Probabilistic Evaluation of Earthquake-induced Liquefaction Potential for Large Region Site Based on Two-dimensional GIS Technique

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ABSTRACT: The empirical equation of cyclic resistance ratio CRR of saturated sands with different probability levels is utilized to evaluate the earthquake-induced sand liquefaction for 2D (two-dimensional) large region site. Based on GIS technique, the evaluation results of earthquake-induced liquefaction potential at the observation boreholes can be adopted as the "elevation" in digital elevation model, therefore the identification of liquefaction locality for 2D large region site can also be evaluated by Kriging interpolation method. The research in this paper shows that: (1) The Kriging interpolation is an effect method for evaluating the latent distribution of the site liquefaction potential beyond the observation boreholes. Thus, it is a preferable way to evaluate the possible liquefaction range for 2D large site. (2) Based on ArcGIS software, the Kriging method embedded in the "Spatial Analyst" extension module can be utilized to evaluate the liquefaction potential of the unidentified points by the evaluated results of the reconnaissance boreholes. Therefore, the 2D distribution map of the liquefaction potential for the large region site can be generated.

1 INTRODUCTION

As the saturated sandy soil liquefaction is one of the most important earthquake-induced geologic disasters. It is necessary to evaluate the earthquake-induced site liquefaction potential to prevent and mitigate the seismic site disasters. The general methods of liquefaction potential evaluation, which based on the test results of SPT (standard penetration test), CPT (cone penetration test) and Vs (shear wave velocity) etc., can just evaluate the actual boreholes. With these evaluated results, the liquefaction potential of the stratums that among the actual boreholes can also be estimated cursorily. However, as the number of reconnaissance borehole in geotechnical investigation is usually limited and the stratum characteristics among these boreholes are unknown. It is uncertain to evaluate the liquefaction extent of the whole engineering site according to the evaluated results of these actual boreholes. Thus, the problem that how to evaluate the earthquake-induced liquefaction potential of the whole engineering site based on the limited boreholes is put forward.

To solve the above problem, two aspects are considered in this paper.

(1) The empirical equation of cyclic resistance ratio *CRR* of saturated sandy soil with different probability levels is utilized to evaluate the earthquake-induced sandy soil liquefaction of the large engineering site region.

(2) Kriging interpolation method is adopted to evaluate the liquefaction potential of the unidentified stratum based on the evaluated results of these observation boreholes. Also, the 2D plot plan of the site liquefaction potential can be given supported by GIS according to the Kriging interpolation results.

2 PROBABILITY EVALUATION MODEL OF THE EARTHQUAKE-INDUCED SITE LIQUEFACTION POTENTIAL

As we all know, the deterministic relationship (H.B.Seed, 1984) between SPT number and the equivalent cyclic stress ratio *CSR* of the saturated sandy soil layer caused by earthquake ground motion, or the cyclic resistance ratio *CRR* has been widely accepted and applied. Whereas, this method can just estimate whether the site soil will be liquefaction case or not but the liquefaction probability can not be given.

Based on 344 liquefaction site data of 25 strong earthquakes, the limit state function CRR_{cri} (Critical Cyclic Resistance Ratio) is constructed to estimate the sandy soil liquefaction based on radial basis function (*RBF*) neural network method by Guo-xing Chen et al. (2006). Therefore, the critical cyclic resistance ratio CRR_{cri} can be calculated by the following formula:

$$CRR_{cri} = 0.0002 N_1^2 + 0.005 N_1 + 0.03 \quad (1)$$

Where, N_1 is the corrected blow count of standard penetration test.

When the equivalent cyclic stress ratio *CSR* of sandy soil layer caused by the earthquake ground motion is greater than CRR_{cri} determined in formula (1), the saturated sandy soil layer will be liquefaction case, otherwise non-liquefied case.

Thus, the probability function to estimate the sandy soil liquefaction can also be set up according to the relationship $F_s = CRR_{cri}/CSR$, F_s is the cyclic resistance safety factor and PL is liquefaction probability.

$$P_L = 1/(1 + F_s^{4.297}) \quad (2)$$

Combine formula (1) and (2), sandy soil liquefaction resistance stress curve under the different probability can be shown as the follows:

$$CRR = [P_L/(1 - P_L)]^{0.233} \cdot (0.0002 N_1^2 + 0.005 N_1 + 0.03) \quad (3)$$

As the Table.1 shows, three grades are classified to identify the liquefaction potential of saturated sandy soil according to the different liquefaction probability level. Therefore, the probabilistic estimation of seismic site liquefaction can be done as the follows: Firstly, one acceptable liquefaction probability level must be confirmed according to the importance of the engineering project. Secondly, the estimation criteria of sandy soil liquefaction (*CRR*) with different probability can be calculated by formula (3). Thus, cyclic resistance safety factor F_s can also be obtained by the formula $F_s = CRR_{cri}/CSR$ when *CRR* is given. Thirdly, based on the calculated F_s, the liquefaction potential under the different liquefaction probability can be given expediently according to the Table 1.

Table 1. Standard for probability evaluation

Liquefaction probability level P_L	Sand liquefaction safety factor F_s	Liquefaction potential evaluation
$0.00 \leqq P_L < 0.30$	$F_s \geqq 1.2$	non-liquefaction
$0.30 \leqq P_L < 0.70$	$0.81 < F_s < 1.2$	possible liquefaction
$0.70 \leqq P_L < 1.00$	$F_s \leqq 0.81$	liquefaction

3 2D GIS EVALUATION OF LIQUEFACTION POTENTIAL BASED ON PROBABILITY MODEL

DTM (Digital Terrain Model) is the digital expression of terrain surface information. Also, it is the digital description of spatial location characteristics and terrain attribute. DTM can describe the spatial terrain information by ordinal array of numerical value. When the terrain attribute is adopted as “elevation” in DTM, it is called DEM (Digital Elevation Model). DEM is utilized to describe the terrain surface in numeric form, and its construction is necessary to the GIS analysis. Based on DEM idea, the probability evaluation results are adopted as the "elevation" in DEM in this paper. Sustained by GIS software-ArcGIS, the 2D distributing plan of earthquake-induced liquefaction potential can be obtained by Kriging interpolation method.

3.1 DEM Applied in 2D Numeric Modeling of Liquefaction Potential

Before the spatial Kriging interpolation based on the evaluated results of the original boreholes, two steps must be completed. Firstly, the site region is divided into regular grids. Secondly, value in every grid can be obtained by Kriging interpolation according to the original data point.

As DEM is one mathematic (or numeric) model to describe the terrain surface, the elevation information of every surface point can be given when DEM is built. GRID model is often utilized to simulate the terrain surface information by dividing the surface into a mount of cells, and only the cell origin, size, numbers and value are stored in GRID.

Therefore, DEM data of GRID structure is adopt in the 2D GIS evaluation of liquefaction potential based on probability model, and every cell in GRID has the same width and height. GRID area is confirmed by the geography coordinate of top left corner, cell size, number of rows and columns.

3.2 Principle of Kriging Method Applied in the Interpolation of Liquefaction Potential

As one of the primary contents in geostatistics, the core technique of Kriging interpolation method based on the regional variable theory in random field is to represent the spatial variation function with the model of variogram function, then the weight coefficients of every sampling position are decided under the condition of unbiased estimation and least estimate variance, and all the sampling positions and the identified weights are assembled linearly in order to obtain the internal estimated value in arbitrary spatial point or block.

3.2.1 Variogram Model

Soil property presents to be special variant. However, there is still certain relativity between the different soil points, viz. the vertical or horizontal relativity and variability. Hence, some soil engineering property, such as ground bearing capacity, the stability of earth slope, soil liquefaction potential and the earth pressure etc., can be generally attributed as regional variable $z(x)$.

Supposing that the covariance of two spatial point (x and $x+h$) is merely depends on h,

$$\begin{aligned} D\{[(z(x)-z(x+h)]\} &= E\{[(z(x)-z(x+h)]^2\}-E^2\{[(z(x)-z(x+h)]\} \\ &= E\{[(z(x)-z(x+h)]^2\} = 2\gamma(h) \end{aligned} \quad (4)$$

Where $\gamma(h)$ is variogram function, then $\gamma(h)$ can be obtained as follows:

$$\gamma^*(h) = \frac{1}{2n}\sum_{i=1}^{n}[z(x_i)-z(x_i+h)]^2 \quad (5)$$

The chart of $\gamma(h)$ is called variogram chart shown in Fig.1. The experimental variogram curve in Fig.1 is generated by variogram $\gamma^*(h)$ which is calculated from the sampling data points. Each sampling point $\gamma^*(h)$ is called experimental variogram. Using linear, Gauss function, power function, exponential function and etc. to fit them, the gained model is called variogram model and the fitting curve is the variogram model fitted by $\gamma^*(h)$, which can be used in the Kriging method directly to calculate the value's weight function. Fig1.shows that: when h is great, the curve extends in level direction, and the level part is called sill. That is to say, data points in this area have no relativity. The delay area between the low $\gamma(h)$ value and sill is called range. Within this range, nearer the distance is, closer the property will be. If the fitting model doesn't get through the origin, there will be a intercept C_0 in the $\gamma(h)$ axis and the intercept is called nugget.

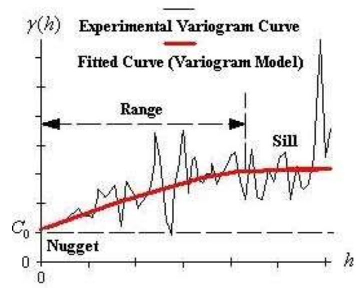

Fig.1. Typical variogram features

3.2.2 Kriging Interpolation Equations

$Z(x_i)$ (i=1,2,…,n)is assumed as a set of scattered liquefaction potential value of sampling points, and these values are second-order stationary. In order to estimate the true liquefaction potential value at arbitrary point x_0 in the identified region, the best appraisal value $Z^*(x_0)$ is assumed as the linear combination of $Z(x_i)$:

$$z^*(x_0)=\sum_{i=1}^{n}\lambda_i z(x_i) \quad (6)$$

Where, λ_i is the weight coefficient; $Z^*(x_0)$ is the appraisal value at the position x_0; i is the serial number of the borehole.

The principle of Kriging interpolation is to make sure this appraisal value $Z^*(x_0)$ is unbiased estimator, and the estimated variance is less than the value of any other linear combination. Therefore, the ideal how to obtain this best value $Z^*(x_0)$ is just to resolve the extreme value problem under the constraint conditions. In conclusion, it can be shown as the follows:

$$\begin{cases}\sum_{i=1}^{n}\lambda_i\gamma(x_i,x_j)+\mu=\gamma(x_j,x_0) \quad (i,j=1,2,\cdots,n)\\ \sum_{i=1}^{n}\lambda_i=1\end{cases} \quad \text{Viz.} \begin{vmatrix}\gamma_{11} & \cdots & \gamma_{1n} & 1\\ \cdots & \cdots & \cdots & \cdots\\ \gamma_{n1} & \cdots & \gamma_{nn} & 1\\ 1 & \cdots & 1 & 0\end{vmatrix}\begin{vmatrix}\lambda_1\\ \cdots\\ \lambda_n\\ \mu\end{vmatrix}=\begin{vmatrix}\gamma_{01}\\ \cdots\\ \gamma_{0n}\\ 1\end{vmatrix} \quad (7)$$

Where, $\gamma_{i,j}=\gamma(x_i,\ x_j)$ is the variogram function value between xi and x_j; μ is the lagrange multiplier.

When the weight coefficient λ_i and Lagrange multiplier μ are obtained after resolving the Kriging equations (7), the best appraisal value $Z^*(x_0)$ can also be estimated according to the formula (6).

3.2.3 Flow Analysis of 2D Numeric Modeling

Fig.2 shows the follow of building the 2D numeric modeling of liquefaction potential and the realization steps are as the follows:

(1) The probability evaluation result of liquefaction potential is adopted as the "Elevation" in DEM, and the GRID model is generated by GIS;

(2) The isoline of liquefaction potential is given by Kriging interpolation

(3) Based on the GRID model, the logic inquiring and algebraic calculation can be operated on multi-cell data;

(4) According to the standard for probability evaluation of liquefaction, several limiting data are chosen accordingly. Thus, the whole site can be divided into several districts with the different liquefaction potential grade.

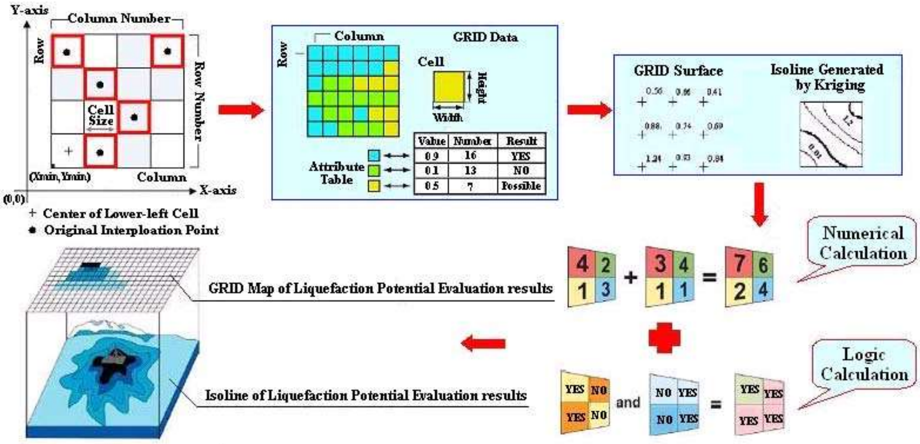

Fig.2. The flow chart of the digital model for the site liquefaction potential

3.2.4 Calculation Example of Kriging Interploation

One basic calculation example is introduced to illustrate the Kriging method applied in the probability evaluation of earthquake-induced liquefaction potential. Table.2 shows the F_s values of the boreholes (x_1, x_2, x_3, x_4 is the original borehole and x_0 is the interpolation borehole). In Kriging interpolation, the most simple model—linear model is adopted here as the variogram model, and the variogram function is $\gamma(h)=4\,h$ when the relative distance h is less than 50 meters.

For example, the distance between x_1 and x_2 is $h=\sqrt{(85.79-120.11)^2+(160.14-161.34)^2}=34.34$ as shown in Table.3. The variogram value between x_1 and x_2 is $\gamma(x_1, x_2) = 4\,h = 4\times34.34 = 137.36$. Thus, the $\gamma(x_i, x_j)$ ($i, j = 0, 1, 2, 3, 4$) value of every two points x_i can be calculated, and the obtained $\gamma(x_i, x_j)$ can also be inputted into the formula (7) to get λ_i. The calculated λ_i is 0.1976, 0.1570, 0.3168 and 0.3286.

Table 2. Value of probability evaluation at the observation points

Borehole	X (m)	Y (m)	F_s	P_L
x_1	85.79	160.14	1.00	0.5
x_2	120.11	161.34	0.65	0.86
x_3	83.13	189.34	1.30	0.24
x_4	114.78	191.29	0.85	0.67
x_0	100.00	180.00	Unknown	Unknown

Table 3. Distance (m) $h_{i,j}$ and semivariance(m^2) $\gamma(x_i, x_j)$ between every two points

	x_1	x_2	x_3	x_4	x_0
x_1	0	34.34	19.32	42.55	24.40
x_2	137.36	0	46.38	30.42	27.43
x_3	77.28	185.52	0	31.71	19.23
x_4	170.20	121.68	126.84	0	18.60
x_0	97.6	109.72	76.92	74.40	0

Remark: In table.3, the above triangle shows the distance $h_{i,j}$ between every two points. Bottom triangle shows the variogram value $\gamma(x_i, x_j)$ between every two points

As seen from Table.2 and 3, the relation between λ_i and $h_{0,j}$ (distance between x_i(i=1, 2, 3, 4) and x_0) is non-linear direct proportion. This non-linear relation reflects the influence extent of the surrounding points act on the interpolation point x_0 impersonally. Finally, F_s value at x_0 can be calculated as $Z(x_0)=0.1976\times1.00+0.1570\times0.65+0.3168\times0.94+0.3286\times0.85=0.88$ according to formula (6) viz. P_L=0.634. According to Table.1 the probability evaluation result of earthquake-induced liquefaction potential at x_0 is "Possible Liquefaction".

3.3 2D Evaluation of Liquefaction Potential Based on ArcGIS

The extension module "Spatial Analyst" embedded in ArcGIS, which is developed by ESRI (Environmental Systems Research Institute, U.S.A), is often utilized to extract and analyze DEM. Also, the visual terrain model can be obtained. For its strong ability of spatial modeling and analysis function, the data collection based on GRID can be built and the corresponding data inquiring, analysis, display can also be realized.

Based on this extension module, the probability evaluation results of liquefaction potential (F_s or P_L) is adopted as the "Elevation" in DEM. The 2D distributing plot of site liquefaction potential can be given by the spatial interpolation when Krigng method, which is attached in the extension module, is utilized.

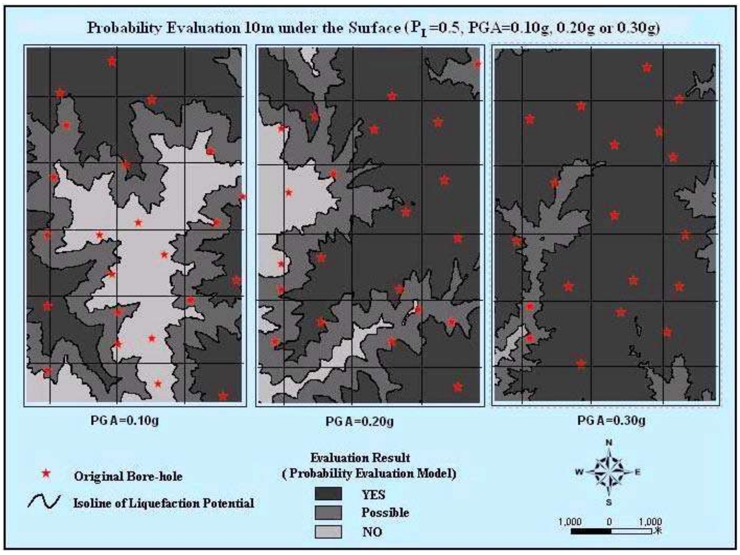

Fig.3. The map of site soil liquefaction potential at depth 10m under different PGA (P_L =0.5)

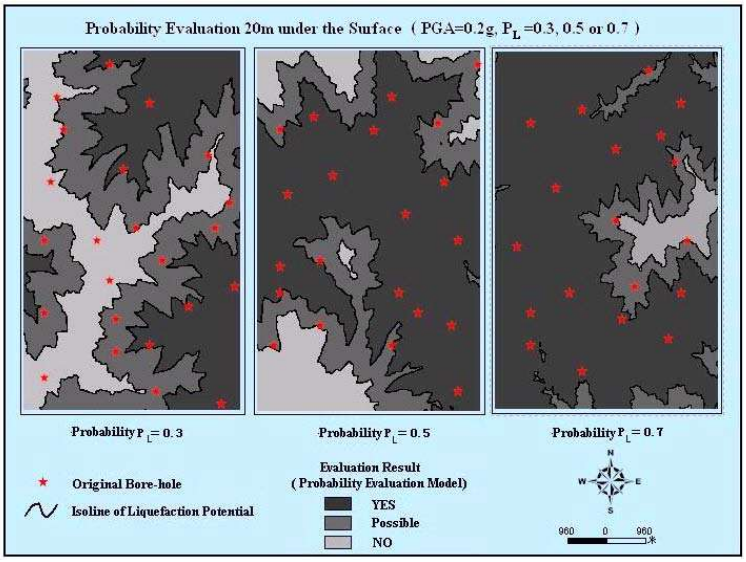

Fig.4. The map of site soil liquefaction potential at depth 20m under different P_L (PGA =0.20g)

In this paper, the cell size of research site is adopted as 50m×50m, viz.250m^2, and the data form is ESRI GRID. Based on the scattered data of original boreholes, the whole site can be interpolated into GRID by Kriging method, and the variogram model is chosen as exponential model. Thus, there are 1259 cells in coordinate *X*, 1548 cells in coordinate *Y* and 1 in coordinate *Z*. According to the standard for probability evaluation in Table.1, the site liquefaction potential can be classified into three grades, viz. liquefaction, possible liquefaction and non-liquefaction. Fig.3,4 are the 2D plot plan of site soil liquefaction potential at depth 10m and 20m under certain probability or PGA.

Iwasaki et al. (1978) proposed a quantitative value of site liquefaction potential index I_{LE} to classify the liquefaction potential grade at a site. The bigger I_{LE} value is, the higher the liquefaction risk grade will be.

As shown in formula (8), ,the soil liquefaction potential index I_{LE} value of every actual borehole at site can be calculated according to Chinese Code for Seismic Design of Buildings, and the site liquefaction risk grade can also be classified synthetically according to the Table 4.

$$I_{LE} = \sum_{i=1}^{n} (1 - N_i / N_{cri}) d_i w_i \quad (8)$$

Where, *n* is the blow count of SPT (standard penetration test) of every actual bore-hole within the evaluation depth. N_i and N_{cri} are the actual and critical SPT value in *i* point respectively. When $N_i > N_{cri}$, N_i is adopted as N_{cri}. d_i(m) is the soil layer depth in *i* point. w_i(m^{-1}) is the influence weight function.

Table 4. Site liquefaction grade classification

Site soil liquefaction risk grade	Slight	Medium	High
I_{LE} value for depth 15*m*	$0< I_{LE} \le 5$	$5< I_{LE} \le 15$	$I_{LE} >15$
I_{LE} value for depth 20*m*	$0< I_{LE} \le 6$	$6< I_{LE} \le 18$	$I_{LE} >18$

Sustained by ArcGIS, every soil layer can be seen as the independent map (the soil is assumed as the homosphere and level stratification in this paper). The map attribute table contains several fields as the follows: N_i, N_{cri}, d_i, w_i and I_i. Thus, the value of $(1-N_i/N_{cri})d_iw_i$ in every soil layer can be calculated. Based on the calculated value, every independent map is expressed in GRID and one new GRID map can also be generated by “GRID Calculator” shown in Fig.5.

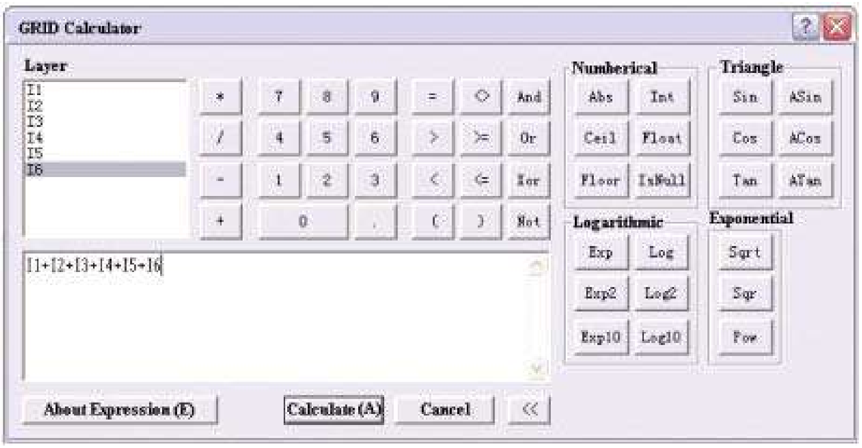

Fig.5. Raster calculator of site soil liquefaction potential by ArcGIS

This new GRID map expresses the sum value of $\Sigma(1-N_i/N_{cri})d_iw_i$, which is composed of all the $(1-N_i/N_{cri})d_iw_i$ value in every GRID cell. That is the I_{LE} value at every cell point. Thereby, the 2D risk evaluation map of liquefaction potential expressed in I_{LE} is shown as Fig.6.

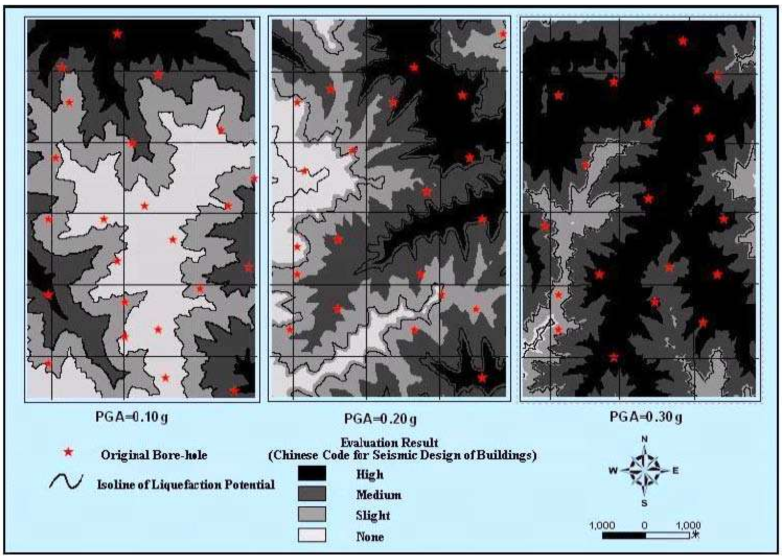

Fig.6. The synthetic map of the soil liquefaction potential I_{LE} for large site

4 CONCLUSIONS

In this paper, the probability evaluation results of earthquake-induced site liquefaction potential at the random soil point can be interpolated and analyzed spatially based on Kriging method and GIS technique. Therefore, some instructive conclusions are obtained as the follows:

(1) When he probability evaluation results are adopted as the “elevation” in DEM, the Kriging interpolation method can be utilized to evaluate the liquefaction potential of the unidentified points by the evaluated results of the observation boreholes. Thus, it is a preferable way to estimate the possible liquefaction risk for engineering site and it is worth to study further in the spatial visibility aspect of liquefaction potential.

(2) The “Spatial Analyst” module embedded in ArcGIS can be adopted to realize the Kriging interpolation and generate the isoline of the liquefaction potential. Also, the advanced GRID analysis can be done to give the 2D plot plan of earthquake-induced site liquefaction potential. Thereby, GIS is a preferable method to realize the spatial visibility of earthquake-induced site liquefaction potential.

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