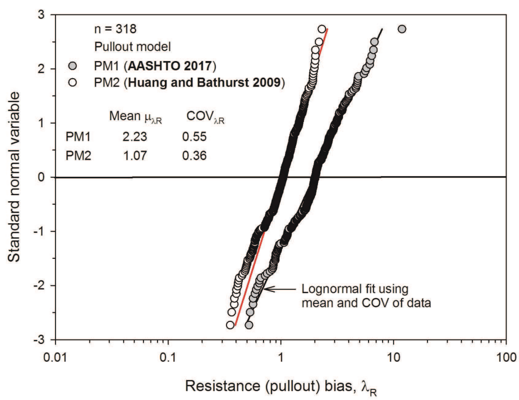

Figure 3. Example CDF plots for pullout bias values for MSE walls (data from Huang and Bathurst 2009).

different load exceedance criteria and different load models will lead to different load factors. Ideally, a different load factor should be selected for each load model to satisfy the same probability of load exceedance regardless of the load model and MSE wall type selected by the designer (e.g., steel or polymeric reinforced MSE wall). However, most important is that the load and resistance factor *combination* achieve the target reliability index for the limit state.

5 Correlation of Nominal Load and Resistance Values

The last parenthetical term in the denominator of Eq. (3) contains the nominal correlation coefficient (ρ_n). A strategy to compute this value is to generate random values of R_n and Q_n using the same values of γ and ϕ sampled from the same distributions in each MC realization (Lin and Bathurst 2018). An example outcome is shown in Figure 5. The value of ρ_n is simply the r value from conventional linear regression analysis.

6 Example Results

Figure 6 shows example LRFD calibration outcomes for the resistance factor in the geogrid pullout limit state using Eq. (3). In these calculations the target reliability index value is β = 2.33 which corresponds to a probability of failure of P_f = 1%. This P_f value may appear high but is reasonable for highly strength-redundant MSE wall systems (Allen et al. 2005). This is because if one reinforcement layer fails the other layers can compensate. The plots show that as the COV of the nominal values decreases from COV = 0.3 to 0.1 (i.e., confidence in calculation of nominal values related to level of understanding increases), the resistance factor becomes greater. Values of COV = 0.1, 0.2 and 0.3 are consistent with Canadian LRFD foundation design practice that is to reward the designer with a larger resistance factor for doing more foundation investigation, more materials testing, adopting technologies that are familiar, proven for similar site conditions and better

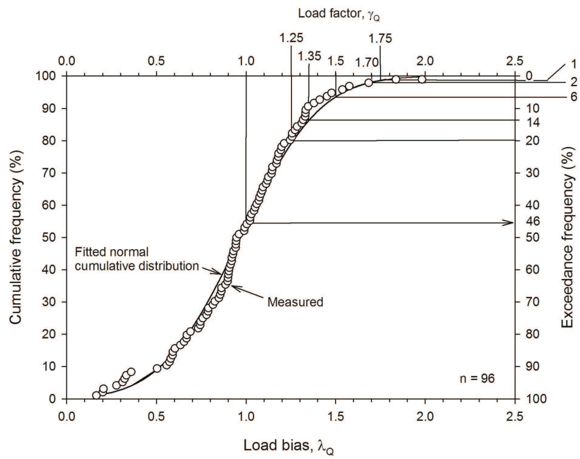

Figure 4. Cumulative frequency distribution of load bias values using load model LM2 (data from Bathurst et al. 2019).

matched to meet structure functions (Fenton et al. 2016). A value of COV = 0 reflects current US practice where the notion of level of understanding to select the resistance factor does not appear, at least not in a formal manner as in Canadian practice. Using values of COV → 0 for the nominal load and resistance values attenuates the influence of bias values and correlation coefficients in Eq. (3) and is responsible for the trend of decreasing resistance factors for COV < 0.10 in Figure 6.

7 Conclusions

This paper provides a brief review of recent efforts by the writers and co-workers to develop methodologies and tools to carry out LRFD calibration for simple limit state equations used in foundation engineering with emphasis on internal stability limit states for MSE walls. The approach explicitly considers the accuracy of the underlying models for load and resistance terms in limit state design equations, uncertainty in the calculation of load and resistance values at time of design, and possible correlations between input parameters. The general approach can be used for other simple soil-structure interaction problems such as external sliding of gravity walls, soil nails and other similar soil stabilization techniques. The general approach can be carried out using conventional Monte Carlo simulation techniques. However, where applicable, the closed-form solution reported in this paper (Eq. 3) offers a convenient method to carry out LRFD calibration and sensitivity analyses using simple spreadsheets.

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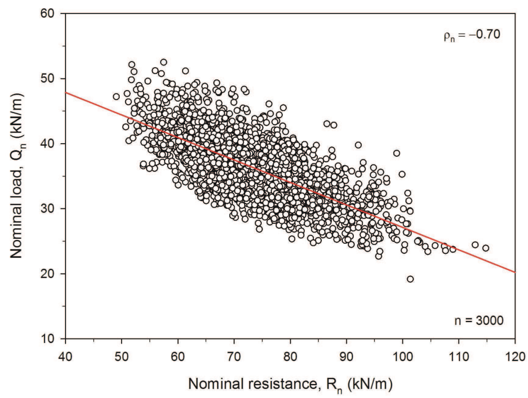

Figure 5. Nominal load versus nominal pullout capacity.

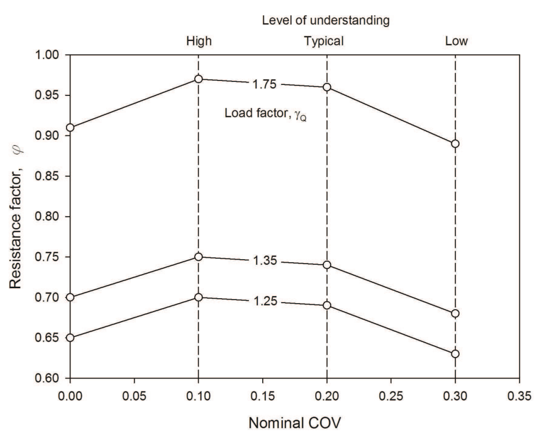

Figure 6. Example computed resistance factor φ for geogrid pullout limit state versus nominal values of COV = COV_{Rn} = COV_{Qn} for target reliability index β = 2.33 and different load factors (load model LM2 and pullout model PM1).