Challenges in Determining Rock Mass Properties for Reliability-Based Design

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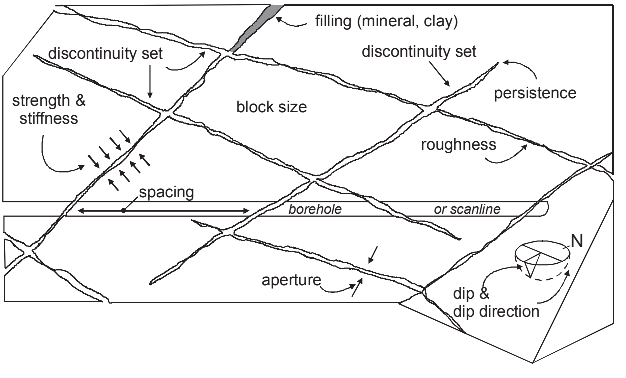
Abstract: In the context of RBD for rock engineering, the principal source of the structural resistance is provided by the rock mass, and in accordance with the principles of RBD this needs to be characterised statistically, on the basis of aleatory variability. Only data that have been obtained by objective quantitative measurement can be used for this, and so rock mass properties currently can only be determined by a synthesis of component properties. Variability appears to exist in all component geometrical and mechanical properties, and the similarity within and between rock types is so slight that reference values are unsuitable, indicating that variability will need to be determined project-by-project. A major challenge in this work is the perennial problem of limited data, but Bayesian approaches may ameliorate this. Modern investigation techniques allow large quantities of discontinuity geometry data to be obtained, and our understanding of discontinuity geometry is advanced; together these bode well for application of RBD. The use of numerical modelling to combine component properties and obtain large scale rock mass properties is proving efficacious, and has confirmed stress and scale dependent variability. Significant variability at the engineering scale has been revealed, which renders large-scale in situ testing unfeasible for characterising rock mass variability. Overall, significant challenges remain in determining the behaviour of rock masses for use in RBD. Appropriate techniques that will allow determination of the necessary properties have largely been developed, but much work remains to be done before routine and simplified design methods appear.

Keywords: Rock engineering; rock mass models; ordinal data; epistemic; aleatory; reliability-based design.

1 Introduction

Most rock masses within which rock engineering takes place contain many mechanical discontinuities of different geological origins; a generally accepted way to conceptualising such masses is to consider them as an assemblage of intact rock blocks dissected by discontinuities (Fig. 1). Both the intact rock and the discontinuities are characterised by various mechanical and geometrical properties, and the engineering behaviour of a rock mass results from the interaction of these. Two approaches are in widespread use for determining the engineering behaviour of a discontinuous rock mass: analytic, in which the properties of the rock mass as a whole are in some way directly or indirectly assessed, and synthetic, in which constituent properties are combined to produce a comprehensive model of a rock mass. Both of these are addressed here.

(a) actual rock mass



(b) conceptual model with key discontinuity properties

Figure 1. Rock masses as an assemblage of intact rock blocks dissected by discontinuities (after Gambino et al. 2019).

The concepts on which reliability-based design (RBD) are well known, and a plot of the elementary R-S case shows these succinctly. The *R-S* plot of Fig. 2, which uses the nomenclature of EN1990 (CEN 2002), illustrates that RBD requires definition of the governing limiting state in terms of actions and resistances, and the establishment of a target probability of unacceptable performance. The plot also shows that both actions and

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resistances are considered to possess aleatory variability. In rock engineering, R is primarily a function of the properties of the rock mass and, regardless of whether we use either an analytical or synthetic approach to determine the rock mass properties, for RBD the challenge is to determine the distribution of R.

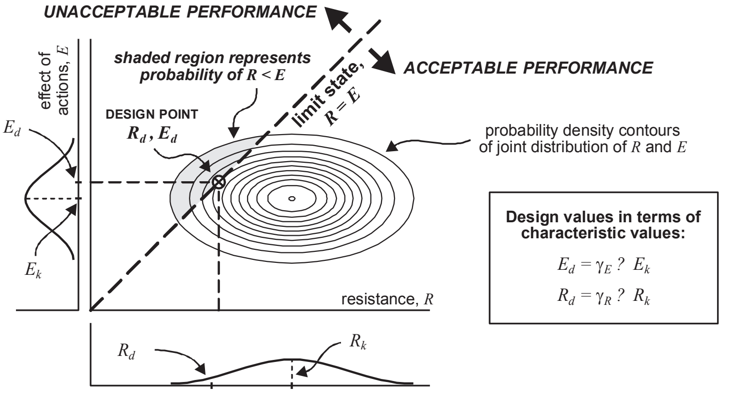


Figure 2. Fundamental R-S plot (after Lemaire 2006).

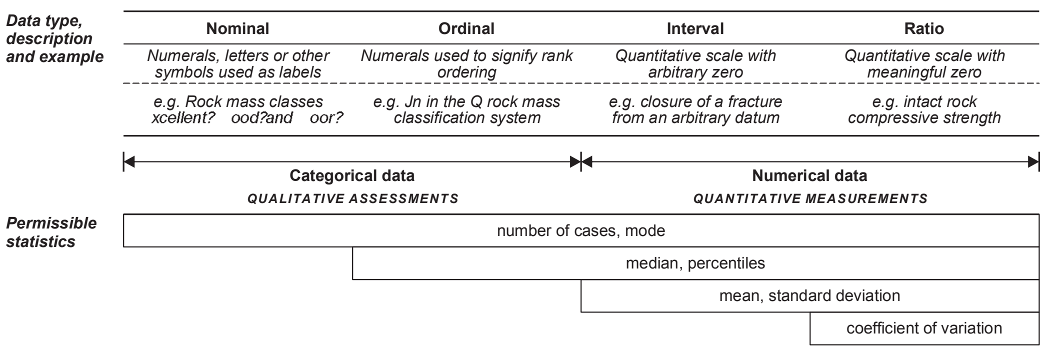


Figure 3. Data types and permissible statistics (after Stevens 1946).

Figure 2 clearly shows that RBD is based on probability, and thus the resistance R needs to be characterized in probabilistic terms. For rock engineering, this has profound implications relating to data type. A relation between data type and permissible statistics was famously proposed by Stevens (1946), and Fig. 3 casts Stevens' concepts into a rock mechanics and rock engineering context. A comparison of Figs. 2 and 3 shows that RBD requires so-called ratio data, i.e. quantitative data referred to a meaningful zero. However, anybody familiar with rock engineering practice will see from Fig. 3 that nominal and ordinal data are commonplace, and will also then conclude that such data are not incompatible with RBD.

That nominal and ordinal data cannot be used to determine means, standard deviations and coefficients of variation is not widely appreciated in rock engineering, and in fact many practitioners, simply following the example of numerous publications, calculate these properties and erroneously undertake probabilistic analyses (say, calculate the mean rock mass class). Literature in other fields clearly shows the error in this. For example, Stine (1989) noted "*The measurement context within which statistics are calculated is of the utmost importance. Performing sophisticated analyses that are appropriate for one scale of measurement on data that reflect a less structured scale yields nonsense. The nonsense might be interpreted, but it will be nonsense nonetheless*". Similarly, when discussing ordinal data, Kampen and Swyngedouw (2000) noted "*Measurement presupposes an objective standard, in the sense that if measurements… are to be compared, [they] have to be obtained either by using the same measuring instrument or at least by carefully calibrated ones. The usual case in social research, however, is that respondents individually answer questions about, for example, the degree to which they agree with one or another statement – and it is hardly tenable that these respondents can be viewed as identically calibrated instruments. In practice, uncalibrated measurement will lead to ambiguous results*". This statement is directly relevant to rock engineering: replacing the words "social research" with "application of rock mass classification schemes" produces a statement that exactly represents the extant conditions in rock engineering.

The issue of ordinal data is of particular significance for analytic approaches to determining rock mass properties, as discussed below.

2 Analytic Approach

As exemplified by the application of rock mass classification schemes, the analytic approach combines assessments of rock mass characteristics (e.g. degree of fracturing and discontinuity condition) to obtain a rating value for the rock mass, and then applies empirical formulae to obtain a value of a parameter of interest, say elastic modulus. All current schemes incorporate ordinal data to a greater or lesser extent and so, as Fig. 3 shows, are unsuitable for calculation of the distribution of R required for RBD. Furthermore, the assessments of these ordinal values are subjective and highly dependent on the experience and bias of the assessor (see the comment of Kampen and Swyngedouw (2000) quoted above). Thus, these assessments display epistemic uncertainty, rather than allowing characterization of aleatory variability as required for RBD. The matter of epistemic versus aleatory attributes has been addressed in the RBD literature (e.g., Der Kiureghian and Ditlevsen 2009), but not in the sense of subjective assessments of ordinal data. Although developments to fully quantify rock mass classification schemes to remove epistemic uncertainty have been reported (e.g., Bedi et al. 2018), we must conclude that at present the analytic approach is not suitable for determining rock mass properties for RBD. Nevertheless, use of these schemes is widespread in rock engineering design, and so quantification of them must be seen as imperative in order to support the growing application of RBD.

3 Synthetic Approach

The synthetic approach to modelling the mechanical behaviour of a fractured rock mass involves combining models of the various contributory phenomena either analytically or numerically to produce a model of the mass. Table 1 indicates those characteristics of rock masses that these models generally include. This approach dates back to at least the 1980s, with models ranging from the simple (e.g. effect of the presence of a single discontinuity on the strength of a rock mass) to the complex (e.g. including the effect of discontinuity spacing and stiffness on mass elastic moduli). The literature on synthetic rock mass models is very extensive, but in the context of RBD it is only the aspect of variability in these characteristics that needs to be reviewed.

Table 1. Principal rock mass constituents included in synthetic approaches.

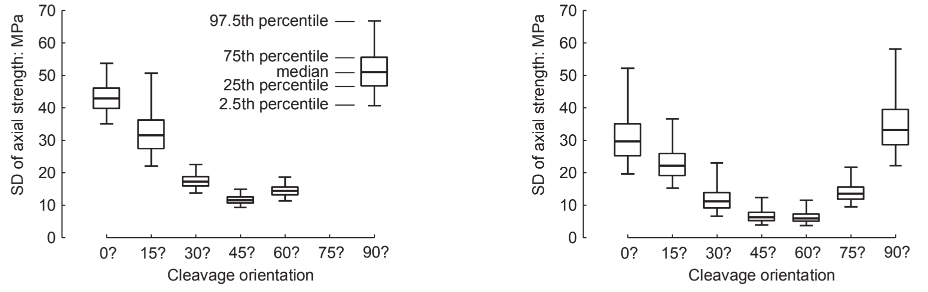
Rock mass						
Intact rock		Discontinuities				
		Geometry			Mechanical characteristics	
Strength	Stiffness	Number of sets	Set orientation	Extent & persistence	Strength	Stiffness

3.1 Intact rock strength and stiffness

The study of rock strength variability can be traced back to the work of Yamaguchi (1970), who showed that the variability of both unconfined compressive strength and direct tensile strength of intact rock is well fitted by a normal distribution. It is only recently that this work has been extended to characterise variability in strength at non-zero confining pressures (Langford and Diederichs 2015; Bozorgzadeh and Harrison 2015). It is now known that variability shows so little similarity within and between rock types that reference values cannot be used for this, and it will need to be determined specifically in testing campaigns (Bozorgzadeh and Harrison 2019).

A major challenge in determining variability of rock strength is the perennial problem of limited data, but it seems that Bayesian data analysis may be able to circumvent this (Bozorgzadeh and Harrison 2019), and a particularly promising approach is to robustly combine properties for different rock types using Bayesian hierarchical modelling (Bozorgzadeh et al. 2019). Even so, further challenges appear in the case of anisotropic rocks: as Fig. 4 shows, there is evidence that heteroscedasticity with respect to loading direction may exist (Bozorgzadeh and Harrison 2014). An area yet to be explored is strength variability in polyaxial stress conditions: although it is widely known that the magnitude of the intermediate principal stress has a significant effect on the strength of rock (e.g. You 2009), there is not yet consensus on an appropriate strength criterion and there seems to have been no investigation of the variability of strength under different stress regimes. As polyaxial stress states are commonplace in underground rock engineering, this lack of knowledge needs to be addressed.

Variability of stiffness within a particular rock type seems not to have received the same attention as has strength. Figure 5 shows recent results investigating variability of tangent modulus, and apart from indicating that this property is approximately normally distributed, two aspects are noteworthy: firstly, the variability is significant – the coefficient of variability is greater than 20% for the metagranite; and secondly, the two rocks display markedly different degrees of variability. This suggests that the earlier comment regarding the lack of similarity of strength variability within and between rock types holds true for stiffness, and it too will need to be determined as part of specific rock testing campaign. These results are for a single component of the elastic compliance matrix; variability of the entire matrix seems not to have been studied.



(a) Delabole slate (b) Angers schist

Figure 4. Heteroscedasticity of anisotropic rock strength (after Bozorgzadeh and Harrison 2014).

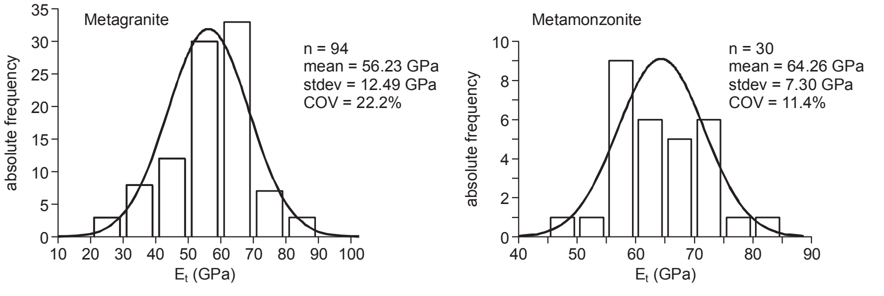


Figure 5. Variability of intact rock tangent modulus (after Pepe et al. 2017).

3.2 Discontinuity geometry

The effect that discontinuities have on the mechanical behaviour of rock masses began to be recognised in the early 1970s. Early work on the statistics of discontinuity occurrence led to the observation that discontinuity spacing x may generally be well represented by the simple exponential model $f(x)=\lambda\exp(-\lambda x)$, in which λ is the discontinuity frequency (Priest and Hudson 1976). This is a single parameter model with the mean equal to the standard deviation, and mean spacing equal to the reciprocal of frequency; in terms of RBD, quantifying the degree of fracturing also directly quantifies its variability. The exponential model is not universally applicable, as there are many geological conditions in which it does not apply well (e.g. columnar jointing in basalt, bedding spacing in rhythmic sedimentary sequences). However, the simplicity of the model ensures it is widely employed, particularly in the generation of discrete fracture network models (see Section 3.4).

Instability mechanisms in discontinuous rock masses often concern displacement of individual blocks of rock along discontinuities, and discontinuity orientation is a critically important parameter in this. The literature on characterisation of variability in discontinuity orientation is extensive, and new approaches continue to be developed and proposed. However, in the context of RBD it is appropriate to examine statistical models of orientation variability. Many distributions exist (see Fisher et al. 1987), but their applicability is questionable: for example, one review (Einstein and Baecher 1983) noted that for 18 out of 22 data sets none of the popular models of uniform, Fisher, elliptical, Bingham or normal passed the χ^2-test. However, the Fisher distribution remains popular: perhaps because its two-parameters of a mean vector and a dispersion measure are both readily computed from measured data.

Discontinuities in rock masses are of limited spatial extent, and this significantly affects the stability of rock structures. Recent work using UAVs and photogrammetry has been used to assess the statistics of discontinuity impersistence on rock slope stability. Fig. 6 shows how impersistence leads to rock bridges, and Fig. 7 presents distributions of discontinuity extent, given as a diameter of a discontinuity with no bridges, as determined using cameras of different focal lengths. This latter figure clearly shows the variability associated with discontinuity extent, but also indicates sampling effects may be present. These results are an important first step to quantifying variability in discontinuity extent, but signify that further work is required to fully understand this property of rock masses.

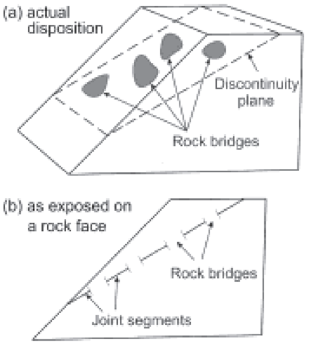


Figure 6. Intact rock bridges due to discontinuity impersistence (after Shang et al. 2018).

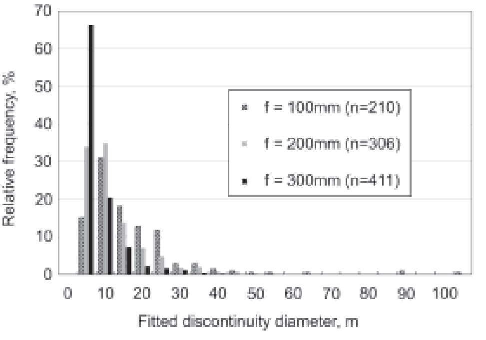


Figure 7. Distribution of discontinuity extent (after Tuckey and Stead 2016).

The past few years has seen rapid developments in obtaining large amounts of high quality discontinuity geometry data (whether from borehole core, borehole wall imagery, or outcrops). Laser scanning and digital stereophotogrammetry (see Fig. 8) systems are commercially available, and data processing techniques continue to increase in capability (e.g., Chen et al. 2017). Robust statistical characterisation of discontinuity geometry is now possible, and random sampling from large actual data sets may soon be more meaningful than attempting to fit and use specific distributions.

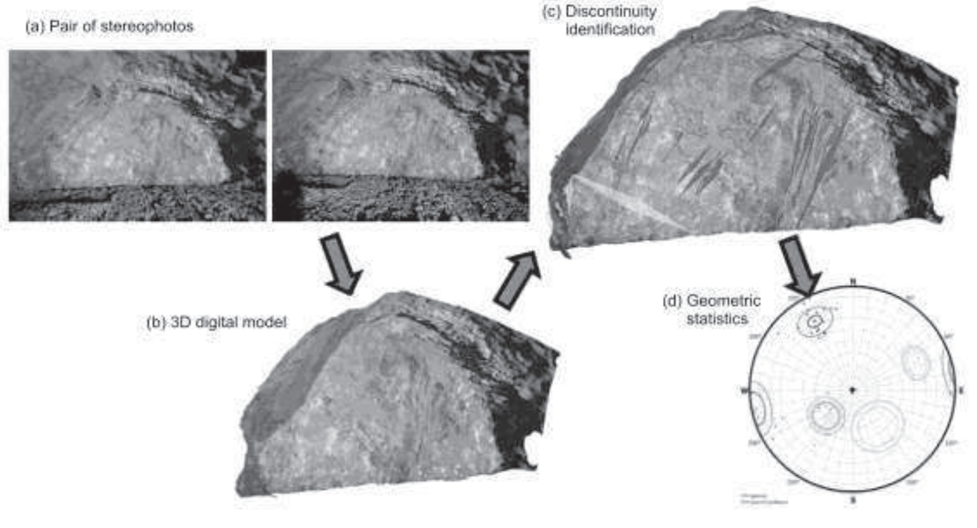


Figure 8. Digital photogrammetry for rock mass geometry data collection (after Gaich and Pischinger 2016).

3.3 Discontinuity strength and stiffness

The shear strength of discontinuities is governed by many parameters, particularly surface roughness, normal stress and intact rock strength, and despite much research a comprehensive strength criterion remains elusive. Characterisation of discontinuity roughness for incorporation in such criteria has a long history, dating from at least the early 1970s, with one of the most popular schemes being joint roughness coefficient (JRC). This can be determined objectively via tilt tests, but most assessments are undertaken by subjective visual comparison to exemplar profiles. This effectively leads to two different coefficients: in the context of Fig. 3 the first is of ratio type (identified here as JRC_r), and the second is ordinal (JRC_o). JRC_o displays significant epistemic uncertainty due to operator bias, as Fig. 9 shows, but perhaps more importantly, the ordinal nature of JRC_o precludes evaluation of variability in terms of COV. Unfortunately, many calculations of this are both reported in the literature and undertaken in practice yet, as we saw above, "*Performing… analyses that are appropriate for one scale of measurement on data [of] a less structured scale yields nonsense*" Stine (1989).

Determining and analysing JRC_r via direct measurement is an active research area, but the matter of quantifying variability of JRC_r on a given discontinuity surface has attracted little attention. One example where a roughness parameter in some way analogous to JRC_r was assessed for variability is shown in Fig. 10. Here, roughness was measured at different orientations, with twelve parallel sample lines being used at each orientation. These results clearly indicate variability, although this was not evaluated as part of this work and the matter seems not to have been pursued by others. We are therefore currently unable to say anything about the likely magnitude of roughness variability, its affect on the variability of discontinuity strength and stiffness, and how it should be accounted for in RBD.

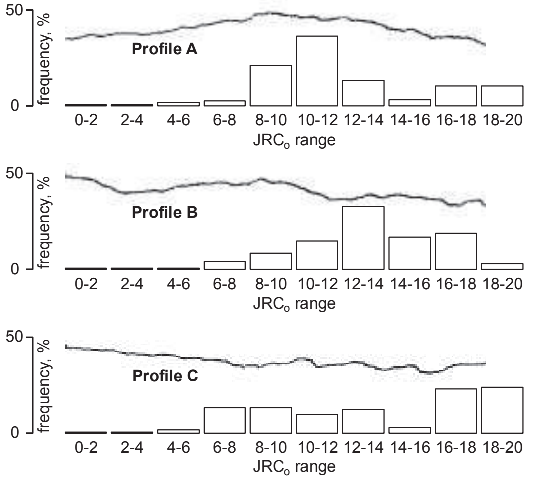


Figure 9. Epistemic uncertainty in visual assessments of discontinuity surface linear profiles (after Beer et al. 2002).

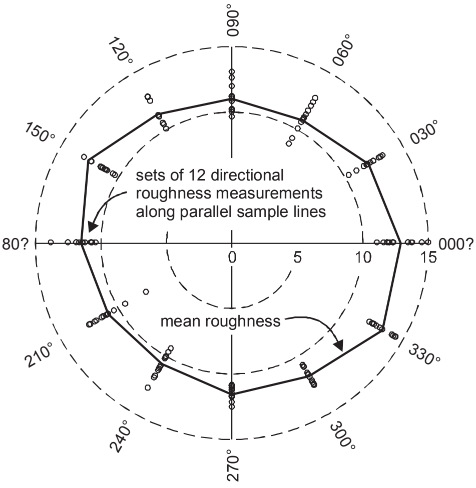


Figure 10. Directional variability of discontinuity surface.roughness measurements (after Tatone and Grasselli 2010).

There also seems to be few reports on variability in shear strength on any one discontinuity. Hencher and Richards (2015) present some results for both natural and saw-cut discontinuities in limestone, and these suggest significant variability in peak strength. As Figure 11 indicates, at any given value of normal stress there is such great variability in the data that it is difficult to discern any meaningful range. For the natural and split surfaces this will partly be due to the inevitable between-specimen variation in surface roughness, but even for the controlled saw-cut surfaces the variability is significant. Given the paucity of work on the matter, it seems that quantifying variability in shear strength urgently needs attention.

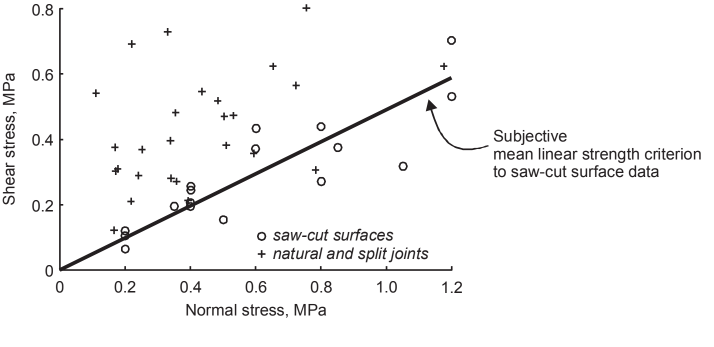
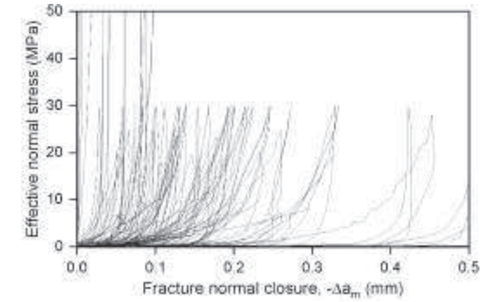


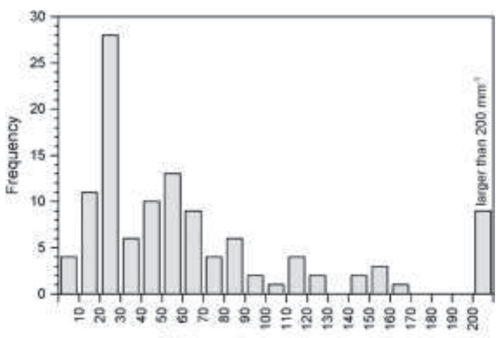
Figure 11. Variability in discontinuity shear strength (after Hencher and Richards 2015).

The stiffness of discontinuities is often decomposed into normal and shear components, and empirical relations are widely used to assess these (Bandis et al. 1983). These relations employ surface roughness, a parameter that as noted above is often assessed subjectively, and in this case they suffer from the problems associated with ordinal data (Fig. 3). Variability in stiffness values was explored in an investigation related to normal stiffness (Zangerl et al. 2008). These authors compiled and analysed more one hundred normal stress-normal displacement results for rocks of a single type. As Fig. 12a shows, stiffness is both highly stress dependent and variable. The variability is presented in Fig. 12b as a histogram of 'stiffness characteristic', which is the gradient of the stiffness vs. normal stress curve. These results show significant variability, despite them being limited to granitic rocks. It is likely that the variability will be at least as significant – and different – for other rock types. There appears to have been no investigation of this matter, and so there are no predictive models for variability in discontinuity stiffness.

The magnitude of between-rock type variability is hinted at in Fig. 13. This represents a compilation of published rock mass stiffness data (presented as the ratio mass stiffness to intact rock stiffness) in terms of rock mass degree of fracturing, and displays an extraordinary degree of variability: at the higher RQDs (i.e. rock masses containing few fractures) the stiffness ratio E_m/E_r ranges from about 0.1 to 1.0 – almost the entire possible range. Such a large scatter suggests it is not meaningful to propose a mean value and COV for stiffness in terms of RQD for use in RBD analyses, and that values and variability of stiffness will need to be determined specifically using means other than a general empirical relation.



(a) variability and nonlinearity of fracture deformation



(b) histogram of stiffness characteristic

Figure 12. Variability in normal stiffness of discontinuities in granitic rocks (from Zangerl et al. 2008).

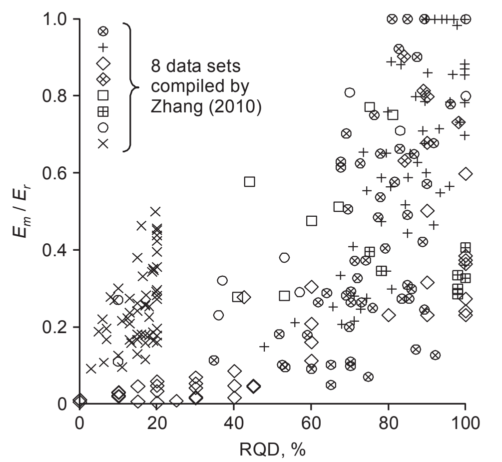


Figure 13. Variability of rock mass stiffness in terms of degree of fracturing (after Zhang 2010).

3.4 Rock mass behaviour: numerical modelling

Finally, all these components need to be combined into a model of the rock mass, and although work on developing analytical models continues (e.g., Yang et al. 2019) numerical modelling is generally used for this. These models are inherently stochastic: commencing with distributions of discontinuity spacing, extent, location and orientation, a two- or three-dimensional network of discontinuities is generated, and to these discontinuities the various mechanical attributes are added. The workflow for generating such discrete fracture network (DFN) models is well established (Fig. 14).

Figures 15 and 16 present results for rock mass elastic modulus obtained from distinct element modelling of DFNs, and indicate that variability of modulus is clearly both scale and stress dependent: Fig. 15 shows that variability reduces to some irreducible value as rock mass volume increases, and Fig. 16 shows that as stress level increases variability reduces and mean modulus approaches an asymptote (cf. the gradient of the curves in Fig. 12a). In both cases the variability of modulus remains non-zero, suggesting that this rock mass property must always be regarded as a stochastic, although how similar the variability is between rock masses of different conditions has not yet been explored. It must also be recognised that these results concern only the elastic modulus acting in one direction: the variability in all components of the elastic compliance matrix remains to be investigated. It should be noted that these DFNS are stochastic in geometry only, but on the basis of the survey presented above it is clear that variability in mechanical properties should also be added. How the addition of these other factors will change the magnitude, scale dependency and non-linearity of the variability remains to be discovered.

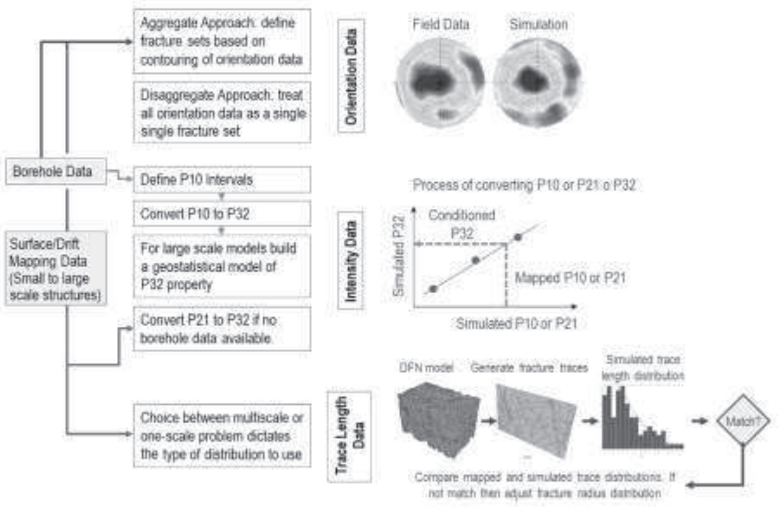
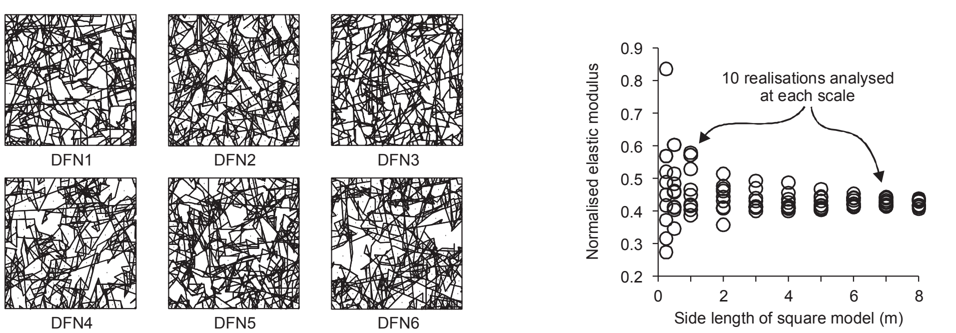
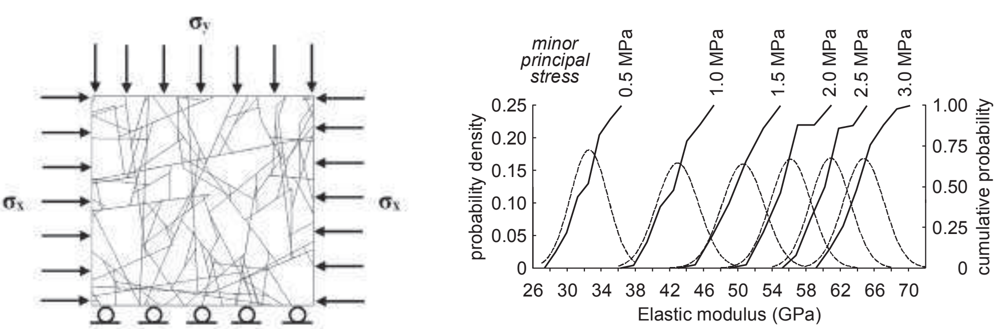


Figure 14. Workflow for Discrete Fracture Network generation (after Miyoshi et al. 2018).



(a) Example DFN models (b) Distributions of horizontal elastic modulus

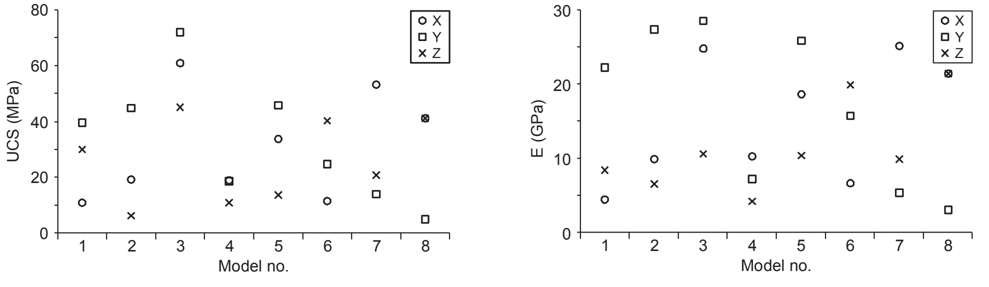
Figure 15. Scale effect in rock mass elastic modulus (after Min and Jing 2003).



(a) Typical numerical model (b) Distributions of elastic modulus based on 50 model realisations

Figure 16. Variability in elastic modulus of a fractured rock mass under biaxial compression (after Noorian-Bidgoli and Jing 2015).

DFNs are also being used as input to synthetic rock mass models (SRMs), a numerical approach that uses large numbers of small spherical particles to represent the rock mass (Mas Ivars et al. 2011). Figure 17 shows the elastic modulus and unconfined compressive strength in each of the *x*, *y* and *z* directions for eight SRM models of a rock mass of size 10m × 10m × 20m. Variability is clearly present, but strikingly there is no discernible trend to the various components of strength and stiffness: once again, we must conclude that the variability of these properties will need to be determined case-by-case. Of even greater significance is that this considerable variability occurs on a volume of rock that is of engineering scale; this indicates that large scale *in situ* testing to determine variability is utterly unfeasible.



(a) Unconfined compressive strength

(b) Elastic modulus

Figure 17. Variability in rock mass strength and stiffness for 10m × 10m × 20m (*x*, *y*, *z*) blocks (after Mas Ivars et al. 2011).

4 Summary

In the context of RBD for rock engineering, the principal source of the structural resistance is provided by the rock mass, and in accordance with the principles of RBD this needs to be characterised statistically, on the basis of aleatory variability. The properties of data types show that aleatory variability can only be characterised using objective, quantitative measurements. Subjective, qualitative assessments, which display epistemic uncertainty, are therefore not suited to RBD. This poses a significant challenge to the widely-used existing analytic methods for determining rock mass properties that are largely based on the application of rock mass classification schemes. It appears that rock mass classification schemes will only be appropriate for use in RBD if they can be developed so that all of their constituents can be determined quantitatively.

The synthetic approach considers rock mass properties to be a combination of component factors associated with both intact rock and discontinuities. Generally, these factors are available in a quantitative form and thus appropriate for RBD. Results for the strength and stiffness of intact rock demonstrate aleatory variability, but that this variability has so little similarity within and between rock types that reference values are unsuitable and it will need to be determined case-by-case. Some aspects of variability, such as variability of polyaxial rock strength, anisotropy of strength and the elastic compliance matrix, seem not to have been explored; these need investigating. A major challenge in this work is the perennial problem of limited data, but it seems that Bayesian approaches may ameliorate this.

Both geometrical and mechanical properties of discontinuities are known to be aleatory. Modern investigation techniques allow large quantities of discontinuity geometry data to be obtained, and our understanding of discontinuity geometry is advanced; together these bode well for application of RBD. With regard to characterising the variability of individual discontinuity properties, much more work is required with regards to roughness, stiffness and strength.

Combining these properties into synthetic models of rock masses using numerical modelling is well established although at an early stage. Results confirm that properties display both stress and scale dependent variability, although how this changes with rock mass conditions remains to be explored. It may be that such numerical modelling will be required project-by-project.

Overall, significant challenges remain in determining the behaviour of rock masses for use in RBD. Appropriate techniques that will allow us to determine the necessary properties have largely been developed, but there is clearly much work yet to be done before routine and simplified design methods appear.

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