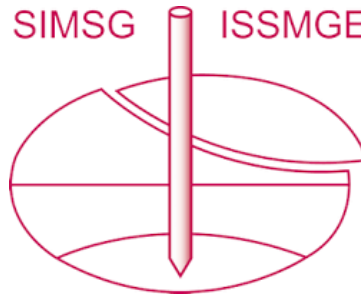


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## **Seismic reliability analysis of underground pipelines based on probability density evolution method**

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**ABSTRACT:** In this paper, a new approach for the dynamic reliability assessment of finite element modelled underground pipeline structures under earthquake excitations with uncertain parameters is proposed. The approach is established based on the thoughts of the newly developed probability density evolution method, which is capable of capturing the instantaneous probability density function (PDF) and its evolution of the responses of nonlinear stochastic structures. Associated with the physics-based stochastic earthquake model and the finite element method, the instantaneous probability density and the evolution process for the seismic response of underground pipeline can be studied effectively. The first-passage reliability problem is investigated from the level-crossing process to analyze the reliability of underground pipelines under seismic excitations. Using the proposed method, numerical examples are investigated considering stochastic input and soil parameters simultaneously. The examples show that the proposed method is of high application in analyzing the reliability of underground pipeline.

### **1 INTRODUCTION**

The seismic reliability analysis of underground pipelines is of paramount importance in lifeline system engineering and has for a long time been widely studied, coming up with a variety of theoretical and numerical methods applicable to engineering practice (Machida and Yoshimura, 2002; Nedjara et al, 2007). However, on account of the calculation limitation in previous studies, the analysis of seismic reliability considering simultaneously the randomness caused by the seismic input and the structural parameters of soil-pipeline system is still an unsolved problem.

The dynamic reliability of stochastic structures was assessed usually by the Monte Carlo simulation, the level-crossing theory through the Rice formula or by the diffusion theory through the Kolmogorov equation (Crandall, 1970). However, for the level-crossing theory, the joint probability density functions of the response and its velocity required in the Rice formula are usually unavailable and assumptions may lead to deviation. Although the diffusion process theory may give more accurate reliability, the application in a practical nonlinear and MDOF system is unfeasible because of the difficulty of solution (Spencer and Elishakoff, 1988). In recent years, a newly developed probability density evolution method for stochastic structures has been proposed. The instantaneous probability density function and the evolution against the time can be obtained precisely (Li and Chen, 2005). For the dynamic reliability problem, the solution can be derived through solving the probability density evolution equation with an initial value condition and an absorbing boundary condition corresponding to a specified failure criterion.

On the other hand, different from the models from the power spectral density function, a new physics-based stochastic earthquake model to reflect the fundamental correlation between the critical factors and the random earthquake motions has been proposed by the authors (Li and Ai, 2006). Several key random parameters are based on to realize the purpose, and a relational expression with physical background can be constructed considering the propagation process of earthquake motion in an engineering site. Using a random Fourier function with adherent probability as the modelling form, the proposed model is hoped to reflect the intrinsic relationship between the random seismic motion and the critical parameters. Based on actual seismic records, different equivalent models of random earthquake motion for different engineering sites are constructed, which correspond to the Chinese Code for Seismic Design of Buildings (GB50011-2001). In addition, according to the conditional probability and the

seismic risk analysis, a developed presentation defined with the conditional random earthquake motion function is proposed. Associated with the probability density evolution method, the proposed random earthquake model can be of comparative advance to provide the basis for seismic reliability analysis.

In this paper, based on the probability density evolution method and the physics-based stochastic earthquake model, a new approach for the dynamic reliability assessment of the finite element modelled underground pipeline structures under random earthquake excitations and uncertain soil parameters is proposed and numerical examples are investigated.

## 2 PROBABILITY DENSITY EVOLUTION METHOD FOR STOCHASTIC STRUCTURES

### 2.1 Principle of Preservation of Probability

The principle of probability conservation can be described as, during a conservative probability transformation process, within the state space, the increment of the probability within a unit volume equals to the inflow probability that gets across this unit.

If the ordinary differential of a state vector  $Y$  can be expressed as,

$$\dot{Y} = G(Y, t) \quad (1)$$

where,  $Y = (y_1, y_2, \dots, y_n)^T$ ,  $G = (g_1, g_2, \dots, g_n)^T$ .

If  $p_Y(y, t)$  is supposed to be the probability density of  $Y(t)$ , based on the principle of probability conservation, the probability density evolution equation can be expressed as followed,

$$\frac{\partial}{\partial t} p_Y(y, t) + \sum_{j=1}^n \frac{\partial}{\partial y_j} [p_Y(y, t) g_j(y, t)] = 0 \quad (2)$$

### 2.2 Probability Density Evolution Equation of the Seismic Response of Stochastic Structures

The dynamic equation of the nonlinear structure can be written as,

$$M(\zeta)\ddot{U} + C(\zeta)\dot{U} + f(\zeta, U) = -M(\zeta)\ddot{U}_g \quad (3)$$

where,  $\zeta$  is the random parameter vector which represents the physical character of a stochastic structure, and its joint probability density function is  $p_\zeta(\mathbf{x})$ ;  $M, C$  are stochastic mass and damping matrices respectively, which include random parameters with the rank of  $n \times n$ ,  $n$  is the dynamic freedom degree;  $U, \dot{U}, \ddot{U}$  are the displacement, the velocity, and the acceleration vectors, respectively;  $f(\zeta, U)$  is the nonlinear restoring force vector; and  $\ddot{U}_g$  is the acceleration vector of input earthquake.

Let a response vector  $X = (U^T, \dot{U}^T)^T$ , the dynamic equation is then changed into a format of state equation including random parameters,

$$X = A(X, \zeta, t) \quad (4)$$

$$A = \left\{ \begin{array}{c} \ddot{U}_g \\ -M^{-1}C\dot{U} - M^{-1}f - \ddot{U}_g \end{array} \right\} \quad (5)$$

If  $\dot{X} = \frac{\partial}{\partial t} X(\zeta, t) = G(\zeta, t)$ , based on the above probability density evolution equation, the joint probability density function of  $X$  and  $\zeta$  will satisfy the following equation,

$$\frac{\partial}{\partial t} p_{X\zeta}(x, x_\zeta, t) + \sum_{i=1}^{2n} \frac{\partial}{\partial x_i} [p_{X\zeta}(x, x_\zeta, t) g_i(x, x_\zeta, t)] = 0 \quad (6)$$

Since the velocity component  $\dot{X}_i$  is only the function of variable  $\zeta$ , the above equation can be re-written as

$$\frac{\partial p_{X\zeta}(x, x_\zeta, t)}{\partial t} + \sum_{i=1}^{2n} \dot{X}_i(x_\zeta, t) \frac{\partial p_{X\zeta}(x, x_\zeta, t)}{\partial x_i} = 0 \quad (7)$$

Having integral at both sides with  $x_1, \dots, x_{l-1}, x_{l+1}, \dots, x_{2n}$ , the probability density evolution equation will be decoupled,

$$\frac{\partial p_{X_l \zeta}(x_l, \mathbf{x}_\zeta, t)}{\partial t} + \dot{X}_l(\mathbf{x}_\zeta, t) \frac{\partial p_{X_l \zeta}(x_l, \mathbf{x}_\zeta, t)}{\partial x_l} = 0 \quad (8)$$

where  $p_{X_l \zeta}(x_l, \mathbf{x}_\zeta, t) = \int p_Y(x, t) dy_1 \cdots dy_{l-1} dy_{l+1} \cdots dy_{2n}$ , is the joint probability density function of  $(X_l, \zeta^T)^T$ .

When the initial displacement and the velocity are independent of the physical parameters of the structure, the corresponding initial condition is described as,

$$p_{X_l \zeta}(x_l, \mathbf{x}_\zeta, t)|_{t=0} = \delta(x_l - X_{l,0}) p_\zeta(\mathbf{x}_\zeta) \quad (9)$$

where,  $X_{l,0}$  is the determinate initial value of  $X_l$ , for a initial static structure, there is  $X_{l,0} = 0$ ;  $\delta(\cdot)$  is the Dirac function; and  $p_\zeta(\mathbf{x}_\zeta)$  is the joint probability density function of the random vector  $\zeta$ .

Solving the above partial differential equation with initial-boundary-values, which include the decoupled probability density evolution equation and the initial condition, the joint probability density function  $p_{X_l \zeta}(x_l, \mathbf{x}_\zeta, t)$  can be calculated. After the integral with  $x_\zeta$ , the probability density function of  $X_l(t)$  can also be solved,

$$p_{X_l}(x_l, t) = \int p_{X_l \zeta}(x_l, \mathbf{x}_\zeta, t) d\mathbf{x}_\zeta \quad (10)$$

The probability density evolution method of the stochastic structures is an effective approach to obtain the instantaneous probability density function and its evolution for the stochastic response.

### 2.3 Computational algorithm of Probability Density Evolution Equation

Step 1: Disperse the value region that corresponds to  $\zeta$ , and disperse the initial condition simultaneously;

Step 2: After dispersing, for each determinate value of variable  $\zeta$ , a determinate dynamic analysis is carried out and the corresponding differential item with time of the target response  $\dot{X}_l(x_\zeta, t)$  can be deduced;

Step 3: Using the method of finite difference to solve the probability density evolution equation, then the joint probability density function  $p_{X_l \zeta}(x_l, \mathbf{x}_\zeta, t)$  and the probability density function of the target response can be calculated.

## 3 SEISMIC RELIABILITY ANALYSIS OF UNDERGROUND PIPELINES

### 3.1 Seismic Reliability Analysis method

The level-crossing process theory is one of the most widely-used theories for the dynamic reliability assessment. In this paper, the seismic reliability problem is investigated by imposing the failure criterion of the first passage problem reliability theory on the probability density evolution equation.

In the time interval  $[0, T]$ , the dynamic reliability  $y$  of the stochastic structures is,

$$R(t) = \Pr\{X(\tau) \in \Omega_s, \tau \in [0, T]\} \quad (11)$$

where  $\Pr\{\cdot\}$  denotes the probability of the random events;  $\Omega_s$  is the safe domain.

Eq.11 indicates that in  $[0, T]$ , only the random events that in the safe domain are reliable events. That means, as long as the random event has crossed the boundary of the safe domain, this event should be identified as a failure event. Since the probability of the random event that has crossed the boundary of the safe domain no longer contributes to the structural reliability, this random event cannot return to the safe domain (Li and Chen, 2005).

Therefore, based on the probability density evolution method through the level-crossing reliability theory, an absorbing boundary condition corresponding to the failure criterion of the first passage problem is imposed on the probability density evolution equation,

$$p_{X_l\zeta}(x_l, \mathbf{x}_\zeta, t) = 0 \quad , \quad x \in \Omega_f \quad (12)$$

where  $\Omega_f$  is the failure domain, and  $\Omega_f \cap \Omega_s = \emptyset$ ,  $\Omega_f \cup \Omega_s = \Omega$ ,  $\Omega$  is the state space of  $X$ .

Using the numerical algorithm to solve the initial-boundary-value partial differential equation problems with the decoupled probability density evolution equation will give the remaining probability density function of the response  $p^*_{X_l\zeta}(x_l, \mathbf{x}_\zeta, t)$ ,

$$p^*_{X_l}(x_l, t) = \int p^*_{X_l\zeta}(x_l, \mathbf{x}_\zeta, t) d\mathbf{x}_\zeta \quad (13)$$

The dynamic reliability of the stochastic structures based on the failure criterion of the first passage can then be assessed through integrating the remaining probability density function over the safe domain,

$$R(t) = \int_{\Omega_s} p^*_{X_l}(x_l, t) dx_l \quad (14)$$

In the case of the symmetric double boundary, the absorbing boundary is,

$$p_{X_l\zeta}(x_l, \mathbf{x}_\zeta, t) = 0 \quad , \quad x_{B1} < x < x_{B2} \quad (15)$$

where  $x_{B1}, x_{B2}$  denote the limited upper and lower boundaries of the response.

Then the reliability reads,

$$\begin{aligned} R(t) &= P_r \{x_{B1} < X_l(\tau) < x_{B2}, \tau \in [0, T]\} \\ &= \int_{x_{B1}}^{x_{B2}} p^*_{X_l}(x_l, t) dx_l \end{aligned} \quad (16)$$

### 3.2 Computational Example

The site is a 50×10m uniform saturated sandy soil space and the ground water is set at the ground surface. And the pipe made of cast iron is buried at the depth of 1m with the diameter is 0.4m.

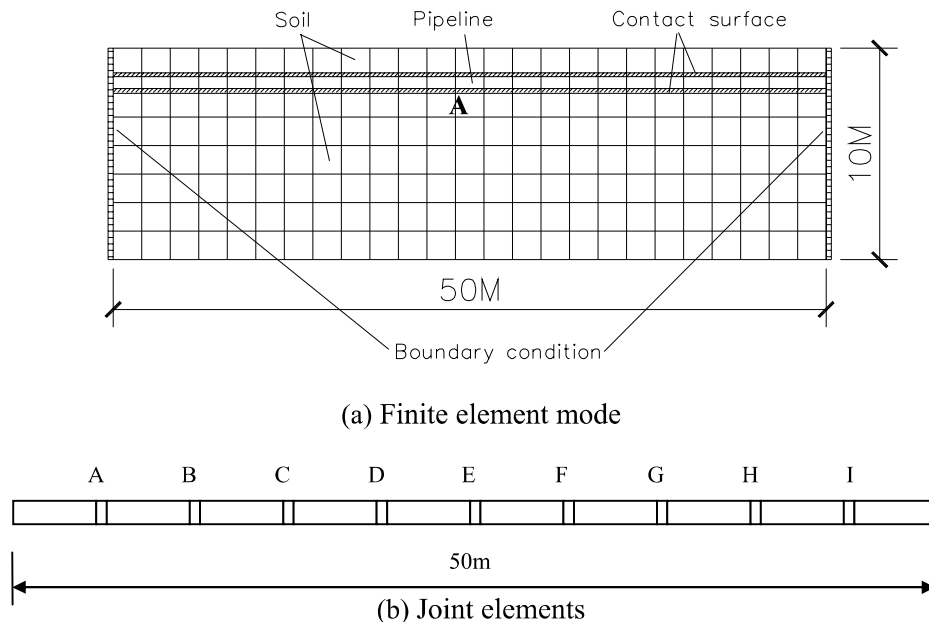
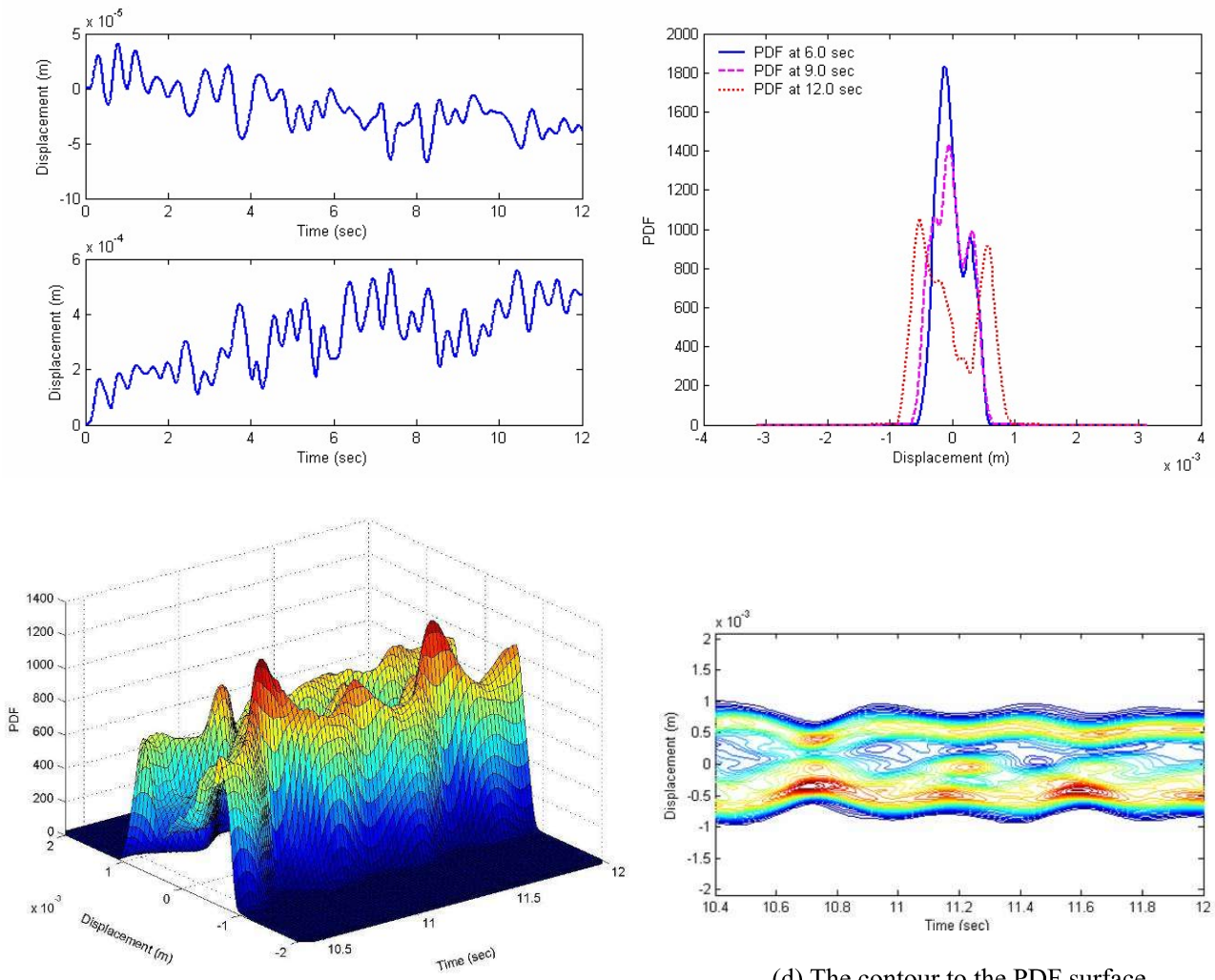


Fig.1 The Structural Mode



(d) The contour to the PDF surface

Fig.2 The PDF of the response for joint E

Table 1 Reliability of Joint E  $P_s$

Boundary (mm)	0.42	3.0
Reliability	0.0605	1

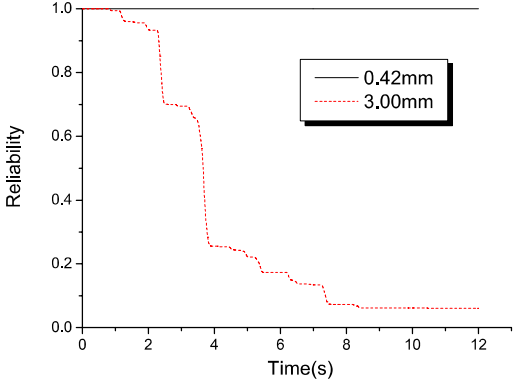


Fig.3 discretionary seismic reliability of joint E

Fig.1(a) shows the finite element mode of the computational example. The structure is subjected to earthquake excitation in the shape of the conditional random earthquake function which is defined subsequently. The earthquake is inputted from the bottom horizontally with the propagation velocity 250m/s and the duration is 20s. The junctions are assumed flexible and joint elements are used to simulate the pipeline junctions shown in Fig.1(b). The joint elements adopt the same form with the pipeline element, while different Young's module and density are defined.

For the deterministic analysis of the seismic response of buried pipelines, the finite element method is adopted to study the seismic response of buried pipelines and the surrounding soil. The soil that surrounds the pipeline is regarded as a solid-liquid two-phase medium. The effective stress method and nonlinear constitutive model of soil are used to study the increase and the dissipation of pore water pressure during the seismic process. At the same time, the contact interface between the pipeline and the surrounding soil is also included. Detailed techniques and the dynamic parameters of the sandy soil and the contact surface can be referred to the related paper of the authors (Ai & Li, 2004).

In this paper, the random inputs and the uncertain soil parameter are both considered and defined according to the reference (Li and Ai, 2006): associated with the seismic risk analysis, in the future definite time range, the conditional random earthquake function is defined as the case of the transcendental probability of the earthquake motion being  $p = 10\%$  and the peak value of the seismic acceleration is  $0.1g$ ; The internal friction angle  $\phi$ , which is defined as the random soil parameter, has a logarithmic normal distribution with the mean  $15^\circ$  and the standard deviation 0.30.

### 3.3 Seismic Reliability Analysis of underground pipeline

Since the material of the pipeline junctions is assumed to be flexible, the Young's modulus and density are defined to be 0.1 of the pipeline's value. The failure mode for the flexible junction is the deformation failure. The target response value is the deformation of the junctions.

Different deformation limits of the displacement are imposed, including the crack deformation limit and the leak deformation limit. For the case of cement material, the limits are respectively chosen with 0.42mm and 3.0mm. According to the seismic reliability problem for pipeline junctions, in the case of the symmetric double boundary, the seismic reliability of the pipeline structure reads,

$$R(t) = P\{|X_j(\tau)| < x_B, \tau \in [0, T]\} \quad (17)$$

Under the proposed stochastic conditions, the probability density function (PDF) of the response for joint E is presented in Fig.2, including the mean and the standard deviation, typical instantaneous PDFs at certain instants of time, evolution of PDF against time and contour to the PDF surface. These indicate the stochastic fluctuation character of a nonlinear random response.

For different boundary values, corresponding reliabilities for junction E are resolved and shown in Table.1. From the Table.1, if the boundary value is the leak deformation limit, the junction reliability is 1 and if the boundary value is the crack deformation limit, the junction reliability decreases sharply. Simultaneously, based on the evolution of PDF, the evolution process of the seismic reliability against time can be obtained. The seismic reliability for junction E by discretionary time is shown in Fig.3

## 4 CONCLUSIONS

A new approach is proposed for the seismic reliability assessment of finite element modelled underground pipeline structures under earthquake excitations and uncertain soil parameters. The approach is established based on the thoughts of the newly developed probability density evolution method. Associated with the physics-based stochastic earthquake model and the finite element method, the instantaneous probability density and the evolution process for the seismic response of underground pipeline can be studied effectively. The seismic reliability is investigated according to the failure criterion of the first passage problem. A computational example is investigated with stochastic input and random soil parameters. Some features of the responses and reliabilities are observed and discussed. It is found that the proposed method is of high application in analyzing the reliability of underground pipeline and the seismic reliability for different structural forms requires further investigation.

## ACKNOWLEDGEMENTS

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