# Probabilistic Assessment of the Uncertainty Associated with the Pullout Capacity of Marquee Ground Anchors

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**Summary:** Marquees are light structures that are connected to the ground by tensile anchors to resist forces imposed by wind pressure acting on the structure. The current methods for predicting the ultimate pullout capacity of marquee ground anchors, based on the axial capacity of a single pile, are inaccurate and uncertain. This paper aims to quantify the uncertainty associated with the methods commonly used for pullout capacity prediction of marquee ground anchors. The study is concerned with a variety of ground anchors currently used in the Australian marquee industry. A series of in situ anchor pullout tests were conducted at six different locations in the Adelaide Region, South Australia, and the actual pullout capacity in each test is determined and compared statistically with the corresponding predicted pullout capacity of four methods currently used in practice. A probabilistic approach is used to determine the probability that the predicted pullout load exceeds a certain value utilising any of the methods considered. This approach is useful in that it enables the designer to make informed decisions regarding the level of risk associated with predicted pullout loads and, consequently, increase the safety of marquee structures.

## INTRODUCTION

Marquees and other temporary light structures are almost exclusively connected to the ground by means of small anchors, often installed vertically, that resist uplift imposed by wind and other forces acting on the structure. The anchors transmit the tensile forces from the structure to the surrounding soil, which resists the tensile force by shear strength, hence providing structural stability. Traditionally, these anchors consist of steel rods, less than one metre in length, that are driven into the ground, usually by means of a sledge hammer. The existing methods for predicting the pullout capacity of small ground anchors are inaccurate and uncertain. A comparative study carried out by Shahin and Jaksa (2003), for methods currently used in practice, indicates that there is inconsistency in the magnitude of the pullout capacity predicted by these methods. Consequently, failures of marquees and other light structures are not rare. As an example of such a failure, in Kapunda, South Australia, an inflatable children's amusement failed, resulting in the death of a young girl (The Advertiser 2001). The inflatable amusement was restrained by ground anchors, and, as a result of a significant wind event, the anchors failed causing the structure to be lifted 10 metres into the air, carrying the child with it (DAIS 2001). In addition, Australian industry sources indicate that when marquees fail, they often need to be repaired or replaced, incurring costs of, sometimes, tens of thousands of dollars (Griggs 2002).

The mechanics of ground anchor behaviour is not well understood (Su and Fragaszy 1988) and available methods for predicting the pullout capacity of ground anchors usually simplify the problem with many assumptions, resulting in inaccurate pullout capacity prediction. A recent study carried out by Shahin and Jaksa (2003) on four methods currently used for pullout capacity prediction indicates that the ratio of predicted to measured pullout capacity can vary from 10% to 510%. The safety of marquee structures can be increased if the uncertainty (i.e. level of risk) associated with the different methods of pullout capacity prediction can be quantified. This can be carried out if sufficient measured and predicted data are available. In this paper, a series of in situ anchor pullout tests that have been conducted at six different locations within Adelaide, South Australia, are used for this purpose. The objective of this paper is to carry out a probabilistic analysis on four methods of pullout capacity prediction, from which the uncertainty associated with particular pullout capacity prediction method can be quantified. In order to demonstrate the proposed probabilistic analysis, a numerical example is provided.

## TRADITIONAL METHODS FOR PULLOUT CAPACITY PREDICTION

In this study, four methods of pullout capacity prediction are chosen for the probabilistic assessment. The methods include: Meyerhof (1973); LCPC (Bustamante and Gianeselli 1982); Das (1995); and Bowles (1997). These methods are selected as they are currently used in practice. Details of the parameters needed to calculate the pullout capacity using each method are given by Shahin and Jaksa (2003).

## PROBABILISTIC ANALYSIS OF PULLOUT CAPACITY PREDICTION

In order to conduct a probabilistic assessment of pullout capacity prediction, Monte Carlo simulation is applied to the deterministic pullout capacity predictions from the methods given above. Monte Carlo simulation attempts to generate a random set of values from known or assumed probability distributions of some variables involved in a certain problem. Full details of the Monte Carlo technique are given by many authors (e.g. Hammersley and Handscomb 1964; Rubinstein 1981). The uncertainty associated with any of the selected pullout capacity prediction methods can be examined by calculating the ratio of the predicted to the measured pullout capacity, $k$. If a set of predicted and measured pullout capacities is available, the pullout capacity ratios can be calculated and used to obtain the probability density function (PDF) of $k$. In this study, a series of in situ pullout capacity tests, that were given by Shahin and Jaksa (2003), are used for this purpose. These tests were conducted at six different locations within Adelaide, South Australia, and the sites were selected so as to cover a variety of soil types and geotechnical conditions. The tests focussed on axial loading of normal rough anchors installed vertically, as these are most commonly used in practice. Three anchor types of different shapes (i.e. circular, hexagonal and star dropper) and different embedment depths (i.e. 400, 600 and 800 mm) were used. Full details of the tests conducted are given by Shahin and Jaksa (2003). For each of the pullout capacity prediction methods considered, predicted and corresponding measured pullout capacities are obtained and used to determine the PDF of $k$. The PC-based software *@Risk* (Palisade 2000) is used to determine the PDF of $k$ that provides the best fit to the available data points. For a given set of data values, *@Risk* can identify the probability distribution that best fits these values from 38 candidate distributions and provides statistical properties that describe the distribution. The theoretical distributions that are found to best match the actual distribution of $k$ for each of the pullout capacity prediction methods are shown in Figure 1. The statistical parameters of these distributions are given in Table 1. A Monte Carlo simulation is then applied to the methods chosen and the uncertainty associated with the predicted pullout capacities of each method is estimated. The detailed procedure of the probabilistic approach is described as follows:

1. For a certain pullout capacity prediction method, the PDF of $k$ and its statistical parameters is estimated, as shown previously (Figure 1 and Table 1);
2. For an individual case of pullout capacity prediction, the deterministic pullout capacity for the method under consideration is calculated, as described by Shahin and Jaksa (2003);
3. A random value of $k$ is generated from the PDF of $k$ obtained in Step 1;
4. From the definition of $k$, the deterministic predicted pullout capacity in Step 2 is divided by the generated random value of $k$ from Step 3 and the corresponding actual pullout capacity is calculated;
5. Steps 3 to 4 are repeated for many iterations (Monte Carlo simulation); and
6. The pullout capacities obtained as part of the Monte Carlo simulation are used to estimate the cumulative distribution function (CDF) or to plot the cumulative probability distribution from which the probability of non-exceedance, or level of risk, associated with a certain pullout capacity prediction, can be estimated.

In order to illustrate the above approach, a numerical example is provided below.

## NUMERICAL EXAMPLE

Consider an anchor having a circular cross section with a diameter of 33 mm and embedment length of 600 mm. The deterministic pullout capacities predicted using the four methods considered are calculated as described by Shahin and Jaksa (2003) and are found to be equal to 0.96, 1.39, 1.22 and 0.87 kN for Meyerhof (1973), LCPC (1982), Das (1995) and Bowles (1997), respectively. What is the uncertainty (i.e. level of risk) associated with the deterministic pullout capacity predicted by each of these methods and what would be the pullout loads if 90% probability of non-exceedance is required for design purposes.

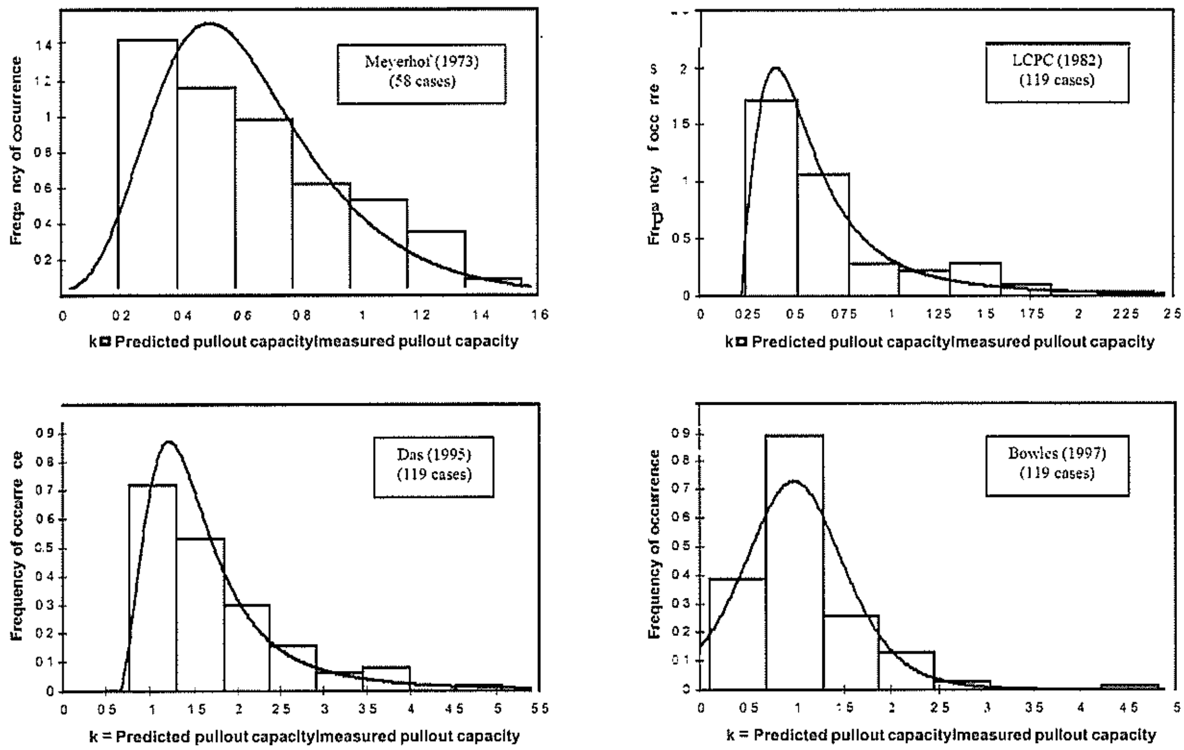


Figure 1. Distributions of *k* for the pullout capacity prediction methods

| Method | Distribution | Mean | σ | a | b | α | β | γ |
|---|---|---|---|---|---|---|---|---|
| Meyerhof (1973) | Gumbel | 0.63 | 0.31 | 0.19 | 1.54 | N/A | 0.24 | 0.49 |
| LCPC (1982) | Loglogistic | 0.74 | 0.61 | 0.24 | 2.40 | 1.96 | 0.32 | 0.22 |
| Das (1995) | Loglogistic | 1.78 | 1.63 | 0.76 | 5.10 | 2.35 | 0.81 | 0.66 |
| Bowles (1997) | Logistic | 0.98 | 0.62 | 0.10 | 4.80 | N/A | 0.98 | 0.34 |

a = lowest possible value
b = highest possible value
α = shape parameter
β = scale parameter
γ = location parameter
σ = standard deviation

For each of the pullout capacity prediction methods, the PDF of *k* is obtained, as shown previously (Step 1). The deterministic solution of pullout capacity prediction is calculated, which is given in the numerical example (Step 2). A random value of *k* is generated from the PDF of *k* using Figure 1 and Table 1 (Step 3). The numerical example is re-calculated by dividing the pullout capacity predicted in Step 2 by the generated value of *k* obtained from Step 3 and a corresponding actual pullout capacity is obtained (Step 4). The above procedure (Steps 3 and 4) is repeated 1500 times (Monte Carlo simulation) until a convergence criterion equal to 1% is achieved (Step 5). In order to determine whether convergence has been achieved, the statistics describing the distribution of the predicted settlements are calculated at fixed numbers of simulations and compared with the same statistics at previous simulations. Convergence is deemed to have occurred if the change in the statistics describing the distribution of predicted pullout capacity is 1% or less. This is carried out automatically with the aid of *@Risk*. The predicted pullout capacities obtained for the 1500 simulations are used to plot the cumulative probability distribution curves from which different probabilities of non-exceedance are obtained (Step 6). The results are shown in Figure 2 and summarised in Table 2.

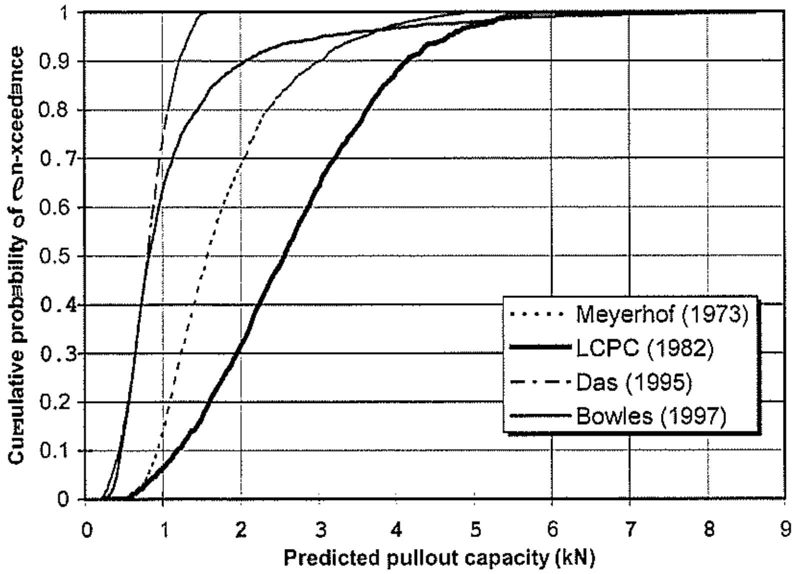


Figure 2. Cumulative probability distributions incorporating prediction uncertainty for the numerical example

Table 2. Results of the stochastic assessment incorporating prediction uncertainty for the numerical example

| Method | Deterministic pullout load (kN) | Probability of non-exceedance at the deterministic value (%) | Predicted pullout load at 90% probability of non-exceedance (kN) |
|---|---|---|---|
| Meyerhof (1973) | 0.96 | 9.3 | 3.1 |
| LCPC (1982) | 1.39 | 13.4 | 4.2 |
| Das (1995) | 1.22 | 87.9 | 1.3 |
| Bowles (1997) | 0.87 | 50.0 | 2.1 |

The results in Table 2 demonstrate that there are probabilities of approximately 9.3, 13.4, 87.9 and 50% for Meyerhof (1973), LCPC (1982), Das (1995) and Bowles (1997), respectively, that the pullout loads will not exceed the deterministic estimated values by each method. In other words, this means that there are probabilities of 90.7, 86.6, 12.1 and 50% for Meyerhof (1973), LCPC (1982), Das (1995) and Bowles (1997), respectively, that the pullout loads could be higher than the deterministic estimated values. This result indicates that the uncertainty associated with the method used to predict pullout capacity can significantly affect pullout loads and thus, should be considered in the analysis and simulation of pullout capacity prediction. The results in Table 2 also show that at 90% probability of non-exceedance (i.e. 10% level of risk), the pullout loads would be equal to 3.0, 4.2, 1.3 and 2.1 kN for Myerhof (1973), LCPC (1982), Das (1995) and Bowles (1997), respectively. This indicates that the uncertainty associated with the method of Das (1995) is minimal, as it gives predictions closer to the actual value. This can be clearly seen in Figure 2, as the method of Das (1995) has a narrow range of predicted pullout capacity over the full range of the cumulative probability of non-exceedance. On the other hand, higher degrees of uncertainties are obtained for the other methods. At 90% probability of non-exceedance, the method of Bowles (1997) gives pullout prediction equal to approximately 2.5 times the actual value, whereas the methods of Meyerhof (1973) and LCPC (1982) give pullout predictions equal to approximately three times the actual values. This can also be seen in Figure 2 as the three methods have a wide range of predicted pullout capacities for the full range of cumulative probability of non-exceedance. The comparative analysis of this example suggests that the method of Das (1995) is the most reliable of the four methods considered. It should be noted that the results obtained are constrained by the data used to determine the distributions of $k$ in Step 1 of the proposed probabilistic approach. If more cases of predicted and measured pullout loads are available, the distributions of $k$ may change and the results could be modified.

## SUMMARY AND CONCLUSIONS

A stochastic approach that applies the Monte Carlo technique has been used to assess probabilistically the pullout capacity of marquee ground anchors of four pullout capacity prediction methods that are currently used in practice. The methods assessed were Meyerhof (1973), LCPC (1982), Das (1995) and Bowles (1997). The proposed approach quantifies the uncertainty associated with each method and incorporates this uncertainty in the analysis of the pullout capacity prediction. In order to illustrate the proposed stochastic approach, a numerical example was given. The results of the numerical example indicated that there were probabilities of approximately 90.7, 86.6, 12.1 and 50% for Meyerhof (1973), LCPC (1982), Das (1995) and Bowles (1997), respectively, that the pullout loads could be higher than the deterministic estimation calculated using each method. The results of the numerical example also indicated that, for 90% probability of non-exceedance (i.e. 10% level of risk), the pullout load predicted using the method of Das (1995) is very close to the actual value, which indicates that the uncertainty associated with the prediction of this method is minimal. On the other hand, it was found that the pullout loads predicted using the other methods ranged from 2.5 to 3.0 times the deterministic values. The above results indicates that uncertainty associated with the method used to predict pullout capacity of marquee ground anchors has a significant impact on its prediction and thus should not be neglected. The results also indicate that the method of Das (1995) is the most reliable of the four methods considered in this study.

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