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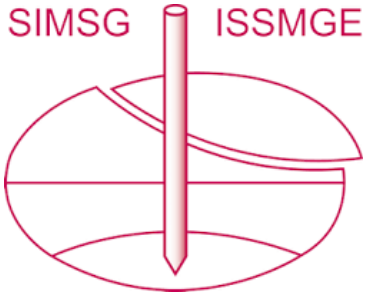

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Probabilistic analysis of seismic slope stability

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ABSTRACT: The paper presents a reliability-based method that can capture the impact of uncertainty of seismic loadings. The proposed method incorporates probabilistic concepts into the classical limit equilibrium and the Newmark-type deformation techniques. The risk of damage is then computed by Monte Carlo simulation. Random process and RMS hazard method are introduced to produce seismic motions and also to use them in the seismic slope analyses. The geotechnical variability and sampling errors are also considered. The results of reliability analyses indicate that in a highly seismically active region, characterization of earthquake hazard is the more critical factor, and characterization of soil properties has a relatively small effect on the computed risk of slope failure and excessive slope deformations.

1 INTRODUCTION

An important step in the slope stability analysis is to identify uncertainties of key design parameter and their statistics. The variability of the various parameters such as material properties, fluctuations of the loads, and the uncertainties of the geometry all contribute the difficulty of evaluating how the slope actually behaves regardless of the computed factor of safety. In spite of extensive effort, there has not been much consensus on how to model these uncertainties systematically, leaving them mostly up to the rather subjective judgments of individual engineers. In the past, there have been numerous studies of the various sources of uncertainty and their impact on the risk of damage of slopes subjected to seismic loading. Most approaches, however, have focused either on material uncertainty or on uncertainty of seismic loading (e.g., Pal et al. 1991; Yegian et al. 1991) partly because of computational convenience, and partly due to the practical difficulty in modeling both material and seismic hazard uncertainties.

In this paper, the authors present a reliability-based method that can capture the impact of uncertainty of key material properties and seismic loading in the assessment of seismic slope stability. The uncertainty of seismic loading is approached by generating a large series of hazard-compatible artificial motions, and by using them in subsequent response analyses. The geotechnical variability and sampling errors are also taken into consideration. This approach incorporates probabilistic concepts into the classical limit equilibrium and the Newmark-type deformation techniques. The risk of damage is then computed by Monte Carlo simulation. Finally, the overall impact of uncertainties and their relative significance on seismic stability of slopes will be judged.

2 GENERATION OF HAZARD-COMPATIBLE GROUND MOTIONS

Due to the random nature of earthquake motions, the definition of the input excitation is one of the major uncertainties in the seismic response analysis. The stochastic representation of the earthquake motion hazard provides, at least theoretically, a systematic way of representing the infinite number of possible scenarios, which correspond, to a certain level of hazard.
A zero-mean process can be represented as:

$$X(t) = \sum_{i=1}^{n} \sqrt{2G(\omega_i)\Delta\omega}\,\sin(\omega_i t + \theta_i) \tag{1}$$

where A_i is the Fourier amplitude, ω_i is the circular frequency, and θ_i is the phase angle of the contributing sinusoid. By fixing an array of amplitudes and randomly generating different arrays of phase angles, we can produce different motions with the same general appearance but different details (Gasparini and Vanmarcke 1976). The study adopts the following widely quoted form for the power spectral density function (PSD) that is based on Kanai (1957) and Tajimi (1960)'s studies.

$$G(\omega) = G_0 \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{\left[1-(\omega/\omega_g)^2\right]^2 + 4\zeta_g^2(\omega/\omega_g)^2} \tag{2}$$

where $G(\omega)$ is the energy content at frequency ω, G_0 is some measure of shaking intensity, and ζ_g and ω_g are the parameters termed as "ground damping coefficient and frequency". Each different array of phase angles can be modeled by statistically independent random phase angles θ_i, which are uniformly distributed between 0 and 2 . Sample stationary earthquake motion can then be obtained as shown in Figure 1

The transient character of the intensity content of the earthquake motion can be added by multiplying the stationary motion by a deterministic modulating (envelope) function $m(t)$. The non-stationary motion then becomes:

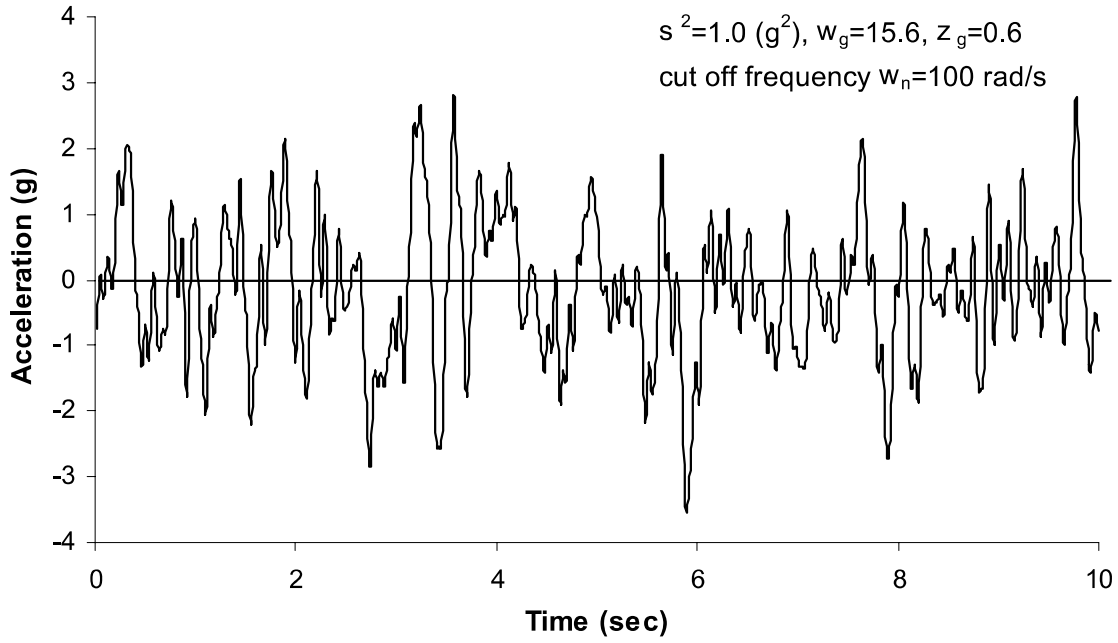


Fig. 1.Sample Stationary Artificial Earthquake Ground Motion for Rock Sites Based on the Frequency Parameters for Firm Soil Proposed by Kanai (1957)

$$Y(t) = m(t)\,X(t) = m(t)\sum_{j=1}^{n} \sqrt{2G(\omega_j)\Delta\omega}\,\sin(\omega_j t + \theta_j) \tag{3}$$

Various modulating functions have been used to incorporate non-stationary aspect of the earthquake ground motions. In their liquefaction-related analytical approaches, Wang and Kavazanjian (1987) proposed a trigonometric modulating function that has two model parameters to define the shape of the modulating function, defined as:

$$m(t) = \sin^{\alpha}\left(\pi(t/t_d)^{\beta}\right) \tag{4}$$

(6)

where α and β are two parameters to determine the shape of the modulating function and t_d is the duration of motion. Wang and Kavazanjian (1987) also developed the statistics of the model parameters α and β, based on their study of 122 Northern California strong ground motions.

The normalized PSD is obtained by dividing the PSD by the variance as:

$$G^*(\omega) = \frac{1}{\sigma^2} G(\omega) \tag{5}$$

When the stationary process $X^*(t)$ is a normalized process with unit variance and PSD function, Equation 3 (non-stationary ground motion) can be rewritten in terms of the normalized PSD function and modulating function as:

$$Y(t) = m(t)X^*(t) = m(t)\sum_{i=1}^{n}\sqrt{2G^*(\omega_i)\Delta\omega}\,\sin(\omega_i t + \theta_i) \tag{6}$$

2.2 Intensity

One of the energy-based parameters is the RMS (Root Mean Square) acceleration, which is defined as (Housner 1975):

$$RMS_a = \left[\frac{1}{T_d}\int_{t_0}^{t_0+T_d} a^2(\tau)\,d\tau\right]^{1/2} \tag{7}$$

where $a(t)$ is an acceleration time history, t_0 is a initial time of interest, T_d is duration of the strong ground shaking. Similarly, temporal RMS can be defined by replacing T_d with a small time interval Δt as:

$$RMS_a(t) = \left[\frac{1}{\Delta t}\int_{t}^{t+\Delta t} a^2(\tau)\,d\tau\right]^{1/2} \quad for\ \Delta t \to 0 \tag{8}$$

It is fairly straightforward to relate time-variant RMS to the deterministic modulating function $m(t)$ for the case when $X^*(t)$ is a normalized process as follows:

$$RMS_Y^2(t) = E\left[Y^2(t)\right] = m^2(t)E\left[X^{*2}(t)\right] = m^2(t) \tag{9}$$

The time-variant RMS of the motion $Y(t)$ is thus identical to the modulating function $m(t)$ in case of the normalized process..

2.3 Frequency Content

There are different methods for characterizing the temporal variation of the spectral content of earthquake motion. The proposal by Saragoni and Hart (1974) is based on dividing the motion into three sections of equal time interval as:

$$Y(t) = m(t)\sum_{j=1}^{3}\left(X_j^*(t_j)\right) = m(t)\sum_{j=1}^{3}\left[\sum_{i=1}^{n}\sqrt{2G_j^*(\omega_i)\Delta\omega}\sin(\omega_i t_j + \theta_i)\right], \qquad t = \sum_{j=1}^{3} t_j \tag{10}$$

Based on the approach by Saragoni and Hart (1974), Wang and Kavazanjian (1987) analyzed 80 out of the same set of 122 earthquake records, which were used to estimate the modulating function parameters, and proposed the statistics (i.e., mean and standard deviation) of the parameter ω_g and ζ_g. Tung et al. (1992) updated the statistics of modulating and PSD function parameters by including an additional set of 36 earthquake records from the 1989 Loma Prieta earthquake. These values are used in our analyses. Figure 2 shows a sample of non-stationary ground motion based on these statistics.

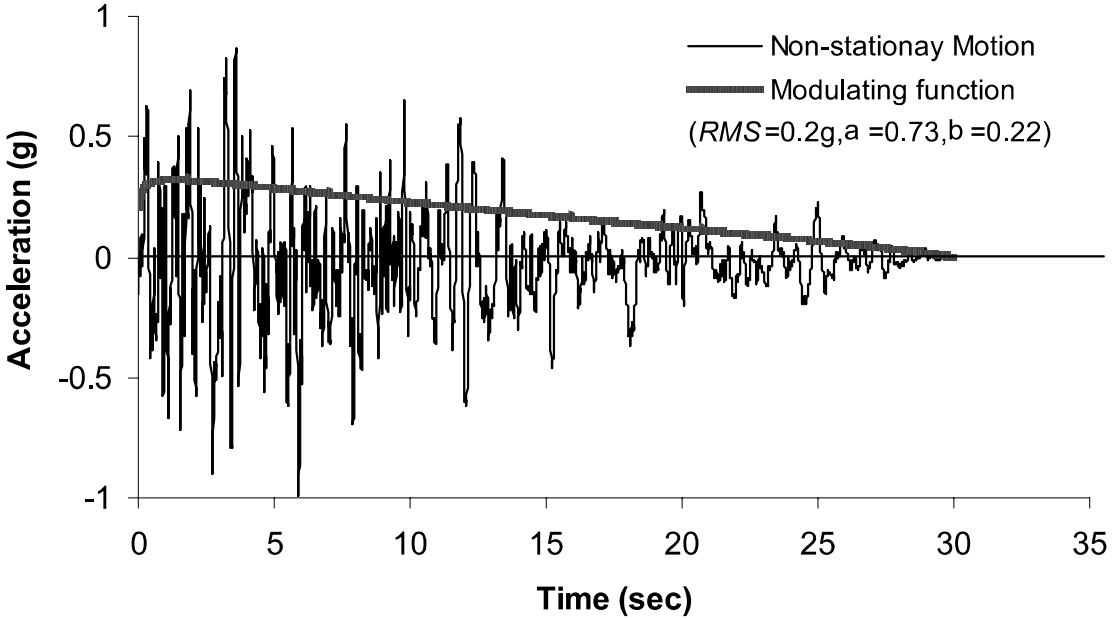


Fig. 2. Sample Time History of Non-Stationary Ground Motion (RMS=0.2 g, a=0.73, b=0.22, td=30 sec)

2.4 Duration

5-95 % Arias duration (often called significant duration) is adapted in this study because it provides clear relationship to RMS hazard. The significant duration is nothing but the duration of the 5-95 % RMS acceleration. Predictive relationships that relate the mean of 5-95 % Arias duration with respect to the distance and moment magnitude (Abrahamson and Silva 1996) are used in this study.

3 LIMIT STATE FUNCTION AND MONTE CARLO SIMULATION

For deformation-based slope stability under static or seismic loadings, the limit state function can be formulated as:

$$g(\mathrm{x}) = D_a(\mathrm{x}) - D_p(\mathrm{x}) \tag{11}$$

where and denote the allowable and incurred deformations respectively.
The numerical implementation and data flow of the proposed approach are illustrated in Figure 3. The author uses the so-called decoupled approach, in which seismic response and deformation analyses are carried out separately, mainly because of its computational efficiency. The analogy of a block resting on an inclined plane (so-called Newmark's method, Newmark 1965) is used for estimating the permanent displacement of the sliding mass due to earthquake shaking. The computation of yield acceleration is performed with newly developed expressions (Kim and Sitar 2004), which enable the calculation of the yield acceleration directly without iterations.
Probability computations were carried out with the aid of CALREL, a general-purpose structural reliability analysis program developed by Liu et al. (1989). The program requires the definition of limit-state functions through user-defined subroutines. These user subroutines are written by the author in such way that once statistics of seismic hazard, soil properties and geometry of the problem are provided, the program performs all sequential computations from the motion generation to deformation estimation without any interruption

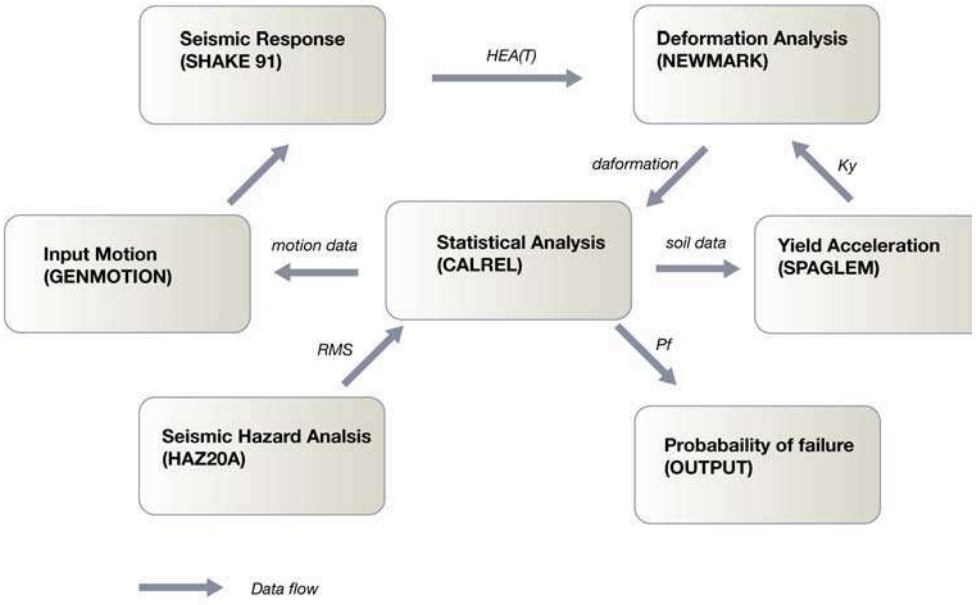


Fig. 3. Numerical Implementation and Data Flow of Probabilistic Seismic Slope Deformation Analysis

4 ILLUSTRATIVE EXAMPLES

The applicability of developed reliability-based methods is examined, starting from the analysis of seismically induced deformations of an idealized waste fill and ending with comparing the results with those of the seismic stability assessment of a homogeneous slope with a circular slip surface. Various levels of available information are examined in terms of their impact on the computed risk of slope failure.

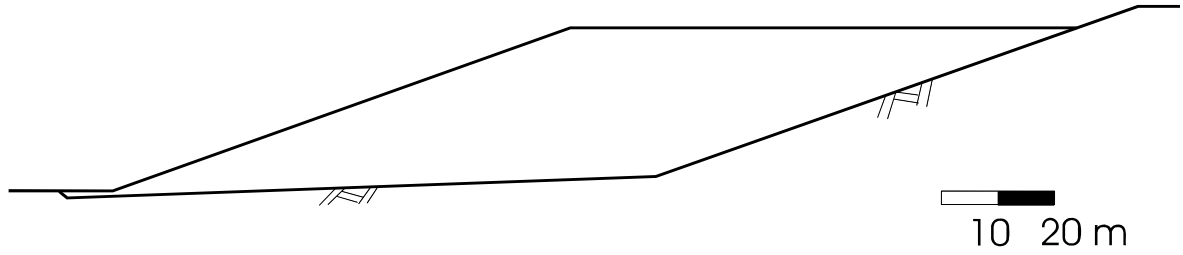


Fig. 4. Geometry of the Landfill Lying on the Gently Sloping Potential Slip Surface

4.1 Landfill (Non-Circular) Slope Stability

The authors consider a problem where we are interested in evaluating the risk of failure of a solid-waste landfill slope along a gently sloping potential slip surface as shown in Figure 4. The key design parameters for both deterministic and probabilistic assessments of the landfill slope stability are given in Table 1. Sliding is assumed to occur only along the liner system underlying the waste-fill, since the foundation soil and waste-fill are judged to be much stronger than the liner interface (i.e., Kim et al. 2005). Accordingly, only the shear-strength properties of the liner interface, and the average density of the waste-fill deposit and soil cover overlying the lining system need to be considered. Expressions for characterization of material properties developed to incorporate sampling errors, spatial variation that were described in details by Kim (2003) are used, but are not repeatedly described here. The potential sliding mass is divided into 40 slices and Spencer method of analysis (as a subroutine of CALREL) is used to evaluate the limiting equilibrium. Figure 5 shows two different samples of generated friction coefficients over the 40 slices of the slip surface.

Table 1. Key design material parameters

Parameter	Distribution	Mean	c.o.v.	No. of Tests	δ_x	δ_y
$\tan(\phi')$	lognormal	0.268	0.14	10	3 m	3 m
γ	deterministic	12 kN/m^3		10		

Table 2. Power spectral density parameters for rock sites (from Tung et al. 1992)

Parameter	Distribution	Segment 1		Segment 2		Segment 3	
		μ	σ	μ	σ	μ	σ
ω_g	Gamma	23.57	3.46	21.12	3.60	18.38	3.50
ζ_g	Gamma	0.352	0.360	0.394	0.380	0.417	0.162

For the purpose of this seismic analysis, the landfill slope is assumed to be in Berkeley, California, in the seismically active San Francisco Bay Area. The major faults affecting the region are the San Andreas, Hayward, and Calaveras faults, all within 30 kilometers of the site. Based on the regional seismicity, seismic hazard analyses, using the empirical RMS attenuation relationship (Kavazanjian et al. 1985), are performed. All the major and minor faults in the vicinity of the site including background sources are considered. The computed RMS hazards are then de-aggregated into several intervals of intensity, magnitude, and distance. In order to generate RMS-compatible ground motion, we need to specify the frequency content and duration in addition to the RMS acceleration. The

author adopts the stochastic ground motion parameters suggested by Wang and Kavazanjian (1987) and updated by Tung et al. (1992). Tables 2 and 3 summarize stochastic ground motion parameters used in this study. Another strong ground motion parameter, which is important in nonlinear deformation analyses, is duration. The hazard compatible duration can be assigned to each generated ground motion by means of de-aggregation of total hazard into appropriate intervals of earthquake magnitude and distance to site.

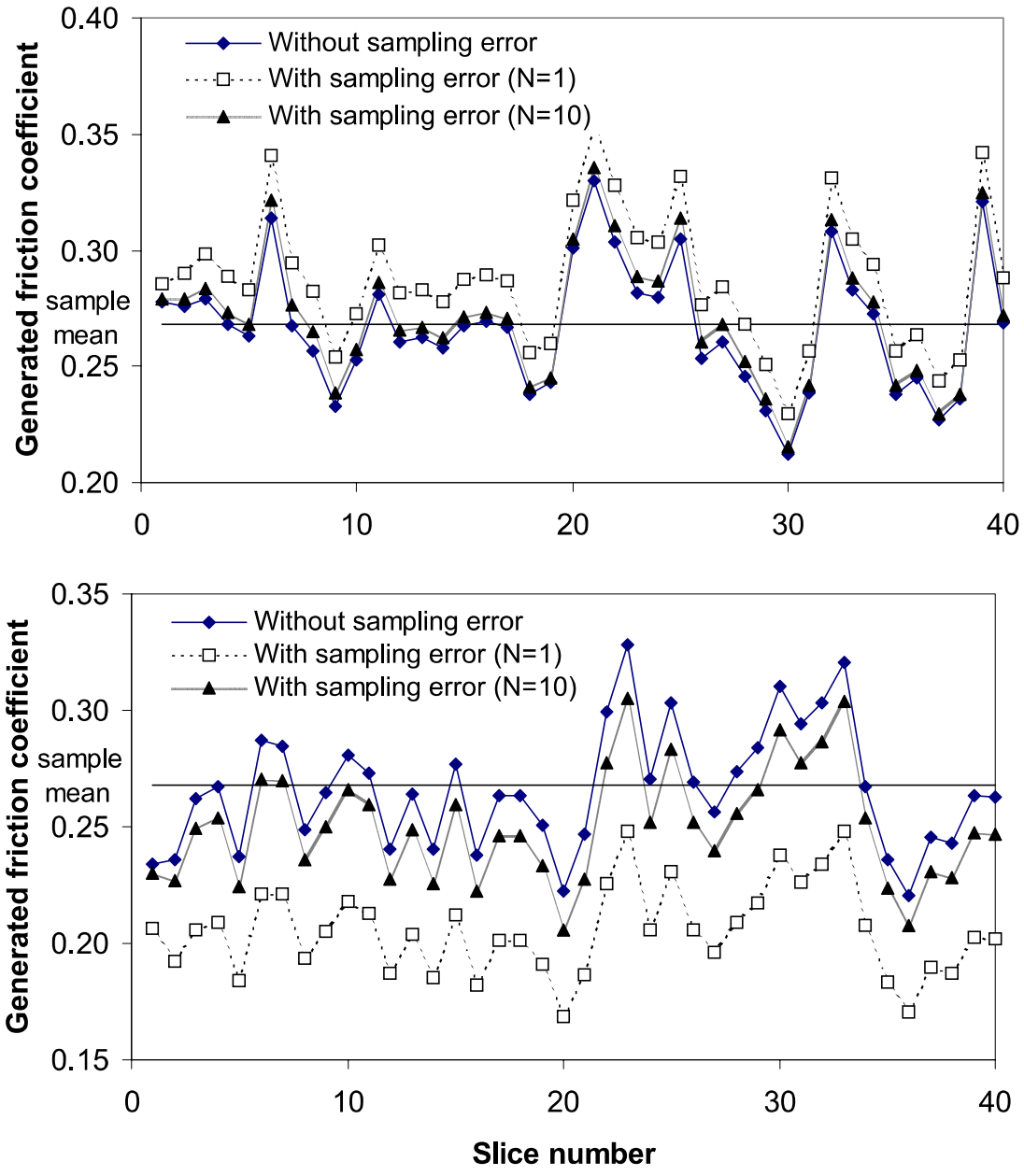


Fig. 5. Samples of Generated Friction Coefficients over the 40 Slices of the Slip Surface: (a) with Trend Higher than the Sample Mean; (b) with Trend Lower than Sample Mean; N: Number of Samples

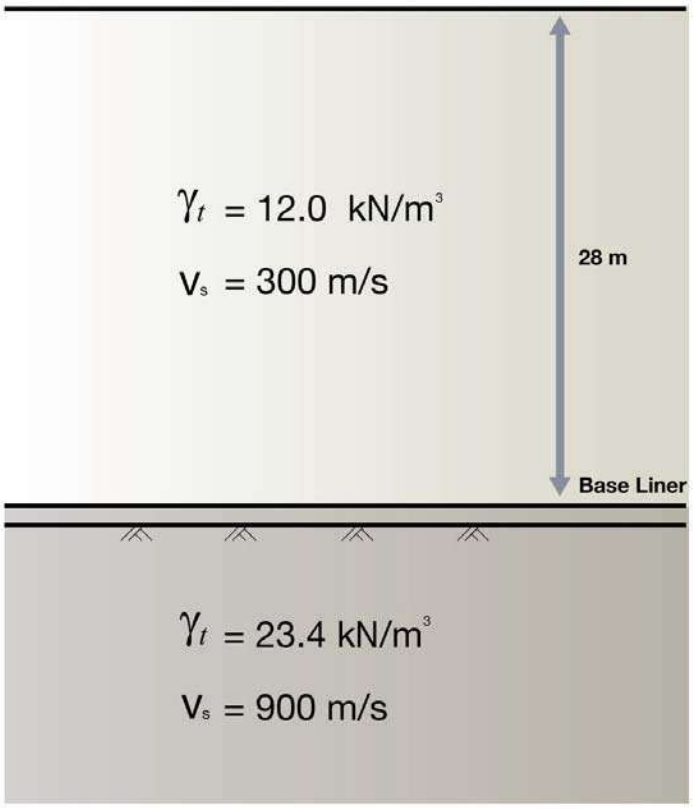


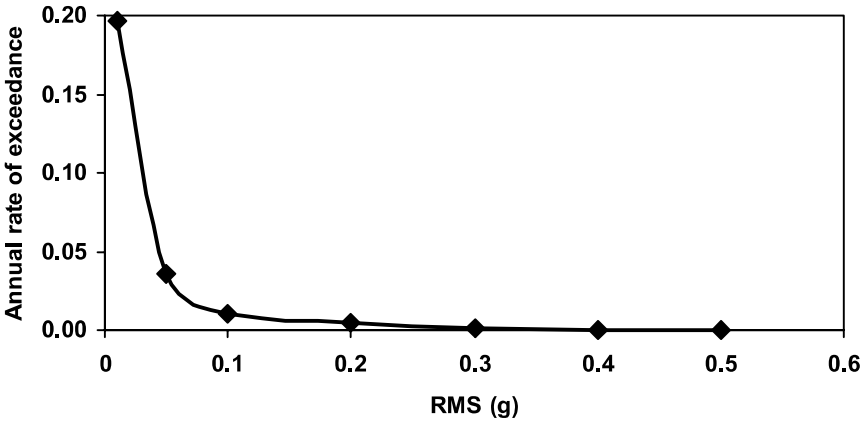
Fig. 6. Geometry of the 1-D Approximation of the Landfill and Dynamic Soil Properties

Table 3. Parameters for modulating function (from Wang and Kavazanjian 1987)

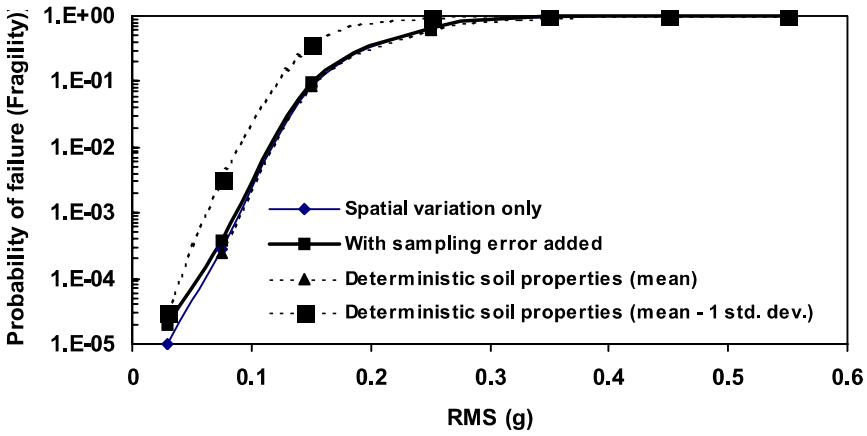
Parameter	Distribution	μ	σ
α	Rayleigh	0.73	0.45
β	Exponential	0.22	0.18

Figure 6 shows the geometry of the problem and the dynamic soil properties, used in the 1-D ground response analyses of the landfill slope, in which the 2-D geometry is approximated by 1-D problem. Modulus reduction and damping curves proposed by Singh and Murphy (1990), and by Schnabel (1973) are used for the waste and base rock respectively. Although shear wave velocity of the solid waste is known to increase with depth, uniform shear wave velocity is assumed so as to be consistent with the assumed homogeneous site condition. The horizontal equivalent acceleration (HEA) time history (or the average seismic coefficient) acting on the sliding mass on slope is estimated through seismic response analyses. Unlike the deformation analyses, the ground response analyses are performed by assuming that the key soil properties are deterministic, hence using mean values of the properties. This approach is adopted, since we employ the decoupled approach, in which ground response and slope deformation analyses are performed separately, by using separate numerical programs (otherwise, in a strict sense, correlations between input parameters of ground response and deformation analyses should be considered). Ideally, the ground response and deformation analyses are performed in so-called coupled mode, thus eliminating the need to provide separate sets of input parameters, as in the case of the decoupled analyses. On the other hand, the coupled analyses such

as dynamic FEM are generally time-consuming, and cannot be used routinely, even in the current fast computing environment. This is even more so in inherently time-consuming probabilistic analyses, which generally involve a significant number of repeated analyses of the problem.



(b)



(c)

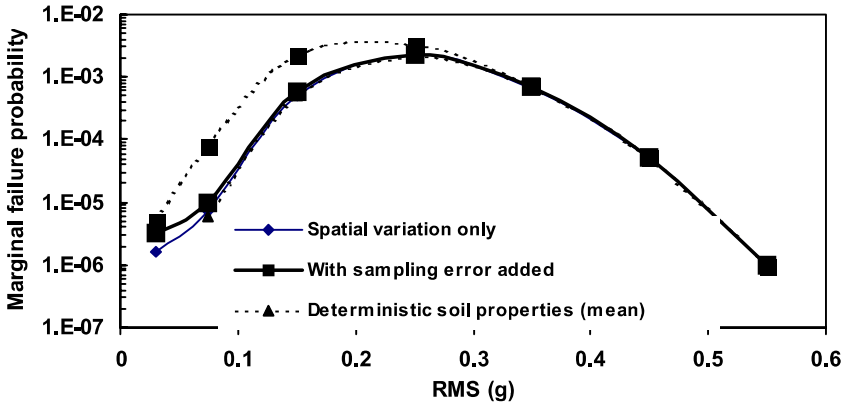


Fig. 7. (a) Annual Hazard Curve (Annual Rate of Exceeding Each Hazard Level); (b) Fragility Curve; (c) Marginal Annual Risk Curve (Annual Probability of Failure with Respect to Each Hazard Level)

The slope is assumed to be able to tolerate up to 300 mm of permanent base displacement. Presently, no standard for acceptable permanent deformation has been established, but in practice it is generally assumed that maximum seismic displacements of 15-30 cm are tolerable for well-designed lined waste fills (i.e., Seed and Bonaparte 1992).

Simulation-based (Monte Carlo) reliability analyses are performed by generating a series of hazard compatible outcrop rock motions, and by using them in subsequent 1-D ground response and slope deformation analyses. The computations are carried out both for the case, where spatial variation is only considered, and the case, where uncertainty arising from sampling is also considered, in order to compare their relative significance on the risk of failure. The results are shown in Figure 7. The seismic hazards (Figure 7a) are same for the four cases. The difference is the level of uncertainty in soil property determination. The deterministic soil properties represents the lowest level of uncertainty in the property determination, the "with sampling error added" the highest level of uncertainty, and the "spatial variation only" in the between these levels. The case of deterministic mean soil properties provides the lower bound of the risk and the mean minus standard deviation yields the upper bound. It can be seen in Figure 7b that all three cases except the case of the mean minus standard deviation are almost identical for all RMS levels larger than 0.1 g. In other words, the risk of failure is not sensitive to the uncertainty of material properties. That is possibly due to significant spatial averaging effects and also relatively large number of samplings. It may be thus argued in this particular case that seismic uncertainty is the dominant factor in the assessment of stability of the slope, and that material uncertainty maybe neglected without significant impact on the computed reliability of the problem. The results suggest that in a highly seismically active region, characterization of earthquake hazard is the most important and that characterization of soil properties, if in a reasonable range, have a relatively small effect on the computed risk of the slope failure.

4.2 Circular Slip Surface

A circular slip slope in Figure 8 is studied in order to see the consistency of findings in the analysis above persist in different types of slope. The results are briefly discussed here only for comparison purpose.

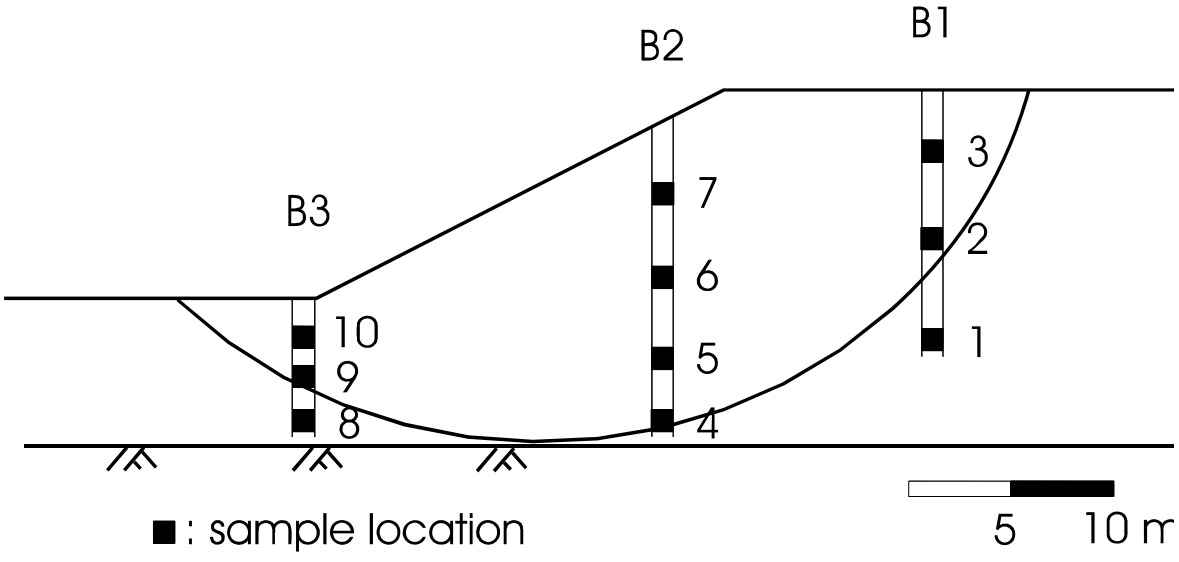


Fig. 8. Geometry and Sample Location of a Slope with a Circular Slip Surface

Three vertical borings are carried out and 10 soil samples are taken at the specific locations shown in Figure 8. Laboratory tests yield a sample mean value with standard deviation for undrained shear

strength, and for soil density. The soil deposit is modeled as a statistically homogeneous random field with two different scales of fluctuation (a measure of correlation distance) and , and and , respectively. In addition to the shear strengths it is assumed that the shear wave velocity measured at the site be around 200 m/sec. The potential sliding mass is divided into 40 vertical soil slices of equal width, for subsequent seismic stability and deformation analyses.
The slope is also assumed to be in Berkeley. Unlike the preceding example, only the Hayward fault that is the closest major fault to the site is considered here.

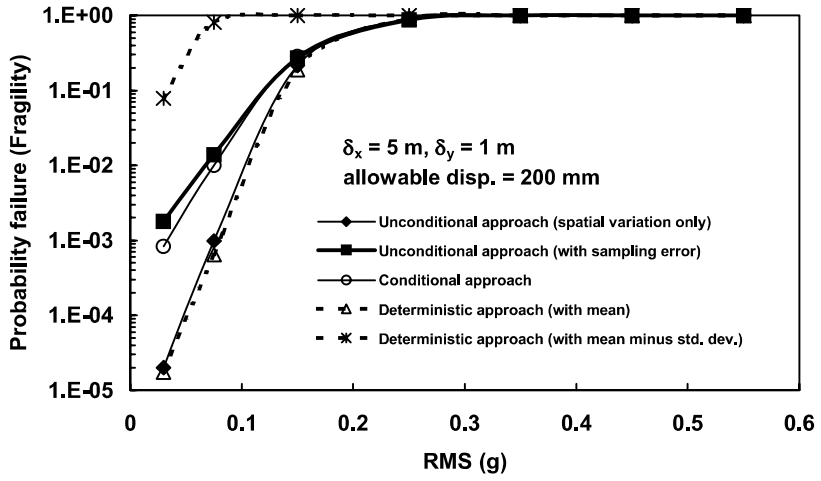


Fig. 9. Probability of Failure Given the Certain Hazard Levels (Fragility Curve)

The computations were carried out with the soil properties modeled with various approaches including conditional, unconditional and deterministic methods. Figure 9 shows the probabilities of failure given the certain hazard levels (often called fragility curve), for five different characterizations of soil properties. It is interesting to observe that the differences in the probabilities of failure for the different cases are significant at the lower level of hazard (< 0.1 g) and gradually decrease with increasing hazard level, being negligible at the higher level (> 0.2 g) except for the case of the mean minus one standard deviation (i.e., conservative assessment of soil properties). For relatively small correlation lengths (and), the difference between the conditional and unconditional (with sampling error) approaches is minimal. Increasing correlation (and) yields a similar trend, but the risk level for the unconditional approaches become higher due to small variance reduction. Thus, in this particular problem, uncertainty of soil properties has a significant impact on the computed risk of slope failure at relatively low levels of seismic hazard, but it has little impact on the computed risk if the slope is exposed to relatively high levels of hazard. In this particular problem, the uncertainty of soil properties arising from the spatial variation and sampling errors does not have much impact on the reliability of the slope for the RMS hazard level higher than 0.2 ~ 0.3 g. The results are consistent with those from the previous non-circular slope case.

5 CONCLUSIONS

The RMS procedure proved to be useful for generation of a large number of hazard-compatible motions, unlike the conventional procedures that usually can generate only a small number of motions that match deterministic targets such as design response spectra.
It is found that for relatively small correlation lengths, the difference between the conditional and unconditional (with sampling error) approach is minimal. Increasing correlation yields a similar trend, but the risk level for the unconditional approaches become higher due to small variance reduction.

Two example problems show that the characterization of material properties has a considerable influence on the probability of failure for a slope that is exposed to relatively low levels of seismic hazard. On the other hand, it shows a little impact on the computed risk in case of relatively high levels of seismic hazard.
The results suggest that in a seismically active region like the Western States of USA, Taiwan and Japan, characterization of earthquake hazard is the more critical factor in the computed risk, and a moderate variability in material properties and its characterization have a relatively small effect on the computed risk of slope failure and excessive slope deformations. On the other hand, in a seismically less active region like the Mid-Western States of USA, Britain and Korea, characterization of material properties may be equally important to that of earthquake hazard, and therefore cannot be neglected.

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REFERENCES

Abrahamson, N., and Silva, W. (1996), Empirical Ground Motion Models, Draft Report, Brookhaven National Laboratory.

Gasparini, D. A., and Vanmarcke, E. H. (1976), Evaluation of Seismic Safety of Buildings Report No. 2: Simulated Earthquake Motions Compatible with Prescribed Response Spectra, Publication No.R76-4, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Housner, G. W. (1975), "Measures of Severity of Earthquake Ground Shaking", Proceedings U.S. National Conference on Earthquake Engineering, Ann Arbor, Michigan.

Kanai, K. (1957), "Semi-Empirical Formula for the Seismic Characteristics of the Ground", Bulletin of the Earthquake Research Institute, Tokyo University, Vol.35, pp.308-325.

Kavazanjian, J., E., Echezuria, H., and McCann, J., M. W. (1985), "RMS Acceleration Hazard for San Francisco", Soil Dynamics and Earthquake Engineering, Vol.4, No.3, pp.106-123.

Kim, J. (2003), "The Importance of Geotechnical Variability in the Analysis of Earthquake-Induced Slope Deformations", Journal of Korean Geotechnical Society, Vol. 19, No. 2, pp.123-133

Kim, J., Bray, J., Riemer, M. (2005), "Dynamic Properties of Geosynthetic Interfaces", Geotechnical testing Journal, ASTM, Vol. 28, Issue 3, pp. 288-296.

Kim, J., and Sitar, N. (2004), "Direct Estimation of Yield Acceleration in Seismic Slope Stability Analyses", Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol.130, No.1, pp.111-115.

Liu, P. L., Lin, H. Z., and Der Kiureghian, A. (1989), CALREL User Manual, Report No. UCB/SEMM-89/18, Department of Civil Engineering, University of California at Berkeley, Berkeley, California.

Newmark, N. M. (1965), "Effects of Earthquakes on Dams and Embankments", Geotechnique, Vol.15, No.2, pp.139-160.

Pal, S. K., Rahman, M. S., and Tung, C. C. (1991), "A Probabilistic Analysis of Seismically Induced Permanent Movements in Earth Dams", Soils and Foundations, Vol.31, No.1, pp. 47-59.

Saragoni, G. R., and Hart, G. C. (1974), "Simulation of Artificial Earthquakes", Earthquake Engineering and Structural Dynamics, Vol.2, pp.249-267.

Schnabel, P. B. (1973), Effects of Local Geology and Distance from Source on Earthquake Ground Motions, Ph.D. Dissertation, University of California, Berkeley, California.

Seed, R. B., and Bonaparte, R. (1992), "Seismic Analysis and Design of Lined Waste Fills: Current Practice", ASCE Special Conference on Stability and Permanent Deformation of Slopes and Embankments, II, Reston, Va, pp.1152-1187.

Singh, S., and Murphy, B. (1990), "Evaluation of the Stability of Sanitary Landfills", Geotechnics of Wastefills - Theory and Practice, ASTM STP 1070, ASTM, pp.240-258.

Tajimi, H. (1960), "A Statistical Method of Determining the Maximum Response of a Building Structure During an Earthquake", Proceedings of the Second World Conference on Earthquake Engineering, Tokyo, pp.781-797.

Tung, A. T. Y., Kiremidjian, A. S., Wang, J. N., and Kavazanjian, J., E. (1992), "Statistical Parameters of AM and PSD Functions for the Generation of Site-Specific Strong Ground Motions", Earthquake Engineering, Tenth World Conference.

Wang, J. N., and Kavazanjian, J., E. (1987), A Nonstationary Probabilistic Model for Pore Pressure Development and Site Response due to Seismic Excitation, Report No.84, The John A. Blume Earthquake Engineering Center, Stanford University.

Yegian, M. K., Marciano, E. A., and Ghahraman, V. G. (1991), "Earthquake-Induced Permanent Deformations - Probabilistic Approach", Journal of Geotechnical Engineering, ASCE, Vol.117, No.1, pp.35-50.