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Quantitative safety prediction of pile foundations

A. Kudzys
KTU Institute of Architecture and Construction, Kaunas, Lithuania

L. Furmonavičius
Geotechnical Group, Ltd, Vilnius, Lithuania

ABSTRACT: A necessity to use probability-based formats in design practice of pile foundations as highly-reliable structures is underlined. The specific features of the bearing capacity of piles in groups and principles of their structural safety prediction are discussed. Probability-based approaches and design equations for structural safety prediction of axially loaded piles in groups exposed to regular and extreme irregular variable actions are presented. The survival probabilities of single piles and pile group blocks are considered. It is shown that the total survival probability of pile foundations may be calculated by the unsophisticated method of transformed conditional probabilities. The numerical example illustrates a merit of the probability-based design of pile foundations and a conditionality of the deterministic method of partial safety factors.

1 INTRODUCTION

The construction of columns of buildings, construction and civil engineering works with heavy axial forces and limited settlement is based on pile raft foundations. The external bearing capacity of single piles is the resistance of their subsoil as a geotechnical support. The internal bearing capacity of piles is their structural resistance as reinforced concrete structures. Thus, the total safety of pile foundations rests on stochastic dependence upon the performance functions of piles characterizing two, different by nature, failure mechanisms.

An external bearing capacity of piles in groups depends on pile-soil and pile-pile interactions. For axially loaded piles in groups, a compressive resistance failure of the single piles and the piles with the soil contained between them acting as a block should be considered. According to Fleming et al. (1992), Katzenbach and Moormann (1997), the compressive resistance of the pile group acting as a block may be calculated by treating the block as a single pile of a large diameter.

Due to significant heterogeneity of soil characteristics, it is difficult to assess and predict the structural safety of pile foundations objectively using some ordinary partial safety factors and ignoring various features of complicated ground conditions and to time-variant non-stationary loads and other actions.

The deterministic approaches and methods may be used only for light and simple structures and small earthworks for which it is possible to ensure that fundamental code requirements may be satisfied on the basis of experience and qualitative geotechnical test data. A wide range of design issues of load-carrying geotechnical structures can be neither formulated nor solved by deterministic analysis methods. Therefore, it is high time that the probabilistic principles and approaches were established in geotechnical design practice.

The presented paper is devoted to demonstrate how feasible it is to introduce probability-based approaches and formats in design practice of pile foundations. These methods ensure the possibility of fulfilling and accomplishing the recommendations of Eurocode 7 (EN 1997-1:2003 E) that each geotechnical design shall be identified along with the risks to property and life.

2 EXTERNAL AND INTERNAL PERFORMANCE OF PILES

The total external compressive bearing capacity of pile foundations (Fig. 1) may be presented in the form:

$$R_n = nR_1 \quad \textbf{(1)}$$

where n and R_1 are the number and external bearing capacity of single piles. This capacity depends on ultimate values of the base resistance q_b and the skin friction q_s. Strengths q_b and q_s depend on the settlement s of the pile head, the diameter of the pile D and the cone penetration resistance q_c or undrained shear strength c_u for cohesionless and cohesive soils, respectively.

The characteristic values of the soil base resistance q_b and the skin friction q_s should be derived with the calculated probability equal to 95 %. The coefficients of variation of the soil mechanical parameters $\boldsymbol{\delta} q_b$ and $\boldsymbol{\delta} q_s$ depend on many factors and may be very great and even equal to 0.65-0.8 (Jardine et al. 2001). The values of their partial safety factors γ_b and γ_s depend on the type of piles (Subsoil and Foundations 1991, Eurocode 7).

Regardless of different load-carrying capacity of single piles, their interaction inside the pile group leads to a block performance and behaviour of the pile group (Katzenbach and Moormann 1997).

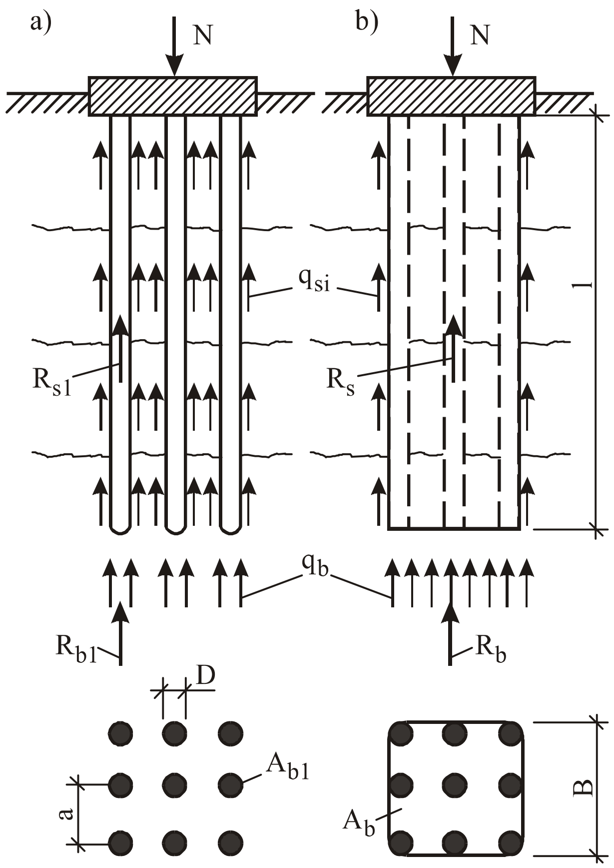

Fig 1. A pile foundation as the group of single piles (a) and the pile block (b)

As is known, the rigid pile cap or raft has a remarkable effect on the behavior of piles and their groups (Iwamoto et al. 2001). Poulos (1993) proved that axial compressive forces decrease from outside to inside of pile groups. When there is a stiff raft, due to redistributions of forces a limit state will be attained only if a significant number of piles fail together. In this case, the compressive bearing capacity of the pile group block may be assessed by treating the block as a single geotechnical structure. In the case of bored piles, the bearing capacity of the block may be smaller than the sum of single pile capacities (Czap 1995).

From ground test results, the external bearing capacity of single piles or their subsoil bearing capacity as a sum of toe an shaft resistances may be expressed as follows:

$$R_1 = A_{b1} q_b + \sum_{i=1}^{n} A_{s1i} q_{si} \quad (2)$$

where A_{b1} and A_{s1i} are the nominal cross-section area of the pile base and the surface area of the pile in a soil layer i, respectively. Analogically to (2), external design resistances of pile group blocks may be written as follows:

$$R = A_b q_b + \sum_{i=1}^{n} A_{si} q_{si} \quad (3)$$

where A_b and A_s are the nominal areas of the block base and surface.

The internal bearing capacity of reinforced concrete single piles as compression columns may be expressed by the formula:

$$R_2 = 0.85 A_c f_c + A_s f_y \quad (4)$$

where A_c and A_s are the areas of concrete and reinforcement cross-sections; f_c is the concrete strength in compression; f_y is the steel yield strength.

For piles in groups, two failure mechanisms are taken into account: compressive resistance failure of the piles individually and of the piles with the soil contained between them acting as a block. According to Eurocode 7, the design resistance shall be taken as the lower value caused by these two mechanisms.

Though are some differences in the values of deterministic factors recommended by Eurocode 7 and French code “Fasicule 62-V”, the design results for axially loaded piles are quite similar (Frank 1997). It corroborated the fact that the actual safety level of designed and existing pile foundations may be determined only by probability-based methods.

3 SAFETY MARGINS OF PILES

The action effects of buildings and works are caused by permanent loads g, sustained $q_1(t)$ and extraordinary $q_2(t)$ components of live floor loads, snow $s(t)$ and lateral wind (or seismic) actions $w(t)$ (Fig. 2). The loads $q_1(t)$ and $q_2(t)$ are assumed to be distributed by the gamma or normal and exponential distribution laws, respectively (ISO 2394, JCSS 2000, Vrowenvelder 2002, Trezos and Thomos 2003).

According to Rosowsky and Ellingwood (1992), the annual extreme sum of sustained and extraordinary components of live loads $q = q_1 + q_2$ may be modeled as a rectangular renewal pulse process and described by a Type 1 (Gumbel) distribution of extreme values with the coefficient of variation $\delta q = 0.58$ and mean value $q_m = 0.47 q_k$, where q_k is its characteristic value.

It is proposed to model the annual extreme wind and snow loads by the Gumbel distribution law (Ellingwood 1981, ISO 2394 1998). Depending on the feature of a geographical area, the coefficients of variation of wind and snow loads are: $\delta w = 0.2 - 0.4$ and $\delta s = 0.3 - 0.7$.

The duration of extreme live loads d_q may be considered as deterministic the value of which are 1-14 days for merchant and 1-3 days for other buildings (JCSS 2000). The durations of extreme wind and snow actions d_w and d_s are equal to 8-12 hours and 14-28 days, respectively. The occurrence rates of changes of sustained and extreme floor and climate actions are equal to $\lambda_{q_1} = 0.2$/year and $\lambda_{q_2} = \lambda_w = \lambda_s = 1$/year [JCSS 2000]. The recurrence number of coincident two extreme actions is:

$$n_{12} = t_n(d_1 + d_2)\ \lambda_{q_2^1}\ \lambda_{q_2^2} \quad \textbf{(5)}$$

Therefore, the recurrence numbers of extreme live loads q simultaneously on two and three storeys of multistorey office and residential buildings during $t_n = 50$ years reference period are equal, respectively, to $n_2 = 0.27\text{-}0.82$ and $n_3 = 0.001\text{-}0.01$ and may be neglected. Thus, in the analysis of foundations of multistorey buildings, floor loads may be used as conventional permanent actions the sum of which is:

$$p_2 = g + q_1 + q_2 \quad \textbf{(6)}$$

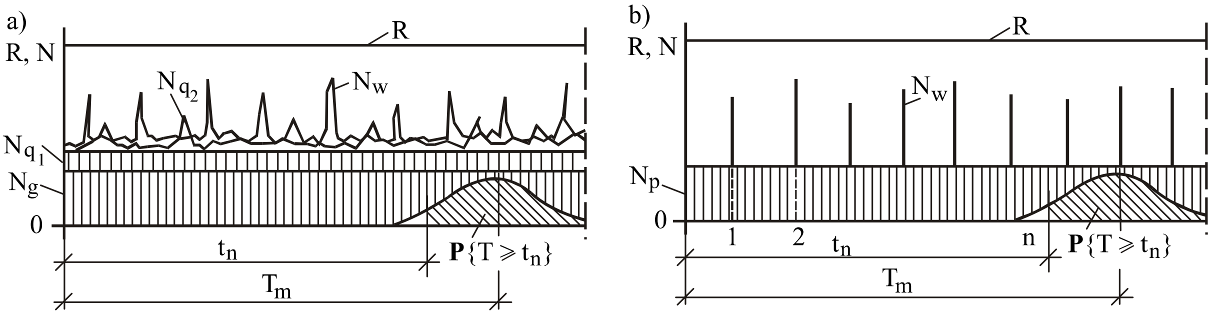

Fig 2. Actual (a) and conventional (b) dynamic models for a safety analysis of piles of multistorey buildings

For the 50-year reference period, the recurrence numbers of extreme actions are: $n_{qw} = 0.2\text{-}2.0$, $n_{qs} = 2.0\text{-}6.0$ and $n_{sw} = 2.0\text{-}4.0$ and must be taken into consideration.

The probability distributions of independent bearing capacities and permanent loads are fairly similar and close to a normal and lognormal ones. Therefore, it is expedient to simplify probabilistic calculations using the conventional resistances of piles and pile blocks. According to probability-based approaches and assumptions, the safety margin processes characterizing the random performance of axially loaded piles of multistorey and other buildings or works may be written, respectively, in the forms:

$$Z(t) = R_{cp} - \theta_s N_s(t) - \theta_w N_w(t) \quad \textbf{(7)}$$

$$Z(t) = R_{cg} - \theta_q N_{q_i}(t) - \theta_s N_s(t) - \theta_w N_w(t) \quad \textbf{(8)}$$

Here

$$R_{cp} = \theta_R R - \theta_p N_p \quad \textbf{(9)}$$

$$R_{cg} = \theta_R R - \theta_g N_g \quad \textbf{(10)}$$

are the conventional resistances of piles or pile bloks, were R is the external or internal load-carring capacity of piles or pile group blocks calculated by Equations (2) and (3) or (4): N_g, N_p and $N_{q_i}(t)$, $N_s(t)$, $N_w(t)$ are the random axial forces caused by permanent and variable actions; θ_R, θ_g, θ_p, θ_q, θ_s and θ_w are the random additional variables characterizing the uncertainties of design models. This format of safety margins is useful in the design practice and context of certain structural analysis (Melchers 1999).

The joint axial force caused by two annual extreme loads as greater value from $\theta_q N_{q_i}(t) + \theta_w N_w(t)$ and $\theta_q N_{q_i}(t) + \theta_s N_s(t)$ must be taken into consideration. The upper tails of density curves of two probability distributions of annual extreme variables coincide. Therefore, the mean and variance of proposed bivariate distribution functions approximately are equal to the sum of their means and variances (Kudzys 2006).

4 SURVIVAL PROBABILITIES OF PILES

The instantaneous survival probabilities of piles at any time t_k, assuming that they were safe at the time less than t_k, are:

$$\mathbf{P}_k = \mathbf{P}(Z_k > 0) = P\{Z(t_k) > 0 \ \ \exists t_k \in [t_1, t_n]\} = \int_0^\infty f_{R_c}(x) F_{N_k}(x)\,dx \tag{11}$$

where $f_{R_c}(x)$ is the probability density function of the conventional resistances by Equations (9) and (10); $F_N(x)$ is the probability distribution function of the variable axial force N_k.

When probability distributions of stochastically independent conventional resistances and regular axial forces are close to the normal one, the probability from Eq. (11) may be rewritten as follows:

$$\mathbf{P}_k = \mathbf{P}(Z_k > 0) = \Phi\left\{\frac{R_{cm} - N_{q_1km}}{(\boldsymbol{\sigma}^2 R_c + \boldsymbol{\sigma}^2 N_{q_1k})^{1/2}}\right\} \tag{12}$$

where $\Phi(\bullet)$ is the cumulative distribution function of the standardized normal distribution; R_{cm}, N_{q_1km} and $\boldsymbol{\sigma}^2 R_c$, $\boldsymbol{\sigma}^2 N_{q_1k}$ are the means and variances of random components of the performance function $Z_k = R_c - N_{q_1k}$.

Very often the variable force N_{q_2} is caused by irregular short-term live loads the probability distribution law of which obeys the Weibull one and is close to the exponential distribution (Vrowenvelder 2002). In this case, the survival probability of piles is:

$$\mathbf{P}_k = \mathbf{P}(Z > 0) = 1 - \boldsymbol{\Phi}\left(-\frac{R_{cm}}{\boldsymbol{\sigma}R_c}\right) - \exp\left(-\frac{R_{cm}}{N_{q_2km}} + 0.5\frac{\boldsymbol{\sigma}^2 R_c}{\boldsymbol{\sigma}^2 N_{q_2k}}\right) \times \left[1 - \boldsymbol{\Phi}\left(-\frac{R_{cm}}{\boldsymbol{\sigma}R_c} + \frac{\boldsymbol{\sigma}R_c}{\boldsymbol{\sigma}N_{q_2k}}\right)\right] \tag{13}$$

When the axial force N_k is caused by recurrent extreme floor and climate loads, the Gumbel distribution function may be expressed in the form:

$$F_{N_k}(x) = \exp\left[-\exp\left(\frac{N_{km} - x}{0.7794\ \ \boldsymbol{\sigma}N_k} - 0.5772\right)\right] \tag{14}$$

For a reference period of t_n (Fig. 2), the components of external and internal long-term survival probabilities of piles and pile blocks can be expressed by the method of transformed conditional probabilities as:

$$\mathbf{P}_i = \mathbf{P}\{T \ge t_n\} = \mathbf{P}\left\{\bigcap_{k=1}^{n} Z_k > 0\right\} = \mathbf{P}_k^n \left[1 + \rho^a \left(1/\mathbf{P}_k^{n-1} - 1\right)\right] \tag{15}$$

Here $\mathbf{P}_k$ is given by Eq. (11) and its modifications (12) and (13);

$$\rho_{kl} = 1/\left(1 + \boldsymbol{\sigma}^2 N_k / \boldsymbol{\sigma}^2 R_c\right) \tag{16}$$

is the coefficient of correlation of cuts of a pile safety margin process;

$$a = [4.5/(1 - 0.98\rho_{kl})]^{1/2} \tag{17}$$

is the bond index of this coefficient; $n = t_n\lambda$ is the number of load changes which is equal to $n_{q_1} = 10$ and $n_{q_2} = n_s = n_w = 50$ for sustained live and annual extreme loads under $t_n = 50$ years service life.

Several safety margins or performance functions exist for foundations because they may fail in different ways (Eloseily et al. 2002). Therefore, the safety analysis of pile foundations must be carried out taking into account a behaviour of single piles or pile group blocks as auto systems (Kudzys et al. 2003). The analysis model can be presented as a series system with two connected stochastically dependent elements.

The survival probability of auto systems may be calculated in an unsophisticated manner by the method of transformed conditional probabilities. The total survival probability of piles may be expressed as follows:

$$\mathbf{P}_{12} = \mathbf{P}_1\mathbf{P}_2 \; [1 + \rho_{12}^{a}(1/\mathbf{P} - 1)] \quad \textbf{(18)}$$

Here **P** is the greater value of the $\mathbf{P}_1$ and $\mathbf{P}_2$, characterizing external and internal safeties of piles;

$$\rho_{12} = \mathrm{Cov}(Z_1, Z_2)/(\boldsymbol{\sigma} Z_1 \cdot \boldsymbol{\sigma} Z_2) \quad \textbf{(19)}$$

is the coefficient of correlation of the external and internal safety margins of piles. The presented Formula (18) reveals the fact that the structural safety of pile foundations may be ensured only using highly-reliable piles.

According to EN 1990 (2002), a conventional measure of the reliability of structures, including geotechnical aspects, is presented as the generalized reliability index using the following expression:

$$\beta_i = \Phi^{-1}(\mathbf{P}_i) \quad \textbf{(20)}$$

where $\mathbf{P}_i$ is the survival probability of structures.

Geotechnical structures including foundations belong to the 3-nd class of their functional working life because it is very difficult to repair or replace their members (Kudzys and Furmonavičius 2003). Thus, in any case, the target value of generalized reliability indices of piles must be no less than $\beta_{min} = 3.8$.

5 NUMERICAL ILLUSTRATION

The pile foundation consisting of nine bored piles for works of the reliability class RC2 was designed according to Eurocode 7 recommendations and directions. Its structural safety was assessed according to the probability-based design methods presented in section 4. The geometrical characteristics of single piles and pile group blocks are: $A_{b1} = 0.096$ m^2 (D = 0.35 m); l = 9.1 m; $A_{s1} = 10.0$ m^2; $A_b = 5.98$ m^2; a = 2D and a = 3D; $A_s = 86.5$ m^2.

The external bearing capacity of axially loaded single bored piles and pile group blocks was determined from the test results on homogeneous ground. The average cone penetration resistance of the cohesionless soil was $q_{ck} = 15.0$ MPa. Therefore, the characteristic values of the base resistance $q_{bk} = 3.0$ MPa and the skin friction $q_{sk} = 0.12$ MPa (Katzenbach and Moormann 1997, Mets 1995). According to Eurocode 7, the partial safety factors for the soil properties of bored piles are: $\gamma_b = 1.6$ and $\gamma_s = 1.3$. Thus, the design soil resistances are: $q_{bd} = 3.0/1.6 = 1.875$ MPa and $q_{sd} = 0.12/1.3 = 0.0923$ MPa.

The design resistances in compression of single piles and pile group blocks are:
$R_{d1} = A_{b1}q_{bd} + A_{s1}q_{sd} = 1.103$ MN, $R_d = A_b q_{bd} + A_s q_{sd} = 19.18$ MN.

The characteristic components of the axial permanent and extraordinary variable forces are: $N_{gk1} = 0.639$ MN; $N_{gk} = 0.639\times9 = 5.75$ MN; $N_{qk1} = 0.1598$ MN; $N_{qk} = 0.1598\times9 = 1.44$ MN. Therefore, the design axial forces are:
$N_{d1} = 0.639 \times 1.35 + 0.16 \times 1.5 = 1.103\,\mathrm{MN} = R_{d1}$, $N_d = 5.75 \times 1.35 + 1.44 \times 1.5 = 9.923\,\mathrm{MN} < 19.18$ MN.

Thus, according to deterministic calculation results, the pile foundation will support the design loads with adequate design safety against compressive failure. Besides, the design of the considered foundations must be based on the limit state analysis of single piles.

The structural safety of designed pile foundations is checked neglecting model uncertainties. The coefficient of correlation of soil mechanical properties was taken as $\rho(q_b, q_s) = 0.9$. The effect of soil uncertainties on the safety of designed piled foundations is considered taking into account three coefficients of variation of soil resistances: $\boldsymbol{\delta} q_b = \boldsymbol{\delta} q_s =$ 0.15; 0.225 and 0.30. To these values correspond the means and standard deviations of soil strengths: q_{bm} = 3.98, 4.76, 5.92 MPa; $\sigma q_b = 0.597$, 1.07, 1.78 MPa and $q_{sm} = 0.159$, 0.1905. 0.237 MPa; $\sigma q_s = 0.0238$, 0.0428, 0.0711 MPa.

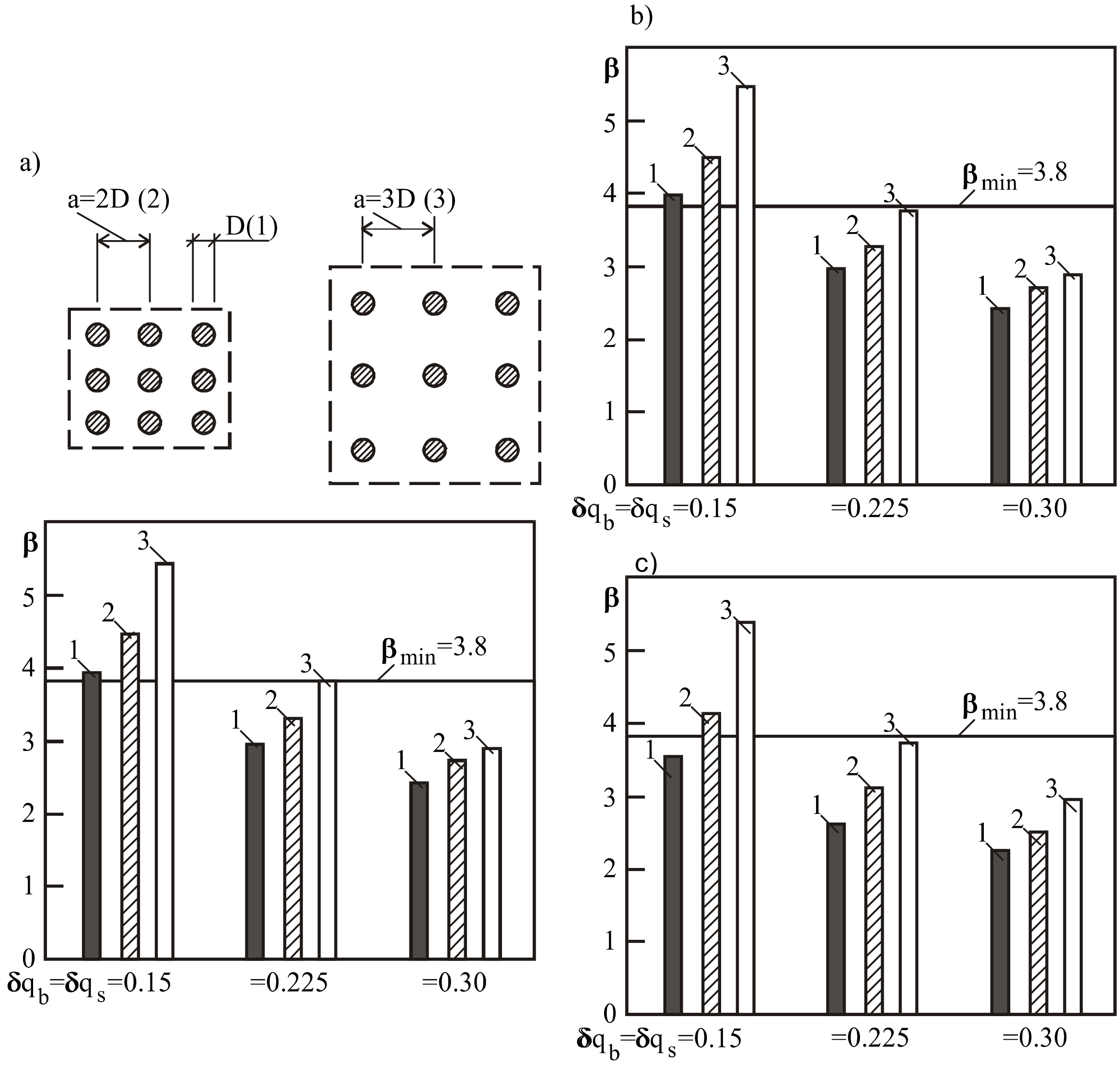

Fig 3. The external reliability index β by (20) versus the coefficient of variation δq of cohesionless soil resistances for single piles (1) and pile blocks with a/D=2 (2) and a/D=3 (3) in the absence (a) and presence of sustained (b) and extraordinary (c) variable actions

The values of instantaneous external survival probabilities of single piles and pile group blocks are based on Formulas (12) or (13) when the coefficients of variation of regular (sustained) or irregular (extraordinary) variable and permanent axial forces, respectively, are $\delta N_{q_1} = 0.30$ or $\delta N_{q_2} = 1.0$ and $\delta N_g = 0.10$. The factor of variable action intensity $\nu = N_{qk} / \left(N_{gk} + N_{qk}\right)$ is equal to 0.2. The long-term external survival probability of piles is calculated by Eq. (15).

Fig. 3 shows that soil resistance uncertainties have a great effect on the external structural safety of pile foundations, and they must be evaluated in deterministic design procedures. When the coefficients of variation of soil resistance parameters are equal to 15 % or less and action effects of piles are caused by regular loads, the results of deterministic and probabilistic design calculations may be identical. However, in the other cases the method of partial safety factors should be used in design practice with great caution. The failure probability of pile foundations designed by Eurocode 7 directions can exceed the specified target values hundred by a factor of and more.

6 CONCLUSIONS

The structural safety of single piles and pile group blocks greatly depends on soil resistance uncertainties. Therefore, it is difficult and sometimes impossible to ensure the sufficient external structural safety of single piles and piles in groups exposed not only to irregular extraordinary variable action effects but also to permanent actions when pile foundations are designed according to Eurocode 7 recommendations.

The structural safety of pile groups in cohesionless soils under axially loaded columns slightly depends on pile spacings. The external and internal survival probabilities of single piles may be taken as the main quantitative reliability parameters of these foundations the target reliability index of which may be ensured only using high-reliable piles.

The structural safety level of piles and their reliability indices may be objectively predicted only by probability-based methods including unsophisticated engineering approaches. Probability-based formats should be used for further development of the Eurocode 7 taking into account the dependence of partial safety factors upon soil resistance uncertainties and calibrating explicitly target values of the reliability indices for various design situations of pile foundations and other geotechnical structures.

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