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Reliability-Based Design of Shallow Foundations Based on Elastic Settlement

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ABSTRACT: The shallow foundation design typically involves two-steps: (1) the ultimate bearing capacity calculation based upon general bearing capacity theories and (2) the maximum contact pressure calculation that produces an allowable total settlement. The allowable bearing capacity for the shallow foundation system is the lesser of the ultimate bearing capacity, divided by a factor of safety, and the computed maximum allowable contact pressure. Currently, these calculations utilize nominal values of soil strength and stiffness parameters and do not explicitly account for their variabilities. This paper develops a reliability-based design methodology for shallow foundations by incorporating variability in the soil parameters. The bearing soil strength and stiffness parameters are assumed to be random variables and the standard elastic settlement equations are combined with the *Monte Carlo* simulation technique to develop a series of probabilistic pressure–settlement curves. Based on an allowable magnitude of total settlement, the pressure-settlement curves are analyzed to develop probability distribution function histograms for allowable bearing capacity. The probability distribution functions can thus be utilized to determine the factor of safety against bearing capacity failure and, ultimately, develop resistance factors that can be included in Load and Resistance Factor Design (LRFD) methodologies.

1 INTRODUCTION

In practice, the design of shallow foundation systems typically follows a two-step approach. First, the ultimate bearing capacity of the soil supporting the foundation is calculated using general bearing capacity theories, such as those given by Terzaghi, Vesic, or Meyerhof (see Terzaghi 1943, Vesic 1973, Meyerhof 1963). An arbitrary factor of safety, determined by the design engineer, is applied to the calculated ultimate bearing capacity to develop the allowable bearing capacity for the shallow foundation system. Second, elastic settlement analyses are conducted to determine the maximum contact pressure that can be applied to the shallow foundation system to limit the total settlement to an allowable magnitude. This maximum contact pressure is assumed to be a second allowable bearing capacity for the shallow foundation system. The design of the shallow foundation system is thus based on the lowest allowable bearing capacity value determined from each of these approaches. In standard geotechnical practice, it is often observed that the settlement limited contact pressure controls the design of the shallow foundation system. In addition, the settlement of a shallow foundation system can be an important design consideration, especially when foundation settlements are critical to the operation of a structure.

In current design methodologies, the maximum settlement limiting contact pressure is determined utilizing nominal values of the soil strength and stiffness parameters and do not explicitly account for design uncertainties. Therefore, it is beneficial to incorporate reliability-based design principles in the elastic settlement analyses in order to rationally include design uncertainty. In the reliability-based design approach, the pertinent bearing soil parameters are assumed as random variables and the variability of the parameters is described using probability distribution functions. The standard settlement equations for shallow foundations based on elastic settlement are combined with *Monte Carlo* simulation techniques to develop a series of pressure-settlement curves for the shallow foundation. Probability distribution functions are developed for allowable bearing capacity utilizing the pressure-settlement curves and a desired allowable total settlement. The probability distribution functions can further be utilized to develop resistance factors for the design of shallow foundation systems at the service limit state based on Load and Resistance Factor Design (LRFD) methodologies.

This paper presents a reliability-based design approach for shallow foundation systems based on the calculation of settlement limiting contact pressures utilizing an elastic settlement approach. The standard elasticity equations are combined with the *Monte Carlo* simulation technique to develop a series of pressure-settlement curves. Utilizing a discrete allowable total settlement, as is common in standard geotechnical practice, the pressure-settlement curves are analyzed to develop probability distribution function histograms for allowable bearing capacity. Finally, resistance factors for use in the design of shallow foundation systems based on Load and Resistance Factor Design (LRFD) methodologies at the service limit state are developed and discussed.

2 PRESSURE VERSUS SETTLEMENT OF SHALLOW FOUNDATIONS

The pressure-settlement behavior of the bearing soil is assumed to behave as an ideal elastic-plastic material, both in the drained or undrained conditions. Figure 1 shows the pressure-settlement behavior of an ideal elastic-plastic bearing stratum, as depicted by the foundation contact stress, q_o, versus settlement, δ, curve. In Figure 1, E_s is the elastic modulus of the bearing soil, q_{ult} is the ultimate bearing capacity of the soil determined from general bearing capacity theory, and δ_{ult} is the foundation settlement at ultimate failure. Ideal elastic-plastic behavior leads to simplification of the settlement analyses. The elastic settlement of shallow foundations is described utilizing the following equation, based on the theory of elasticity and Hooke's law (see Das 2004):

$$\delta = \frac{1}{E_s} \int_0^H \left(\Delta\sigma_z(z) - \mu\Delta\sigma_x(z) - \mu\Delta\sigma_y(z) \right) dz \qquad \textbf{(1)}$$

where, H is the thickness of the bearing soil (distance from bottom of foundation to an incompressible stratum), $\Delta\sigma_x(z)$, $\Delta\sigma_y(z)$ *and* $\Delta\sigma_z(z)$ are the stress increase in the x, y, and z-directions, respectively, due to the net applied foundation loading, and μ is the Poisson's ratio of the bearing soil.

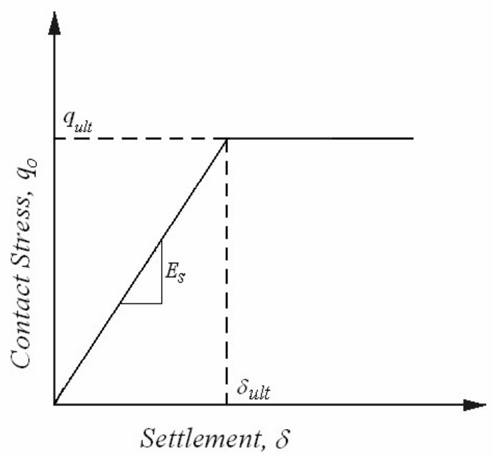

Fig 1. Bearing soil pressure-settlement relationship for ideal elastic-plastic behavior.

The ultimate bearing capacity of the soil, q_{ult}, can be determined from the following equation, based on general bearing capacity theory given by Meyerhof (1963):

$$q_{ult} = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2}\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \tag{2}$$

where, c' is the soil cohesion, q is the effective stress at the bottom of the foundation, γ is the unit weight of the bearing soil; F_{cs}, F_{qs}, and $F_{\gamma s}$ are shape factors; F_{cd}, F_{qd}, and $F_{\gamma d}$ are depth factors; F_{ci}, F_{qi}, and $F_{\gamma i}$ are load inclination factors; and N_c, N_q, and N_γ are bearing capacity factors. Because Eq. 2 was originally derived for continuous foundations (i.e. plane-strain case), the shape, depth, and load inclination factors have been empirically derived utilizing experimental data by numerous researchers. For this paper, the shape factors utilized were based on equations developed by De Beer (1970), the depth factors were based on equations developed by Hansen (1970), the load inclination factors were based on equations developed by Meyerhof (1963) and the bearing capacity factors were based on equations developed by Prandtl (1921), Reissner (1924), Caquot and Kerisel (1953), and Vesic (1973). When the foundation contact stresses on the soil exceed the ultimate bearing capacity given by Eq. 2, the bearing soil is assumed to have failed in general shear and the settlement calculated by Eq. 1 will theoretically approach infinity.

In general, the strength and stiffness parameters of the bearing soil are related to the properties of the in-situ soil strata. Soil improvement techniques that are conducted on the bearing soil prior to foundation construction are often reflected in modification of the parameters. In addition, for cohesionless materials, the strength and stiffness parameters are typically observed to increase with an increase in confining stress (i.e. increase with depth). Experimental studies conducted by Janbu (1963) indicate that the elastic modulus of the bearing soil, E_s, is proportional to the confining stress and may be expressed by the following equation:

$$E_s = K \sigma_{atm} \left(\frac{\sigma'_z K_o}{\sigma_{atm}} \right)^x \tag{3}$$

where, x is a exponent determining the rate of variation of E_s with respect to $\sigma'_z K_o$ (i.e. σ_3) and K is a modulus number. Parameters x and K are constants and the horizontal earth pressure is normalized with respect to atmospheric pressure, σ_{atm}, expressed in the same pressure units as E_s. The value of the unknown parameters, x and K, are determined experimentally from the results of drained triaxial tests conducted under various confining pressures by plotting the values of E_s against $\sigma'_z K_o$ on a log-log scale and fitting a straight line to the data (see Duncan and Chang 1970, Duncan et al. 1980). The value of K is thus the y-intercept of the line and the value of x is the slope of the line. Lambe and Whitman (1969) report that the value of x can vary from 0.4 to 1.0, with the larger values applying to loose sands, but that a reasonable value can be taken as 0.5. Duncan and Chang (1970) performed a series of drained triaxial tests on silica sand under varying confining pressures and reported values for x and K equal to 0.65 and 295, respectively, for a loose sand and 0.54 and 2000, respectively, for a dense sand.

For simplicity of analysis that can benefit engineers in shallow foundation design, a homogeneous bearing layer is often assumed by utilizing an average magnitude of the strength and stiffness parameters. Although this approach does not account for vertical fluctuation of the parameters, it is consistent with the standard geotechnical practice. The advantage of this assumption is that constant values of the strength and stiffness parameters may be used as representative of the bearing soil at a given site. In this paper, reliability-based design solutions are presented assuming both a homogeneous bearing layer and a bearing layer in which the elastic modulus, E_s, increases with confining stress based on Eq. 3.

3 VARIABILITY OF STRENGTH AND STIFFNESS PARAMETERS

In typical geotechnical design, nominal values of the strength and stiffness parameters are utilized without quantification of uncertainties that exist within the soil itself and the design process. These uncertainties are often known to exist and result from the inherent variability of the soil, testing and measurement errors, and transformation fluctuations from developed correlations (Kulhawy and

Phoon 1996). Traditional settlement analyses methodologies do not incorporate a consistent approach to account for these uncertainties and conservative estimation of the strength and stiffness parameters, along with a global factor of safety, is often the only manner to ensure some level of safety in the design. By integrating a reliability-based design process, the magnitude of the uncertainty within the elastic model parameters can be quantified and systematically accounted for in the overall analysis.

In recent years, a reliability-based design process called Load and Resistance Factor Design (LRFD) has been increasingly advocated for the design of foundations (AASHTO 2004). In the LRFD method, the foundation may be evaluated for a number of load combinations, including those for normal strength and those for extreme-event loading requirements. Service limit state requirements (i.e. settlement limiting requirements) are also evaluated. The rationale behind the LRFD method is the replacement of the traditional global factor of safety with a probabilistically developed parameter called a resistance factor, ϕ_R. Because the resistance factor is developed using probabilistic techniques, uncertainty in the model parameters is rationally incorporated in the overall design process.

Several recent papers have examined the probabilistic distribution of settlement and ultimate bearing capacity of shallow foundations by modeling the bearing soil as a spatially varying medium (Fenton and Griffiths 2002, 2003, 2005, Nobahar and Popescu 2001). The pertinent strength and stiffness parameters of the bearing soil, along with the correlation distance between the parameters, are randomly varied in a finite element analysis to develop probabilistic distribution functions for settlement and/or ultimate bearing capacity. The results of these analyses demonstrate how uncertainties in the model parameters can ultimately affect the design of shallow foundation systems and further justify the importance of developing reliability-based design methodologies for use by geotechnical practitioners.

As mentioned previously, the development of the stress-settlement curve given in Figure 1 involves utilizing Eq. 1 and Eq. 2. To that end, the bearing soil parameters that must be considered as possessing uncertainty in the settlement analysis include the elastic modulus, E_s, the Poisson ratio, μ, the drained friction angle, ϕ', and the unit weight, γ. To account for these probable uncertainties, the bearing soil parameters must be assumed as random variables and described statistically using probability distribution functions based on a mean and standard deviation (or coefficient of variation, COV). Since the bearing soil parameters are always non-negative, the use of non-Gaussian probability distribution functions, such as the lognormal or beta distribution, are necessary. The lognormal probability distribution has been widely utilized to describe the uncertainties in a number of probabilistic analyses in geotechnical engineering, including those conducted by Duncan (2000), Nobahar and Popescu (2001), and Fenton and Griffiths (2002), and is thus utilized in this paper.

Typically, the statistics of the bearing soil parameters can be established at a site by back-calculating the bearing soil parameters and the quantifying the magnitude of uncertainty within the parameters using actual pressure-settlement data (see Misra and Roberts 2006, Misra et al. 2007). For shallow foundations, pressure-settlement data is rarely obtained, and instead, the bearing soil parameters are typically quantified using site exploration techniques such as CPT and SPT. In this paper, it was determined that the magnitude of the mean of the bearing soil parameters be based on typical values for a medium to medium-dense sand and that the uncertainty in the parameters be based on research conducted by others using various site exploration techniques. Therefore, the mean of the elastic modulus, E_s, was assumed to be 30 MPa (refer to Das 2004) and the mean of the drained friction angle, ϕ', was assumed as 32°. Because the Poisson's ratio does not vary significantly in granular soils and is relatively difficult to assess, it was not considered as a random variable and was assumed as 0.30. The unit weight of the soil was also not considered as a random variable due to its relatively low COV (<10%) as reported in Phoon and Kulhawy (1999) and Foye et al. (2006a) and was therefore assumed as 18 kN/m^3.

For the probabilistic analysis, the magnitude of uncertainty in the bearing soil parameters, E_s and ϕ, was based on approximate guidelines for design soil variability given by numerous researchers. The range of the COV for the drained friction angle, ϕ', was reported to be between 2% and 13% by Harr (1996) and Kulhawy (1992) and therefore, a value of 5% was assumed for this paper. This resulted in a randomly generated drained friction angle ranging from 27° to 38°, which appeared to be fairly reasonable for medium to medium-dense sand. The range of the COV for the elastic modulus, E_s, was reported to be much higher and was given to be between 20% and 70% (Phoon and Kulhawy 1999). For this study, however, the COV for the elastic modulus was assumed as 30%. It should be noted that a COV of 30% signifies that a random variable can assume values 90%-100% lower or higher than the mean value (Lacasse and Nadim 1996). This range can be quite large, producing values of the bearing soil elastic modulus that may not be representative of the magnitude that the elastic modulus may vary for the given soil type. Therefore, a COV of 30% may not only account for inherent variability with the bearing soil, but may also incorporate other sources of variability, such as measurement errors.

Probabilistic analyses were conducted assuming that the elastic modulus was both constant and increasing with depth. In the case where the elastic modulus of the bearing soil was assumed to increase with depth, the elastic modulus was taken to be equal to 30 MPa at a depth below the bottom of the foundation equal to one-half the foundation width, *B/2*. At this depth, the vertical stress in the bearing soil is approximately 50% of the foundation stress (see Corps of Engineers 1990). Therefore, using Eq. 3 and assuming constant values for K_o and x equal to 0.5 and 0.5, respectively, along with the depth and elastic modulus values as described above, the mean value for the modulus number, K, was calculated to be approximately 800. The values for the elastic modulus were thus randomly generated by a *Mathcad* spreadsheet assuming a COV of 30% for the modulus number in order to describe the parameter behavior with relation to the confining pressure (see Misra et al. 2007). This resulted in a randomly generated modulus number ranging from approximately 280 to approximately 2100. These values appear to correspond well with the magnitudes given by Duncan et al. (1980) for poorly graded sands, sandy gravels and sandy silts.

Vertical spatial correlation in the bearing soil parameters was not directly examined in this paper. Research conducted by Fenton and Griffiths (2003) demonstrated that the vertical correlation length of the random fields of the bearing soil parameters did not significantly affect the mean of the performance function when the COV of the parameters was assumed as 30%. This was especially true when the bearing soil parameters were assumed to be uncorrelated (statistically independent), as they are in this paper. Vertical spatial correlation appears to affect the probabilistic results only when the COV of the bearing soil parameters becomes greater than 100% (see Fenton and Griffiths 2003), which is significantly greater than COV values generally reported in the literature (see, e.g., Phoon and Kulhawy 1999). Therefore, the statistics of the bearing soil parameters in this paper are assumed to account only for horizontal fluctuation in the parameters on any given site.

4 PRESSURE-SETTLEMENT ANALYSIS USING *MONTE CARLO* SIMULATION

There are many methods for conducting a probabilistic analysis to determine the statistics of the performance function must be determined. Typical methods to calculate the statistics of the performance function are the first-order, second moment method (FOSM), the point estimate method, and *Monte Carlo* simulation. For reliability-based design applications in geotechnical engineering, the *Monte Carlo* simulation method appears to be widely utilized (see Fenton and Griffiths 2002, Fenton and Griffiths 2003, Chalermyanont and Benson 2004). In addition, because closed-form probabilistic solutions were not possible to obtain from the elastic settlement equation, only the *Monte Carlo* simulation technique is feasible and was thus utilized in this paper. The *Monte Carlo* simulation method is a "brute-force" method in which the general approach is to randomly generate a large set of values for the bearing soil parameters based on the statistics of the parameters (Fishman 1995). The values are then substituted into the pressure-settlement equation given previously, resulting in a large number of pressure-settlement curves. The number of simulations conducted is based on achieving a particular level of reliability (Fishman 1995, Harr 1996). For this paper, 5,000 *Monte Carlo* simulations were conducted. For two random variables in a system, this number of simulations equates to confidence level of 90%, with a maximum allowable system error of 10% (see Harr 1996).

Initial *Monte Carlo* simulations were conducted assuming the following magnitudes for the foundation geometry: L = 3m, B = 3m, and D_f (depth to bottom of foundation below grade) = 1m. Because this paper is specifically focused on the service limit state design of shallow foundations, the thickness of the bearing soil stratum, H, was assumed to be equal to $7B$. At this depth, the vertical stresses in the bearing soil due to the foundation loading are generally less than 1% of the applied foundation stress and therefore, the stresses do not significantly affect the settlement magnitude of the foundation at this depth or greater (see Army Corps of Engineers 1990).

5 ALLOWABLE BEARING CAPACITY PROBABILITY DISTRIBUTION

Settlement criteria are seldom given in terms of probability distributions or specified in terms equivalent to load factors. Although some work has recently been reported in this area (see, e.g., Zhang and Ng 2005), allowable settlement criteria are typically given as discrete magnitudes in standard design applications. While a single allowable settlement value that encompasses all types of structures or all design situations can be impossible to define, the magnitude of allowable settlements for various structures has been discussed in the literature (Skempton and MacDonald 1956, Burland and Wroth 1974, Barker et al. 1991, Zhang and Ng 2005) and must be based on the experience of the designer and on observations with similar structures. Therefore, serviceability design of foundations cannot be performed in a manner analogous to that for strength, with the probability distribution for estimated settlement of the foundation being compared with the probability distribution for tolerable movement of the structure. It is therefore useful to determine the bearing capacity histogram at which the settlement of the foundation is equal to an allowable settlement (see Misra and Roberts 2005, 2006).

It is possible to determine the pressure from each pressure-settlement curve that corresponds to the allowable total settlement once the generation of the pressure-settlement curves has been completed. These pressures can be described as the bearing capacity of the shallow foundation corresponding to the allowable total settlement or the nominal service limit state bearing capacity. For example, assuming an allowable total settlement of 25mm, which is relatively standard in general geotechnical design of shallow foundations, the nominal service limit state bearing capacities can be presented as a cumulative distribution function histogram as shown in Figure 2. Figure 2 presents a bearing capacity cumulative distribution function histogram assuming the stiffness of the bearing soil parameters is constant with depth (solid line) and increasing linearly with depth (dashed line). The pertinent statistics of each bearing capacity cumulative distribution function histogram have been reported in Table 1. As observed from the pertinent statistics, the COV of the service limit state bearing capacity for the shallow foundation is approximately 30% assuming either a constant modulus of elasticity or linearly varying modulus of elasticity of the bearing soil.

Table 1. Pertinent statistics of the shallow foundation bearing capacity at an allowable total settlement.

	Load Capacity (kPa)		
Elastic Modulus	**μ**	**σ**	**COV**
E_s constant with depth	288	85	0.295
E_s linearly increasing with depth	271	80	0.295

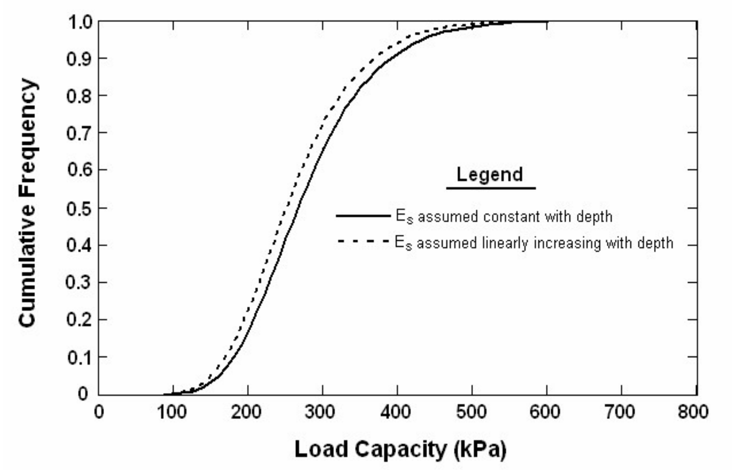

Fig 2. Cumulative distribution function histograms for bearing capacity corresponding to an allowable total settlement on the shallow foundation system.

The standard design process requires the engineer to calculate the factored load on the foundation system given the dead and live loads. Load factors are applied to each load to account for load uncertainties (AASHTO 2004). Therefore, once the factored load on the shallow foundation system is determined, along with the foundation bearing pressure, Figure 2 can be used to calculate the probability of service limit state failure corresponding to the factored contact pressure. The probability of failure at the service limit state is generally desired to be less than 0.5%, which has been reported in the literature (see Phoon et al. 1995, Misra and Roberts 2006).

A family of cumulative distribution function histograms can be generated for various shallow foundation sizes to assist the design engineer in determining the most efficient shallow foundation for a given factored load. In Figure 3, a series of cumulative distribution function histograms have been developed for the bearing capacity at an allowable total settlement of 25mm. The curves are based on the results of *Monte Carlo* simulations conducted for various shallow foundation sizes (all square foundation geometries). In all the simulations, the elastic modulus was assumed constant with depth and the mean values of the bearing soil stiffness and strength parameters were assumed as previously given. The bearing pressure on each shallow foundation is calculated based on the factored load and appropriate cross-sectional area of the foundation system and is then utilized with Figure 3 to determine the corresponding probability of service limit state failure.

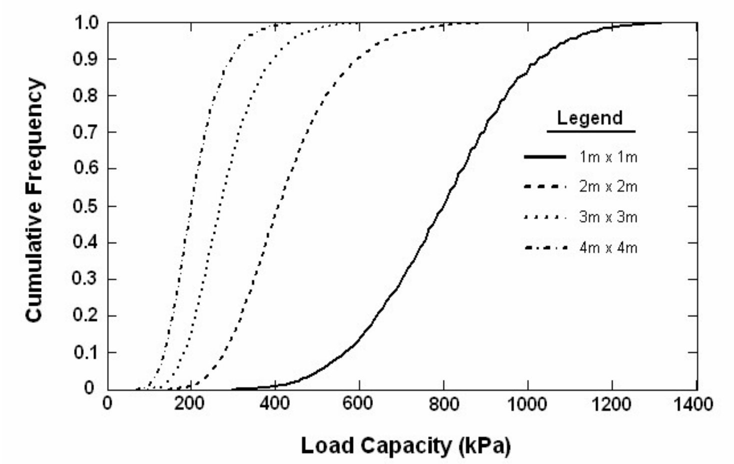

Fig 3. Cumulative distribution functions for bearing capacity of various shallow foundation sizes corresponding to an allowable total settlement.

Due to the difficulty with utilizing Figure 3 at small magnitudes of cumulative frequency, the probability of service limit state failure can be computed directly using standard statistical methods based on the factored bearing pressure and the pertinent statistics of the cumulative distribution function histograms. For example, assuming a factored load of 1000kN and assuming that the cumulative distribution function histograms follow the lognormal distribution, the probability of service limit state failure corresponding to an allowable total settlement of 25mm can be computed based on the subsequent bearing pressure on each foundation using standard statistical tables. The results of these calculations are given in Table 2. It is observed from Table 2 that for an allowable total settlement of 25mm, the 3m x 3m shallow foundation is the most efficient for the given factored bearing pressure, assuming that the target probability of failure is taken as 0.5%. Table 2 also provides service limit state failure probabilities for different magnitudes of allowable total settlement assuming a factored load of 1000kN and using the same *Monte Carlo* simulation load-settlement curves used to develop Figure 3. Therefore, using the *Monte Carlo* simulation technique, it is possible for a design engineer to develop a series of tables for the probability of service limit state failure corresponding to an infinite combination of foundation sizes, factored loads, bearing pressures, and allowable total settlement magnitudes. These tables can ultimately assist the engineer in choosing the most efficient and safe shallow foundation system for the given design, while assuring consistency from one shallow foundation system design to the next.

Table 2. Probability of service limit state failure for various foundation sizes and allowable total settlement magnitudes assuming a factored load of 1000kN.

	Allowable Total Settlement		
Foundation Size	**12.5 mm**	**25 mm**	**50 mm**
1m x 1m	100%	87%	48%
2m x 2m	76%	4%	0.0001%
3m x 3m	24%	0.1%	0.000001%
4m x 4m	5%	0.002%	≈ 0%

6 RESISTANCE FACTORS FOR SERVICE LIMIT STATE DESIGN

It is economically desirable in design to achieve a consistent probability of service limit state failure from one shallow foundation system to the next. This ensures that the probability of service limit state failure is always known and is consistent with the designer's intentions and the operation of the system. However, the presented probabilistic computation method can be cumbersome and time consuming for everyday shallow foundation design. In addition, the probabilistic analyses require that numerous *Monte Carlo* simulations be performed in order to design the most efficient shallow foundation system. The *Monte Carlo* simulations require a relatively high speed computer and the designer must possess some knowledge of statistics. It is therefore desirable to develop a reliability-based design method that does not require conducting an in-depth probabilistic analysis for every design. For routine designs, a simplified approach using a probabilistically computed parameter called a resistance factor, ϕ_R, can be utilized. Resistance factors are applied to the nominal bearing capacity of the shallow foundation to account for uncertainties in the design process that would normally be represented by a lumped global factor of safety.

The use of resistance factors in design has many advantages. First, most engineers are familiar with the use of resistance factors and can directly measure how a global factor of safety for the shallow foundation system is accounted for by the resistance factor. Second, by using resistance factors, the engineer does not need to perform the complicated probability computations for each design. Finally, the resistance factors may be calibrated to produce designs that consistently achieve a desired level of reliability (Phoon et al. 1995).

Calculating resistance factors requires that the probabilistic computation method presented previously be conducted one time in order to determine the pertinent statistics of the bearing capacity histogram developed using an allowable total settlement. It is also necessary to define the desired probability of service limit state failure of the system in terms of a target reliability index, β_T. The target reliability index is an alternative way to express the probability of failure on a more convenient scale because the probability of service limit state failure is typically very small (Phoon and Kulhawy 1996).

Figure 4 provides the results of the probabilistic simulations in terms of the service limit state resistance factor for an allowable total settlement equal to 25mm. The resistance factor calculations are presented with respect to various COV magnitudes of the elastic modulus, along with various assumed levels of bias of the bearing capacity. Based on a site characterization investigation conducted by the design engineer using standard geotechnical exploration and testing techniques, the level of site variability can be assessed. The level of site variability can be characterized ranging from "low" to "high", based on recommendations by Paikowsky et al. (2004) using both engineering judgment and observed subsurface conditions. Once the site variability has been assessed, the appropriate resistance factor for use in service limit state design of the shallow foundation system is selected using Figure 4. As observed from Figure 4, the bias of the bearing capacity does not significantly affect the magnitude of the resistance factor that should be used in design, especially when the values of the resistance factors are rounded to the nearest 0.05, which is consistent with standard practice (Baecher and Christian 2003).

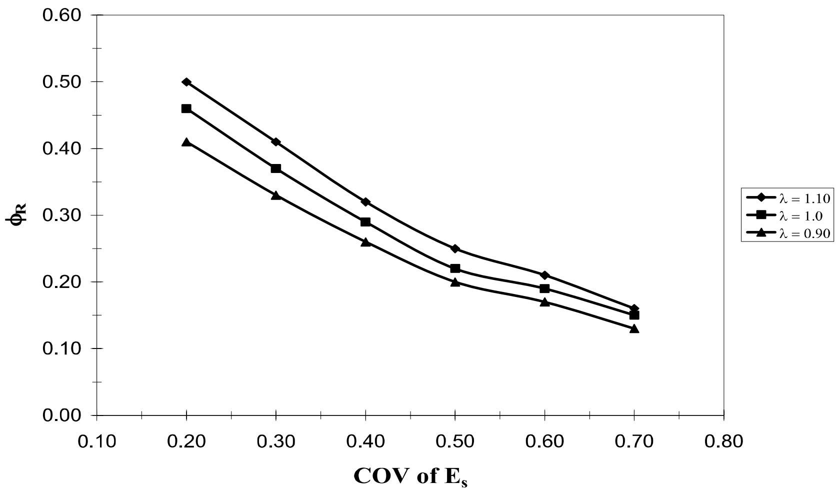

Fig 4. Service limit state resistance factors assuming an allowable total settlement of 25mm for various COV magnitudes of the elastic modulus and various levels of bias of the bearing capacity.

It is important to observe in Figure 4 that the magnitude of the resistance factor decreases as the COV of the elastic modulus increases. This is expected as the global factor of safety of the shallow foundation design must increase with an increase in site variability. This is also different from standard reliability-based design approaches utilized in structural engineering where a single "global" resistance factor is often specified. This provides strong evidence that resistance factors for use in geotechnical design must be developed for various levels of uncertainty and should be based on the characteristics of the bearing soil, foundation construction methods, and design approach.

It is noted that a study conducted by Foye et al. (2006b) determined the magnitude of resistance factors for use in shallow foundation design at the ultimate limit state. The resistance factors were developed based on general bearing capacity theory with the pertinent bearing soil properties characterized using either CPT or SPT data. The load factors used by Foye et al. (2006b) in the development of the resistance factors corresponded to the AASHTO load factors for the ultimate limit state. The resistance factor calculations conducted by Foye et al. (2006b) did not consider any settlement criterion. Although calculated using service limit state load factors (i.e. load factors equal to unity), it is interesting to note that the resistance factors reported herein compare favorably to those reported in Foye et al. (2006b). In general, assuming a bias of the bearing capacity equal to unity, the resistance factors reported in this paper for service limit state design varied from 0.46 to 0.15, while the resistance factors reported by Foye et al. (2006b) for the design of rectangular shallow foundations on sand at the ultimate limit state varied from 0.60 to 0.40 and 0.45 to 0.30 based on CPT and SPT data, respectively, at a target reliability index of 2.5.

7 CONCLUSIONS

The design of shallow foundations involves a two-step process in which the nominal bearing capacity of the foundation is calculated based on ultimate strength methods and allowable settlement criteria. It is often observed that the nominal bearing capacity is controlled by the allowable settlement criteria. Due to variability in the strength and stiffness parameters of the bearing soil, it is beneficial to develop a reliability-based design methodology for shallow foundations. In this paper, a methodology for the reliability-based design of shallow foundations based on an allowable total settlement magnitude has been presented. Using a standard elastic settlement approach, the strength and stiffness parameters of the bearing soil were assumed as random variables and a probabilistic analysis was conducted using the *Monte Carlo* simulation technique. Separate simulations were conducted assuming the elastic modulus of the bearing soil was constant with depth and increased linearly with depth. A series of pressure-settlement curves were developed and analyzed to determine the contact pressures from each curve that corresponded to a desired allowable total settlement. These contact pressures were assumed to be equal to the service limit state bearing capacity of the shallow foundation and were presented in the form of a cumulative distribution function histogram. It was observed that the cumulative distribution function histograms did not vary significantly based on the assumption of a constant elastic modulus with depth or a linearly increasing elastic modulus.

It was further demonstrated that a series of cumulative distribution function histograms for service limit state bearing capacity at an allowable total settlement can be developed for different square foundation sizes. This would allow the engineer to optimize the shallow foundation design by examining the probability of service limit state failure for each foundation at the given factored load and calculated contact pressure. By assuming a target probability of failure equal to 0.5% (i.e. a target reliability index, β_T, of 2.6), as is typical in service limit state design, the engineer can select the most appropriate shallow foundation system. This procedure will ensure that both safety and efficiency are achieved in the shallow foundation design process.

Although the method of conducting the *Monte Carlo* simulations and developing the probability histograms for service limit state bearing capacity are relatively straightforward, the process can become cumbersome and time consuming for standard day-to-day design. Therefore, the development of resistance factors that can be used for the design of shallow foundation systems in lieu of the cumulative distribution function histograms appear to be very beneficial as demonstrated herein.

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